LECTURE NOTES

ON

NETWORK THEORY

3rd SEMESTER

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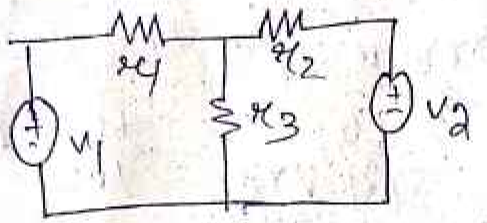
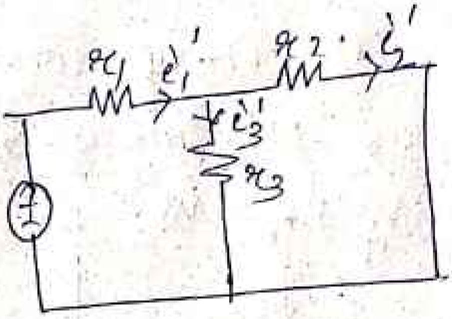
Superposition Theorem

In a linear bilateral network containing more than one source of energy the overall effect of all source considered simultaneously is same as the algebraic sum of individual effect of each source considered one at a time and being independent of all other sources

Explanation

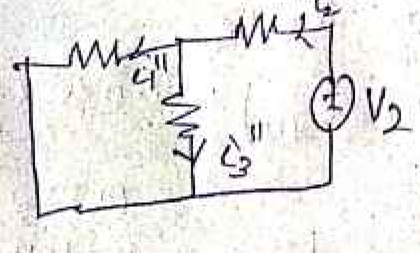




$$i_1' = \frac{V_1}{\frac{r_2 r_3}{r_2 + r_3} + r_1} \qquad \text{Again } i_2' = i_1' \frac{r_3}{r_2 + r_3}$$

$$i_3' = i_1' - i_2'$$

Next by removing V_1 by short circuit, let the circuit be energized by V_2 only



$$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1 + r_3} + r_2}$$

$$i_1'' = i_2'' \frac{r_3}{r_1 + r_3}$$

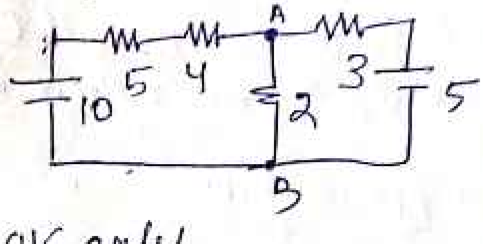
$$i_2'' = i_2'' - i_1''$$

As per superposition $\boxed{i_3 = i_3' + i_3''}$

* Direction of current calculated of each source should be taken care of.

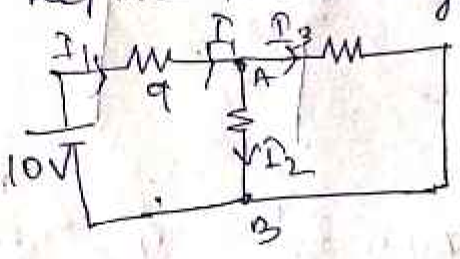
Ex Find the current flowing in the 2Ω resistor of the circuit shown in fig by applying Superposition theorem

Sol: By using source conversion



S-1 Consider voltage source 10V only

Replace the voltage (5 V) source by a short circuit

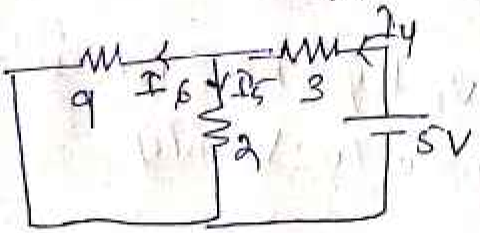


$$I_1 = \frac{10}{9+\frac{2\times3}{2+3}} = \frac{10}{10.2} = 0.98A$$

By current division rule.

$$I_2 = I_1\left(\frac{3}{3+2}\right) = 0.98\times\frac{3}{5}$$
$$= 0.58\ A$$

S-2: Consider voltage source 5V only.



$$I_4 = \frac{5}{3+\left(\frac{2\times9}{2+9}\right)} = \frac{5}{4.636}$$
$$= 1.078A$$

By current division. Rule

$$I_5 = I_4\left(\frac{9}{2+9}\right) = 1.078\times\frac{9}{11}$$
$$= 0.882\ A$$

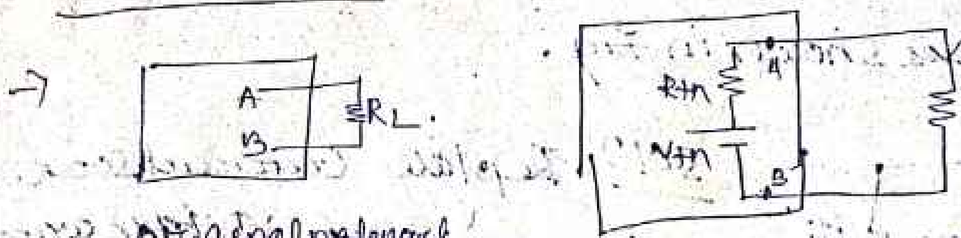
(from A to B)

S-3 According to superposition theorem

Total current in 2Ω Resistor due to the presence of both source become $(I_2+I_5) = 1.47A$

(from A to B)

Thevinin's Theorem



original network → (Thevinin equivalent circuit)

$$I_L = \frac{V_{th}}{R_{th}+R_L}, \quad V_{AB} = I_L R_L$$

statement

In order to find the responce through any particular element connected across a pair of terminal A&B of a linear active N/W, then the rest of the N/W may be replaced by a Thevenin voltage (V_{th}) & a series resistance called thevenin's resistance R_{th}.

Procedure

S-1 Replace current source if any with equivalent voltage source.

S-2. Identify a pair of terminal A&B across the desired element and mark it as R_L

S-3. Find voltage across A&B in absence of R_L and mark as V_{th}.

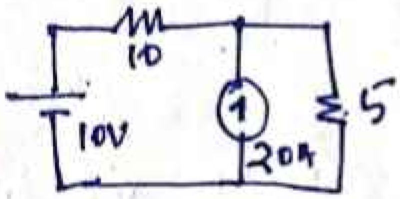
S-4. Replace all the voltage source with short circuit. calculate the equivalent resistance by looking into the network from the open terminal A&B and mark this as R_{th}

S-5. Draw Thevenin equivalent circuit.

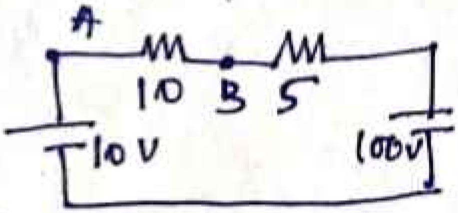
S-6 Solve for current and voltage as per equation

$$I_L = \frac{V_{th}}{R_{th}+R_L} \qquad V_{AB} = I_L R_L$$

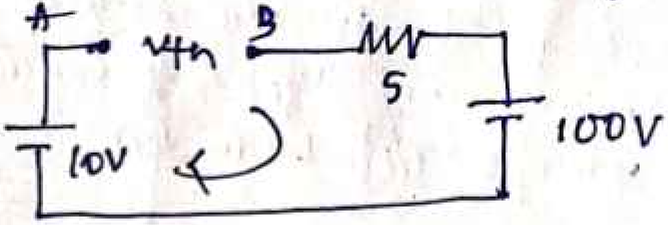
Ex: Find the current flowing in the 10Ω resistor of the ckt as shown in fig.



Sol^n Replace current source with equivalent voltage source

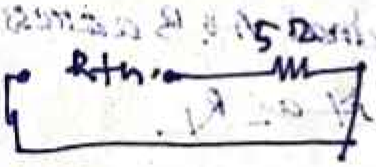


Then 10Ω is removed from the circuit and its open terminal are marked as A & B



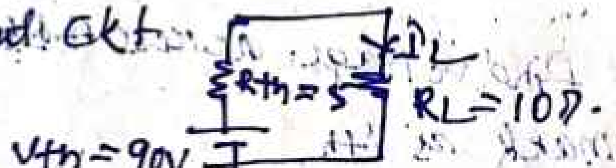
~~Applying~~ KVL $V_{th} = V_{AB} = -10 + 100 = 90V$

Calculation of R_{th}



$R_{th} = 5Ω$

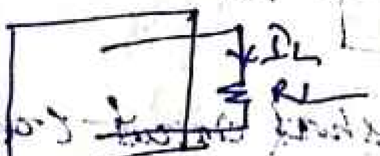
Thevenin equivalent ckt

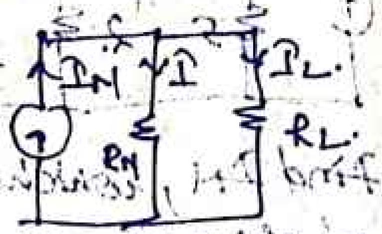


$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{90}{10 + 5} = \frac{90}{15} = 6\ Amp$$

Norton's theorem

Statement → In order to find the responce through any particular element connected accross a pair of terminals A & B of a linear active D.C. Network the rest of the network may be replaced by a Norton equivalent circuit containing a current source called Norton current I_N & a resistor called Norton Resistance R_N





$$I_L = I_N\left(\frac{R_N}{R_N+R_L}\right) \quad V_{AB} = I_L R_L$$

Procedure

→ Replace voltage source if any equivalent current source.

→ Identify a pair of terminal A & B across the desired element and mark it as R_{AB}

→ Find the current that would flow through a shorted link replacing AB and mark it as I_N

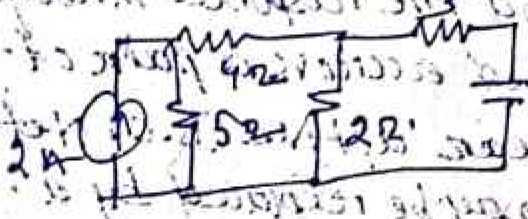
→ Replace all current source with open circuit & calculate the equivalent resistance by looking the network from the open terminal A & B and more the R_N

→ Draw equivalent resistance.

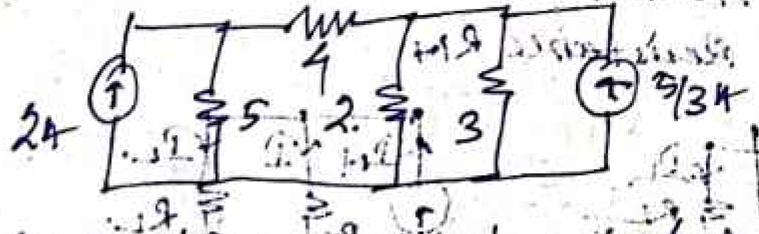
→ Slove the current and voltage as per eq?

$$I_L = I_N\left(\frac{R_N}{R_N+R_L}\right) \quad V_{AB} = I_L R_L$$

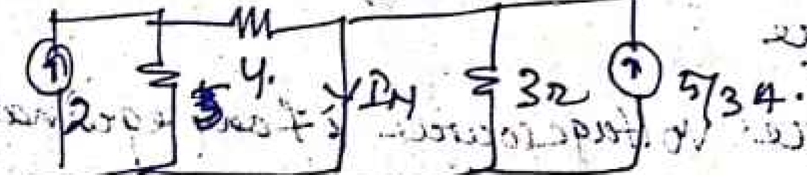
Q: Determine the current through 3Ω resistor by apply Norton theorem



Convert voltage source into current source.

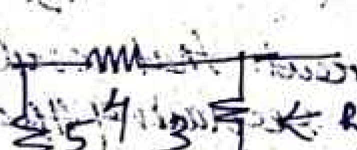


→ to find I_N, which is a short circuit current that would flow through A&B due to the rest of the network with R_L replaced by short.



$$I_N = 2\left(\frac{5}{5+4}\right) + 5/3 = 2.77 \text{ Amp}$$

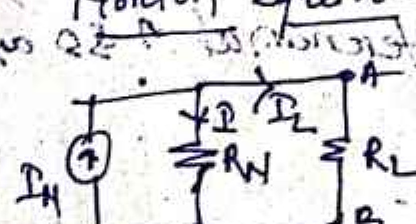
Calculation of R_N



$R_N = 9 \| 3$

$$R_N = \frac{9\times3}{9+3} = 3.25$$

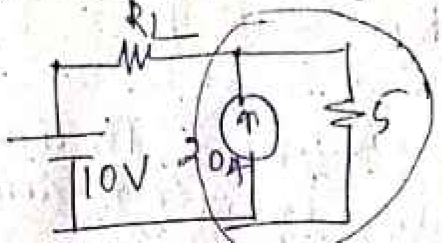
Norton equivalent circuit



$$I_L = I_N\left(\frac{R_N}{R_N+R_L}\right) = \cancel{1.47 \text{ Amp}}$$

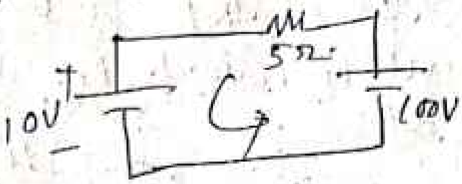
Hence ~~1.47~~ Amp pass through I_L by Norton theorem

Ex-21 Find the current flowing in the 10Ω resistor of the circuit by Norton's theorem

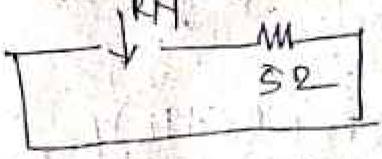


Soln

→ convert current source to voltage source

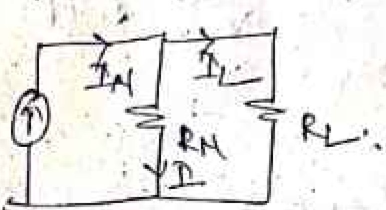


$$I_N = \frac{100+10}{5} = 18A$$



$R_N = 5Ω$

Norton equivalent

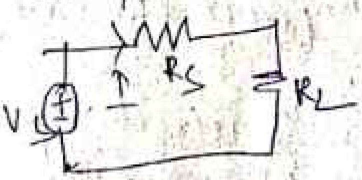


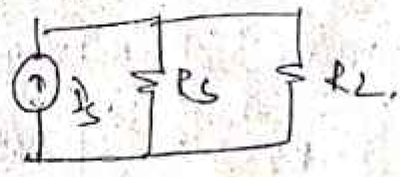
$$I_L = I_N\left(\frac{R_N}{R_N+R_L}\right)$$

$$= 18\left(\frac{5}{5+10}\right) = 6\,Amp$$

Maximum Power transfer theorem

→ In a linear bilateral network containing an independent voltage source in series with resistance (R_S) delivers maxm power to the load resistance (R_L) when R_L is equal to R_S. $(R_S = R_L)$





→ Similarly the independent current source parallel with source resistance R_S delivers maxm power to the load resistance $R_L = R_S$

→ Let's consider the above circuit

$$I = \frac{V_S}{R_S+R_L} \quad \text{—— (1)}$$

Power delivered to the $P_L = I^2 R_L$ — (2)

$$= \left(\frac{V_s}{R_s + R_L}\right)^2 R_L \quad -(3)$$

to find the value of R_L for max^m power transfer differentiate P_L w.r.t R_L and equate to zero

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left\{\left[\frac{V_s}{R_s + R_L}\right]^2 \cdot R_L\right\} \quad -(4)$$

$$= V_s^2 \left[\frac{(R_s + R_L)^2 - 2R_L(R_s + R_L)}{(R_s + R_L)^4}\right] = 0 \quad -(5)$$

$$\Rightarrow (R_s + R_L)^2 = 2R_L(R_s + R_L) \quad -(6)$$

$$\Rightarrow R_s + R_L = 2R_L \quad -(7)$$

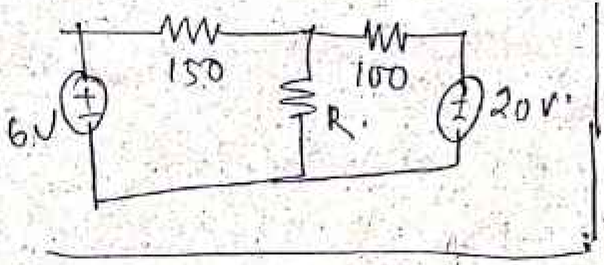
$$\Rightarrow \boxed{R_s = R_L} \quad -8$$

we can find that max^m power

$$P_{max} = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \cdot R_{th}$$

$$= \frac{V_{th}^2}{4R_{th}}$$

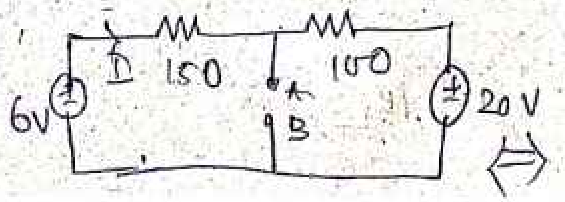
Q: Find the value of 'R' for max^m power transfer



Sol^n open circuit the resistance R and find V_{th} & R_{th}.

Apply KVL to the loop

$$6 - 150I - 100I - 20 = 0$$



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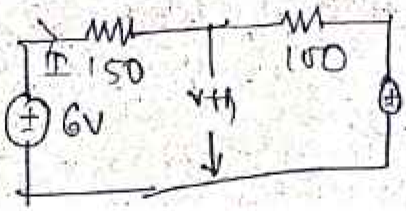
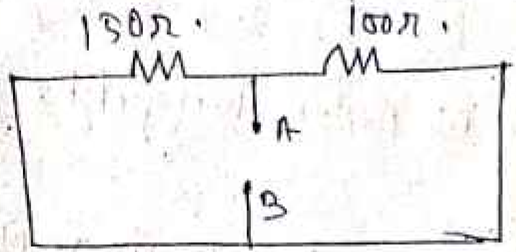


$-250I = 14$

$I = \frac{-14}{250} = \frac{-7}{125}$ Amp

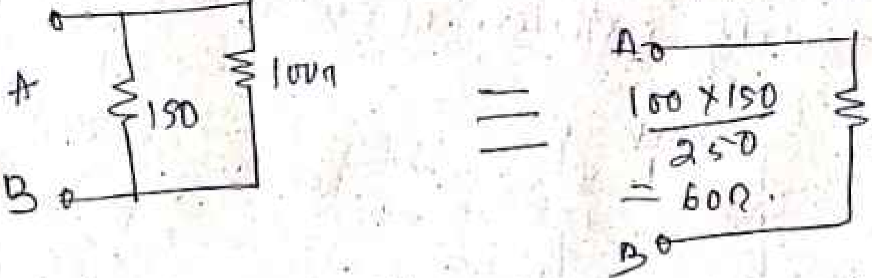
$V_{th} = V_{AB} = 100I + 20 = 100\left(\frac{-7}{125}\right) + 20$

$= 14.4\,V.$

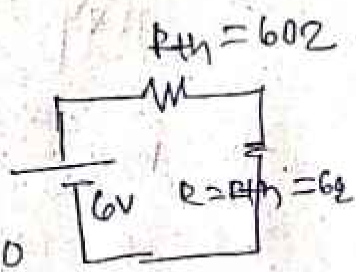


to find R_{th} short circuit the voltage source and simplify to get

$R_{th} = 60\Omega.$



For maximum power transfer



$R = R_{th} = 60\Omega$

$P_{max} = I^2 R = \left(\frac{14.4}{60+60}\right)^2 \times 60$

* This theorem is not applicable if there are 2 or more source in a network in series or parallel.

Reciprocity theorem

statement → If a source of emf located at one point in a network composed of linear bilateral network circuit elements produced a current I at a selected point in the network. the same source of emf acting at the 2nd point will produce. the same current at the first point.

Procedure.

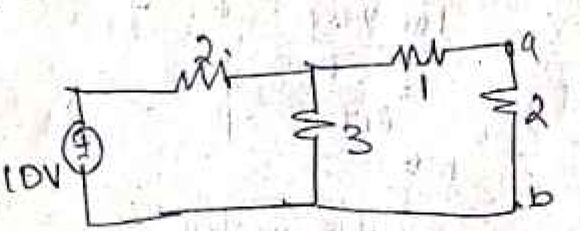
S-1 – The branch between which reciprocity theorem is to be applied is selected first.

S-2. → The current in the branch is obtained using ~~reciprocity~~ conventional network analysis ~~theorem is to be applicable~~ ~~calculated~~.

S-3. – The voltage source is interchanged beth the branch concerned.

S-4. The current in the branch where voltage source exsiting earlier is calculated

Prob.

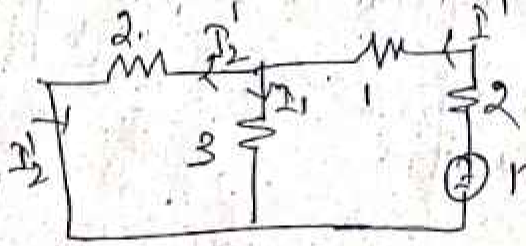


Ans $R_{eq} = \{(2+1) \parallel 3\} + 2 = \frac{9}{6} + 2 = \frac{21}{6} = 3.5$

$I = \frac{V}{R_{eq}} = \frac{10}{3.5} = \frac{20}{7}$ Amp

$I_2 = I \times \frac{3}{3+1+2} = \frac{20}{7} \times \frac{3}{6} = \frac{10}{7}$ Amp

Fig. 2.



$R_{eq} = (2 \parallel 3) + 1 + 2 = 6/5 + 3 = 21/5$

$I' = \frac{V}{R_{eq}} = \frac{10}{21/5} = \frac{50}{21}$

$I_2'' = \frac{50}{21} \times \frac{3}{3+2} = \frac{10}{7}$ Amp

Application of Reciprocity theorem

→ This theorem is applicable to linear, time invariant n/w consisting of passive n/w element

→ This theorem is applicable in dc as well as ac circuit.

→ This theorem allows interchange the position of excitation and response.

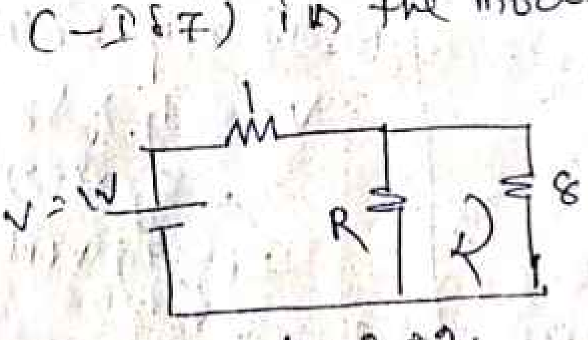
→ It provide bilateral property of the network

→ It provide great convenience in design and measurement.

Compensation Theorem

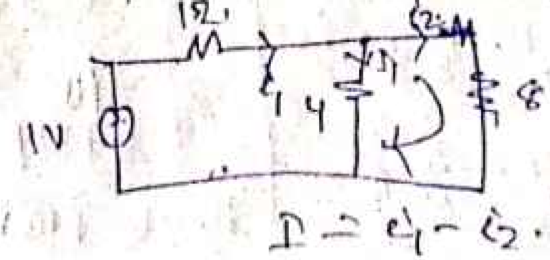
In any linear bilateral active network. If any branch carrying a current 'I' has its impedance 'Z' changed by an amount δZ the resulting changes that occurs in the other branch are the same as those which would have been caused by the injection of a voltage source of (−I δZ) in the modified branch

Q



4 − 2 − 7.

Soln step-1



$I = i_1 - i_2$.

Apply KVL in Loop-1

$-i_1 - 4(i_1 - i_2) + 1 = 0$

$\Rightarrow -i_1 - 4i_1 + 4i_2 = -1$

$\Rightarrow -5i_1 + 4i_2 = -1$

$\Rightarrow 5i_1 - 4i_2 = 1$ — (1)

Apply KVL in Loop-2

$-8i_2 + 4(i_1 - i_2) = 0$

$\Rightarrow -8i_2 + 4i_1 - 4i_2 = 0$

$\Rightarrow 4i_1 - 12i_2 = 0$

$\Rightarrow -4i_1 + 12i_2 = 0$ — (2)

Solving eqn (1) & (2)

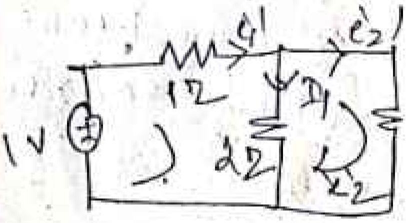
$5i_1 - 4i_2 = 1$ — (1)

$-4i_1 + 12i_2 = 0$ — (2)

$i_1 = \frac{3}{11} A \quad i_2 = \frac{1}{11} A$

$I_1 = i_1 - i_2 = \frac{3}{11} - \frac{1}{11} = \frac{2}{11}$ Amp

Step-2, After changing the value of the resistance from 4Ω to 2Ω



$I_1' = i_1' - i_2'$

$3i_1' - 2i_2' = 1$ — (3)

$-2i_1' + 10i_2' = 0$ — (4)

By solving eqn (3) & (4)

$i_1' = \frac{5}{13} A \quad i_2' = \frac{1}{13} A$

$I_1' = i_1' - i_2' = \frac{4}{13} A$

Change in current

$\delta I = I_1' - I_1$

$= \frac{4}{13} - \frac{2}{11} = \frac{18}{143}$ Amp

Using compensation theorem

$V_C = I_1 \times \delta Z$

$= \frac{2}{11} \times (-2)$

$= \left(-\frac{4}{11}\right)$ volt

$\delta I = \frac{-V_C}{2 + \frac{8}{9}}$ ⇒ Equivalent resistance

$= \frac{4/11}{2 + 8/9} = \frac{18}{143}$ Amp

Hence proved.

Table of dual element

Electrical Resistance → conductance.

Inductor → capacitor.

Capacitor → Inductor.

Series Branch → Parallel Branch.

Switch closed → switch open

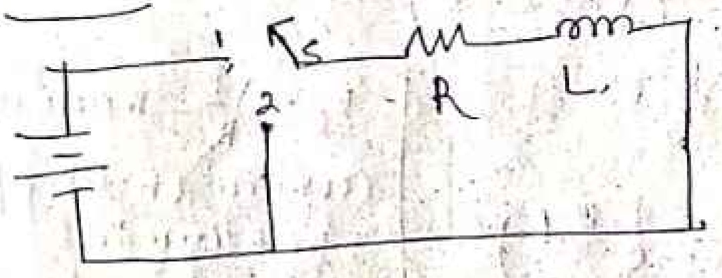
charge → flux linkage.

Mesh → Node.

Module-2

1st order circuit → It contains resistance and one energy storing element i.e one inductor or capacitor. This 1st order circuit, during its transient state of operation is governed by 1st order linear differential eqn.

R-L circuit



Consider some instant t seconds after the voltage is applied.

i = current flowing through the circuit at instant 't' second

$\frac{di}{dt}$ = rate of growth of current at the instant

$V_R = iR$ = voltage across R

$V_L = L\frac{di}{dt}$ = voltage across L

Applying KVL $\quad V = Ri + L\frac{di}{dt}$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad (1)$$

$\Rightarrow$ eq^n(1) is a non homogenous eq^n

so $$i = e^{-\frac{R}{L}t}\int e^{(R/L)t}\cdot\frac{V}{L}\,dt + Ke^{-\frac{R}{L}t} \quad -(2)$$

$$\Rightarrow i = i_p + i_c$$

Sol of eq^n(2) is $i = \frac{V}{R} + Ke^{-\frac{R}{L}t}$

At $t = 0^+$ (just after switching) eq^n(3) becomes

$$0 = \frac{V}{R} + Ke^{-\frac{R}{L}\times 0} = \frac{V}{R} + K$$

$$K = -\frac{V}{R}$$

Putting in eq^n(3) we get

$$i = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$i = I_0\left(1 - e^{-t/\tau}\right)$$

$I_0 = \frac{V}{R}$ = maximum current or steady state current

$\tau = \frac{L}{R}$ = time const

we put $t = \tau = \frac{L}{R}$ in eqn (4) we get

$$i = I_0(1-e^{-1}) = 0.632 I_0 = 63.2\% \text{ of } I_0.$$

we put $t = \infty$ in eqn (4) we get

$$i = I_0(1-e^{-\infty/\tau}) = I_0$$

Thus current in R-L circuit would attain max value (I_0) only after infinite time.

voltage drop across inductor $V_L = L\frac{di}{dt}$

$$\Rightarrow V_L = L \cdot \frac{d}{dt}\left[I_0(1-e^{-t/\tau})\right] = L \cdot \frac{d}{dt}\left[I_0 - I_0 e^{-t/\tau}\right]$$

$$\Rightarrow V_L = L\left[\frac{d}{dt}I_0 - \frac{d}{dt}I_0 e^{-t/\tau}\right] = L\left[0 + \frac{1}{\tau}I_0 e^{-t/\tau}\right]$$

$$\Rightarrow V_L = L\left[\frac{1}{4R}I_0 e^{-t/\tau}\right] = I_0 R e^{-t/\tau}$$

$$\boxed{V_L = V e^{-t/\tau}} \quad \boxed{\text{where } V = I_0 R}$$

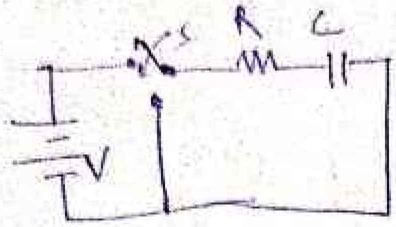
voltage drop across resistor is

$$V_R = iR = I_0(1-e^{-t/\tau})R = V(1-e^{-t/\tau})$$

→ In transient period voltage across resistor exponent rising and voltage across inductor exponentially decaying. One the transient dies out within a short time the steady current ($I_0 = V/R$) remain in the circuit.

R.C

Case-1 (charging of R-C circuit)



Consider a R-C circuit connected in series with a battery of voltage V and a switch 'S'

→ Initially capacitor is uncharged and voltage across it is zero ($V_C = 0$)

⇒ $V_R = V$ → whole supply voltage.

Initial current in the circuit $\boxed{I_0 = \frac{V}{R}}$

→ As current flows in the capacitor starts charging and capacitor voltage V_C increase.

→ Consider some instant 't' second after the voltage is applied

i → current flowing through the circuit at instant 't' second.

$V_R = Ri$ = voltage drop across R

$V_C = \frac{1}{C}\int I.dt$ = voltage across 'C'

Applying KVL $\boxed{Ri + \frac{1}{C}\int i.dt = V}$

Differentiating both sides w.r.t time 't' we get

$$R.\frac{di}{dt} + \frac{i}{C} = 0 \Rightarrow \boxed{\frac{di}{dt} + \frac{1}{RC}i = 0} \quad \text{—(1)}$$

Eq(1) is a homogeneous eq^n sol^n of this eq^n(1)

$$i = ke^{-t/RC} \quad \text{--- (2)}$$

where k is a constant, whose value can be calculated from the initial condition.

$$t = 0^+$$

current $i(0^+) = \frac{V}{R}$

eq^n(2) becomes $\boxed{\frac{V}{R} = ke^{-0/RC}}$

$$\Rightarrow \boxed{K = \frac{V}{R}}$$

putting this value of 'k' in eq^n (2)

$$i = \frac{V}{R} e^{-t/RC}$$

$$\Rightarrow \boxed{i = I_0 e^{-t/\tau}} \quad \text{--- (3)}$$

$I_0 = \frac{V}{R}$ i.e. max^m current in the circuit

$\tau = RC$ (time const. or capacitive time constant)

Voltage across the resistor:

$$V_R = iR = I_0 R e^{-t/\tau} = Ve^{-t/\tau} \quad \text{--- (4)}$$

Voltage across the capacitor:

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_0^t I_0 e^{-t/\tau}$$

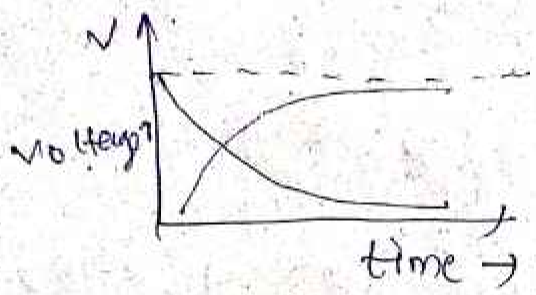
$$\Rightarrow \boxed{V_C = V(1 - e^{-t/\tau})} \quad \text{--- (5)}$$

→ The charge stored in capacitor during charging

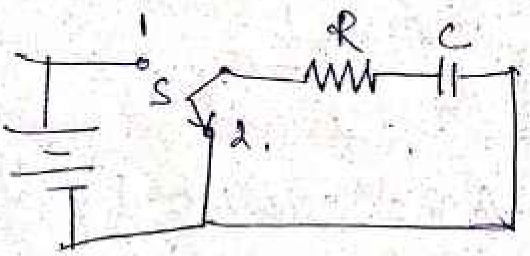
$$q = CV_C = CV(1-e^{-t/\tau})$$
$$= Q(1-e^{-t/\tau})$$

If we put $t = \tau = RC$ we get

$$V_C = V(1-e^{-1}) = 0.632V = 63.2\% \text{ of } V$$



Discharging of R.C circuit



i = discharging current at any instant

Applying KVL

$$Ri + \frac{1}{C}\int i \, dt = 0$$

Differentiating both sides w.r.t we

$$R\frac{di}{dt} + \frac{i}{C} = 0 \quad -(6)$$

Eqn (6) is a homogeneous differential eqn

$$i = Ke^{-t/RC} \quad \text{——(7)}$$

where K is a const., whose value can be calculated from intial condition

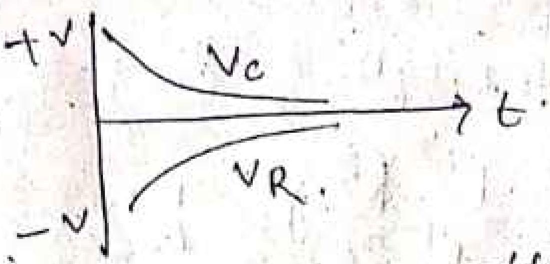
$t = 0$ $i = -V/R$ eqn (7) becomes

$$-\frac{V}{R} = Ke^{\frac{0}{RC}}$$

$$\Rightarrow \boxed{K = -\frac{V}{R}}$$ putting this value in eqn (7)

$$i = -\frac{V}{R} e^{-t/RC} = -I_0 e^{-t/\tau}$$

$$\boxed{i = -I_0 e^{-t/\tau}} \quad \text{—(8)}$$



$$V_R = Ri = -I_0 Re^{-t/RC} = -Ve^{-t/\tau}$$

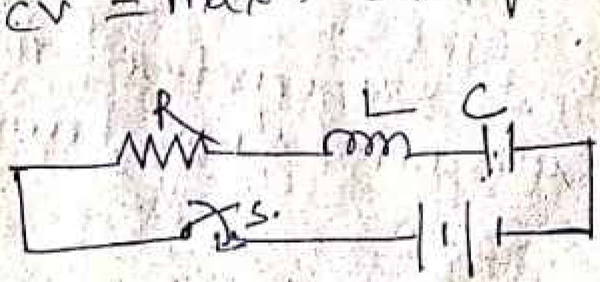
$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int -I_0 e^{-t/\tau}$$

$$\boxed{V_C = I_0 Re^{-t/\tau} = Ve^{-t/\tau}}$$

The charge in capacitor during discharging is $q = CV_C = CVe^{-t/\tau} = Qe^{-t/\tau}$

$Q = CV$ = max^m charge in capacitor.

R-L-C



Applying KVL

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$

$$\Rightarrow R\cdot\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{1}{LC} i = 0 \quad -(2)$$

eqn (2) is a second order linear homogeneous eqn

Its characteristic eqn is

$$\boxed{p^2 + \frac{R}{L} p + \frac{1}{LC} = 0} \quad -(2) \quad \left[\because p = \frac{d}{dt}\right]$$

The root of this eqn

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= \alpha \pm \beta$$

$$p_1 = \alpha + \beta \qquad p_2 = \alpha - \beta$$

$$\alpha = -\frac{R}{2L} \qquad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

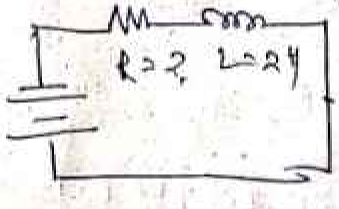
The soln of the differential eqn

$$\boxed{i = K_1 e^{p_1 t} + K_2 e^{p_2 t}} \quad -(3)$$

where K_1 & K_2 are constant whose values are calculated from boundary condn

Q. A constant voltage is applied to R.L series ckt at t=0 by closing a switch. The voltage drop across 'L' is 25V. at t=0. & drop to 5V at t = 0.025 sec, L=2H find (a) the applied voltage (b) the value of R

Ans



(a) At t=0, the entire voltage is dropped across inductor & no voltage is droped across the resistor. so applied voltage

V=25 Volt.

(b) At any instant voltage across inductor is 5V

$$V_L = Ve^{-t/\tau}$$

$$\Rightarrow 5 = 25e^{-\frac{0.025}{\tau}}$$

$$\Rightarrow \tau = 0.01553$$

$$\Rightarrow \frac{L}{R} = 0.01553$$

$$\Rightarrow R = \frac{L}{0.01553} = \frac{2}{0.01553} = 128.78\Omega$$

$$\Rightarrow \boxed{R = 128.8\Omega}$$

Q. The winding of an electromagnet has an inductance of 3H & resistance of 15Ω. when it is connected to 120V dc supply. calculate

(a) The steady state value of current flowing in the winding

(b) The time const. of the circuit

(c) The value of induced emf after 0.1 sec.

(d) The time for the current to rise to 25%.

of its final value.

(e) The value of current after 0.3 sec

Ans

(a) Given $R = 15\Omega$, $L = 3H$ $V = 120V$.

Steady state current $I_0 = \frac{V}{R} = \frac{120}{15} = 8A$

(b) $\tau = \frac{L}{R} = \frac{3}{15} = 0.2 \, sec$.

(c) The value of induced emf after 0.1 sec.

$$V_L = Ve^{-t/\tau} = 120 \left(e^{-\frac{0.1}{0.2}}\right) = 72.800$$

(d) current at any instant during rise is

$$i = I_0 (1 - e^{-t/\tau})$$

$$\Rightarrow \frac{85}{100} I_0 = I_0 (1 - e^{-t/0.2})$$

$$\Rightarrow \boxed{t = 0.379 \, sec}$$

(e) $$i = I_0 (1 - e^{-t/\tau}) = 8 (1 - e^{-\frac{0.3}{0.2}})$$

$$\Rightarrow \boxed{i = 6.21A}$$

Q An $8\mu F$ capacitor is connected in series with $0.5 M\Omega$ resistance across 200V supply calculate (a) initial charging current (b) the current and power developed across capacitor, 4 second after it connected to supply.

Ans
$$C = 8\mu F = 8 \times 10^{-6} F$$
$$R = 0.5 M\Omega = 0.5 \times 10^{6} \Omega$$
$$V = 200 \, volt.$$

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(a) initial charging current $= \frac{V}{R}$

$$= \frac{200}{0.5\times10^{6}} = 4\times10^{-4}A$$

(b) time constant $= \tau = RC = 0.5\times10^{6}\times8\times10^{-6} = 4sec$

* the current across capacitor is 4 sec, after it's connected to the supply is

$$\boxed{i = \frac{V}{R}e^{-t/\tau}} \quad i = 4\times10^{-4}e^{-\frac{4}{4}}$$

$$\boxed{i = 1.47\times10^{-4}A}$$

* The power developed across capacitor in connected to the supply is 4 sec after it is connected to the supply is

$$V_C = V(1-e^{-t/\tau})$$

$$\Rightarrow V_C = 200(1-e^{-4/4}) = 126.44\ volt.$$

Q: The time constant of a coil was found to be 2.5ms with a resistance of 100Ω added in series, a new time constant of 0.5ms was obtained. Find R & L of the coil.

Soln Given time constant $\tau = \frac{L}{R} = 2.5ms$
$= 2.5\times10^{-3}$

when resistance (R) added the time constant.

$$\tau' = \frac{L}{R+100} = 0.5$$

$$\frac{\tau}{\tau'} = \frac{2.5}{0.5}$$

$$\Rightarrow \frac{\frac{L}{R}}{\frac{L}{R+100}} = 5 \Rightarrow \frac{R+100}{R} = 5R$$

$$\Rightarrow \boxed{R = 25\Omega}$$

But given $\frac{L}{R} = 2.5 \times 10^{-3}$

$$\text{so } L = 2.5 \times 10^{-3} \times R = 2.5 \times 10^{-3} \times 25$$
$$= 62.5 \times 10^{-3} H$$

$$\boxed{L = 62.5 \times 10^{-3} H}$$

Sl.No	Name of the element	Symbol	At $t=0$	At $t=(\infty)$
1	Resistor	R	R	R
2.	Inductor	L	O.C	S.C
3.	Capacitor	C	S.C	O.C
4.	Inductance with initial current I_0	$I_0 \rightarrow$	I_0 O.C	I_0 S.C.
5.	Capacitance with initial voltage V_0	C, V_0	V_0 S.C.	O.C.

AC Analysis

→ Admittance is defined as where Y is the admittance in siemens Z is the impedance measured in ohm.

→ Resistance is a measure of the opposition of a circuit to the flow of a steady current while impedance takes into account not only the resistance but also dynamic effect, known as Reactance.

$$\boxed{Y = \frac{1}{Z}}$$

In R.L series ckt $\rightarrow$ $Z = \sqrt{R^2 + X_L^2}$

$X_L = \omega L = 2\pi f L$

In R.C. series ckt $\rightarrow$ $Z = \sqrt{R^2 + X_C^2}$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

In R.L.C $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\rightarrow$ Average value = $\frac{\text{Area under the curve}}{\text{Length of the base of the curve}}$

$\rightarrow$ R.M.S value = $\sqrt{\frac{\text{Area of half cycle wave squared}}{\text{Half cycle base}}}$

$\rightarrow$ Form factor $\rightarrow$ The ratio of rms value ~~too~~ to ~~current~~ quality ~~avg~~ average value of an alternating quantity is known as form factor. It is represent by K_f

$$K_f = \frac{\text{R.M.S value}}{\text{Avg. value}}$$

$\rightarrow$ Peak factor $\rightarrow$ The ratio of maximum value to the rms value of an alternating quantiter is known as peak factor or amplitude factor. It is represent by ka.

$$Ka = \frac{\text{maximum value}}{\text{R.M.S value}}$$

Types of power

1) Active power / Real power / True power

2) Reactive power / pulsating power

3) Apparent power = $|V| |I|$

4) complex power = VI^*

($I^* \rightarrow$ complex conjugate of I)

* Instantaneous & Average power

→ In an A.C ckt the power at any instant is called so.

→ It is equal to the product of the value of voltage and current at any instant.

→ In an a.c ckt let the instantaneous value of the voltage & current

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t - \phi)$$

where ϕ = phase angle between V & I

→ Instantaneous power $P = vi$

$$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m \cdot I_m \sin\omega t \cdot \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$$

$$\because \cos A \cdot \cos B = 2\sin\left(\frac{A+B}{2}\right) - \sin\left(\frac{A-B}{2}\right)$$

$$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

⇒ The 2nd term of right hand side of the above. eqn contains a double frequency 2ω. So the magnitude of the avg. value of the 2nd term is zero. It is because average of sinusoidal quantity of double frequency over a complete cycle is zero. Hence the 2nd term is avoided.

$$P_{av} = \frac{1}{2} V_m I_m \cos\phi.$$

→ The instantaneous power is the avg. power in A.C circuit $P_{av} = \frac{1}{2} V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P_{av} = VI\cos\phi$$

$V = \frac{V_m}{\sqrt{2}}$ = rms value of the voltage in A.C ckt

$I = \frac{I_m}{\sqrt{2}}$ = r.m.s value of current in A.C circuit

$$\boxed{P = VI\cos\phi}$$

P. It's Active power

Reactive power:

→ Reactive power generates from reactive element (inductor & capacitor)

→ The product of RMS value of voltage & current with the sine of the angle between them is called the reactive power in A.C circuit.

$$\boxed{Q = VI \sin\phi}$$

→ Reactive power for purely inductive ckt.

$$Q_L = V_L I_L = I^2 X_L = \frac{V_L^2}{X_L}$$

where X_L = Inductive reactance

$$\boxed{X_L = \omega L = 2\pi f L}$$

→ Reactive power for purely capacitive ckt

$$Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C}$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

Complex power

→ The product of R.M.S value of the voltage current in a ckt is called complex power.

→ It is represented by 'S' & its unit VA

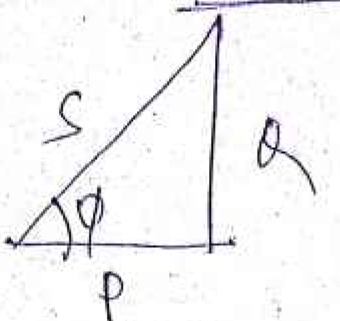
→ In complex form

$S = P + jQ$ (for inductive ckt)

$S = P - jQ$ (for capacitive ckt)

→ magnitude of the complex power $S = \sqrt{P^2 + Q^2}$

Power triangle



$\tan\phi = (Q/P)$

$\phi = \tan^{-1}(Q/P)$

Analysis of coupled circuit

Self inductance

When a current changes in a circuit, the magnetic flux linking the same circuit changes (or vice versa) and emf is induced in the circuit. This induced emf is proportional to the rate of change current.

$$V = L\frac{di}{dt} \quad -(1)$$

V = induced voltage

$\frac{di}{dt}$ = Rate of change of current

L = const. of proportionality called self inductance.

We know $\boxed{L = \frac{N\phi}{i}} \quad -(2)$

N = no. of turns in the circuit

ϕ = flux linkage.

Substituting eqn(2) in eqn(1) we get

$$V = \frac{L\, d\left(\frac{N\phi}{L}\right)}{dt}$$

$$= L \times \frac{1}{L} \times N \frac{d\phi}{dt}$$

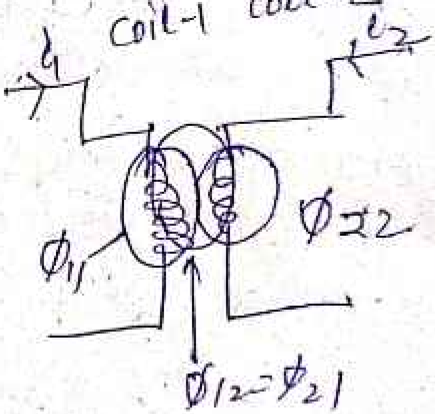
$$\boxed{V = N\frac{d\phi}{dt}} \quad \text{---}(3)$$

comparing eqn(1) & (2)

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\Rightarrow \boxed{L = N\frac{d\phi}{di}} \quad \text{---}(4)$$

Mutual Inductance:



when 2 coils carry current i_1 & i_2 each coil will have leakage flux ϕ_{11} & ϕ_{22} coil 1 & coil-2 respectively as well as mutual flux ϕ_{21}.

the flux of coil-2 links coil-1 or ϕ_{12}, the flux of coil-1 links coil-2.

The induced voltage of coil-2 is

$$V_{L_2} = N_2\frac{d\phi_{12}}{dt} \quad \text{---}(1)$$

Again since ϕ_{12} is related to the current of coil-2 & the induced voltage is proportional to the rate of change of i_1

$$V_{L2} = M_1 \frac{di}{dt} \quad \text{---(2).}$$

where M is the mutual inductance between the 2 coils

comparing eqn (1) & (2)

$$M \frac{di}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

$$\Rightarrow \boxed{M = N_2 \frac{d\phi_{12}}{di_1}} \quad \text{---(3)}$$

Similarly $\boxed{M = N_1 \frac{d\phi_{21}}{di_2}}$ ---(4).

when the coils are linked with air medium the flux & current are linearly related

$$\left.\begin{aligned} M &= N_2 \frac{\phi_{12}}{i_1} \\ M &= N_1 \frac{\phi_{21}}{i_2} \end{aligned}\right\} \quad \text{---(5)}$$

Co-efficient of coupling (K)

It is defined as the fraction of total flux that links the coils

$$\boxed{K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}}$$

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The max^m value of K is unity

from eq^n (1)

$$M^2 = N_1 N_2 \frac{\phi_{12}\,\phi_{21}}{i_1\, i_2}$$

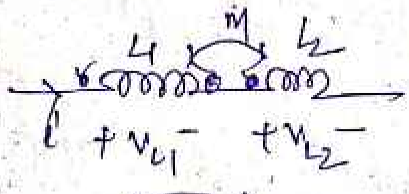
$$= N_1 N_2 \frac{K\phi_1\, K\phi_2}{i_1\, i_2}$$

$$= K^2 \frac{N_1\phi_1}{i_1} \cdot \frac{N_2\phi_2}{i_2}$$

$$M^2 = K^2 L_1 L_2$$

$$\Rightarrow M = K\sqrt{L_1 L_2}$$

<u>for</u> series connection of coupled coil



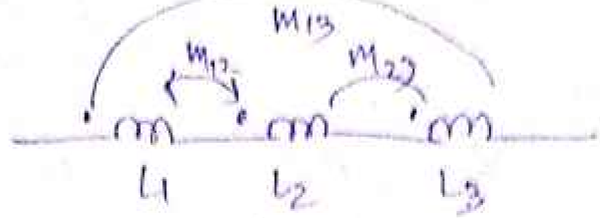
$L_{eq} = L_1 + L_2 + 2M$

$L_{eq} = L_1 + L_2 - 2M$

for <u>parallel connection</u>

(1) For adding $\rightarrow$ $L_{eff} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

(2) parallel opposing $L_{eff} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$



for coil ~~eq~~ coil 1 → $L_1 + M_{12} + M_{13}$

coil 2 → $L_2 + M_{12} + M_{23}$

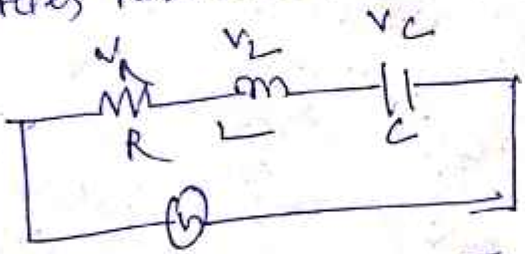
coil 3 → $L_3 + M_{13} + M_{23}$

The net ~~eq~~ inductance =

$$L_1 + M_{12} + M_{13} + L_2 + M_{12} + M_{23} + L_3 + M_{23} + M_{13}$$

$$= L_1 + L_2 + L_3 + 2(M_{12} + M_{23} + M_{13})$$

Series Resonance.



$$I = \frac{V}{Z}.$$

$$Z = R + jωL + \frac{1}{jωC}$$

$$= R + j\left(ωL - \frac{1}{ωC}\right)$$

$$= R + j(X_L - X_C) \quad \text{+ve}$$

Let f_0 or $ω_0$ be the frequency at $X_L = X_C$.

$$ω_0 L = \frac{1}{ω_0 C}$$

$$ω_0^2 = \frac{1}{LC} \Rightarrow ω = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

Q factor $Q = \frac{V_L}{V} = \frac{V_C}{V_C}$

Q.1 Bandwidth of series Resonating circuit & it relation with Q.

Ans The frequency band width in the Limits of lower and upper power frequency is called the bandwidth of the resonant circuit.

$$X = \pm (X_L - X_C) = R$$

$$R = \pm \left(\omega L - \frac{1}{\omega C} \right) = \pm X$$

Let f_1 be the frequency when the net circuit reactance be -ve. and f_2 be frequency when the net circuit reactance is +ve.

Then $\left(\omega_2 L - \frac{1}{\omega_2 C} \right) = R$ —(1)

$$\left(\omega_1 L - \frac{1}{\omega_1 C} \right) = -R \quad —(2)$$

Adding eqn (1) & (2)

$$(\omega_2 + \omega_1) L - \frac{1}{C} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1} \right) = 0$$

$$\Rightarrow (\omega_2 + \omega_1) L - \frac{1}{C} \left(\frac{\omega_2 + \omega_1}{\omega_1 \omega_2} \right) = 0$$

$$\Rightarrow L = \frac{1}{C} * \frac{1}{\omega_1 \omega_2}.$$

Again subtracting eqn (2) from eqn (1).

$$(\omega_2 - \omega_1) L + \frac{1}{C} \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) = 2R.$$

$$\Rightarrow (\omega_2 - \omega_1) L + \frac{1}{C} \left(\frac{\omega_2 - \omega_1}{\omega_2 \omega_1} \right) = 2R \quad —(3)$$

Dividing eqn (3) by L

$$(\omega_2-\omega_1)+\frac{1}{LC}\frac{(\omega_2-\omega_1)}{\omega_2\omega_1}=\frac{2R}{L}$$

$$\Rightarrow (\omega_2-\omega_1)+\frac{1}{LC}\left(\frac{\omega_2-\omega_1}{1/LC}\right)=\frac{2R}{L}$$

$$\Rightarrow 2(\omega_2-\omega_1)=\frac{2R}{L}$$

$$\Rightarrow (\omega_2-\omega_1)=\frac{R}{L} \quad \text{---}(4)$$

$$\Rightarrow \boxed{f_2-f_1=\frac{1}{2\pi}\frac{R}{L}}$$

we know $Q=\omega_0\frac{L}{R}$, $\Rightarrow \frac{R}{L}=\frac{\omega_0}{Q}$ ---(5)

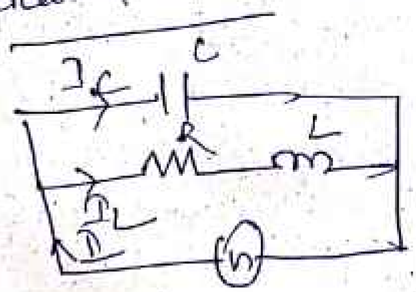
comparing with eqn (4)

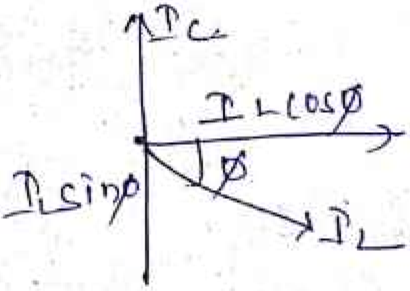
$$\frac{\omega_0}{Q}=\omega_2-\omega_1$$

$$\Rightarrow Q=\frac{\omega_0}{\omega_2-\omega_1}=\frac{f_1}{f_2-f_1}$$

$$\Rightarrow \boxed{Q=\frac{f_0}{\text{Bandwidth}}=\frac{\text{Resonant frequency}}{\text{Bandwidth}}}$$

Parallel Resonance





$$Y=Y_1+Y_2$$

$$=\frac{j}{X_C}+\frac{1}{R+jX_L}$$

$$=\frac{j}{X_C}+\frac{R-jX_L}{R^2+X_L^2}$$

$$= \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right)$$

At resonable the imaginary part must be zero

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$\Rightarrow \omega_0 C = \frac{\omega_0 L}{R^2+\omega_0^2 L^2}$$

$$\Rightarrow R^2+\omega_0^2 L^2 = \frac{L}{C}$$

$$\Rightarrow \omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}}$$

$$f = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$Z = R + j\omega L$$

$$Z_L = \sqrt{R^2+(\omega L)^2} = \sqrt{R^2+\frac{L}{C}-R^2}$$

$$Z_L = \sqrt{\frac{L}{C}}$$

Resonance current $I = \frac{V}{L/CR}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

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Laplace Transform

let f(t) be be a function of time which is zero for t<0 and which is arbitrary defined for t>0 subject to some condition. Then the laplace transform of the function f(t). denote by F(s) is defined as

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \cdot dt.$$

1 → $U(t)$ → unit step → $1/s$

2 → $U(t-T)$ → unit step by function delay by T → $\frac{e^{-sT}}{s}$

3 → $\delta(t)$ → unit impulse → 1

4 → e^{at} → exponential → $\frac{1}{s-a}$

5 → e^{-at} → exponential → $\frac{1}{s+a}$

6 → $\sin\omega t$ → sine function → $\frac{\omega}{s^2+\omega^2}$

7 → $\cos\omega t$ → cos function → $\frac{s}{s^2+\omega^2}$

8 → $t^n (n=1,2,3..)$ → ramp function → $\frac{n!}{s^{n+1}}$

9 → t → unit ramp function → $\frac{1}{s^2}$.

Convolution integral.

→ Convolution of 2 real function corresponds to multiplication of their respective function.

→ If $L[f_1(t)] = F_1(s)$ & $L[f_2(t)] = F_2(s)$

Convolution is defined by

$$L[f_1(t) * f_2(t)] = F_1(s) \cdot F_2(s) \quad \text{---(1)}$$

→ The 2 function $f_1(t)$ & $f_2(t)$ are multiplied in such a manner that one is continously moving with time 'τ' relative to other.

i.e $$f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) \cdot f_2(\tau)\, d\tau \quad \text{---(2)}$$

The statement of the mathematical expression given in expression (1) is called convolution theorem.

let $L[f_1(t) * f_2(t)] = F(s)$

$$F(s) = \int_0^\infty [f_1(t) * f_2(t)]\, e^{-st}\, dt$$

$$\rightarrow F(s) = \int_{t=0}^{\infty} \left[\int_{\tau=0}^{t} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau \right] \cdot e^{-st} \cdot dt \quad \text{---(3)}$$

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Inverse laplace theorem. (I.L.T)

→ The inverse laplace theorem is the transformation of a laplace transform into a function of time

→ $L^{-1}[F(s)] = f(t)$

Q: Determine I.L.T for $F(s) = \frac{s+1}{s(s+2)}$

given $F(s) = \frac{s+1}{s(s+2)}$

$= \frac{A}{s} + \frac{B}{s+2}$

$A = s \times F(s) \Big|_{s=0} = s \times \frac{s+1}{s(s+2)} \Big|_{s=0} = \frac{1}{2}$

$B = (s+2) \times F(s) \Big|_{s=-2} = s+2 \times \frac{s+1}{s(s+2)} \Big|_{s=-2} = \frac{1}{2}$

$F(s) = \frac{1}{2} \cdot \frac{1}{s} + \frac{1}{2} \cdot \frac{1}{s+2}$

Taking inverse laplace of above function.

for unit step $\boxed{f(t) = \frac{1}{2} + \frac{1}{2} \cdot e^{-2t}}$ $t>0$

Q: $F(s) = \frac{2s+1}{(s+1)(s+2)(s+3)}$

sol: $F(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$

$A = (s+1) \times F(s) \Big|_{s=-1} = (s+1) \times \frac{(2s+1)}{(s+1)(s+2)(s+3)} = \frac{-1}{2}$

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$$B = (s+2) \times F(s) \Big|_{s=-2} = (s+2) \times \frac{2s+1}{(s+1)(s+2)(s+3)} \Big|_{s=-2} = 3.$$

$$C = (s+3) \times F(s) \Big|_{s=-3} = (s+3) \times \frac{(2s+1)}{(s+1)(s+2)(s+3)} = -5/2.$$

$$F(s) = \frac{(-1/2)}{s+1} + \frac{3}{s+2} + \frac{(-5/2)}{s+3}$$

we take I.L.T of above eq^n we get

$$f(t) = -\frac{1}{2}e^{-t} + 3e^{-2t} - 5/2\, e^{-3t}.$$

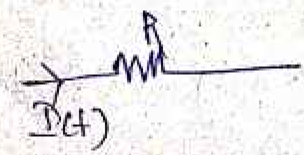
Transfer function Representation

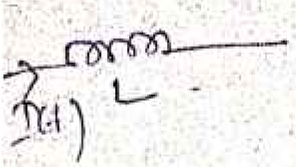
→ It is the ratio of laplace transform of output signal or responce to the laplace transform of input signal or responce taking all initial condition are zero.

$$T.F = G(s) = \frac{V_2(s)}{V_1(s)}$$

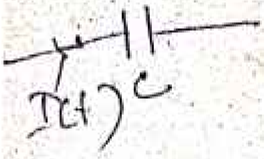
$$Z(s) = \frac{V(s)}{I(s)} \rightarrow \text{impedance function}$$

$$Y(s) = \frac{I(s)}{V(s)} \rightarrow \text{Admittance function.}$$

	In time domain	In frequency domain
	$v(t) = RI(t)$	$V(s) = RI(s)$

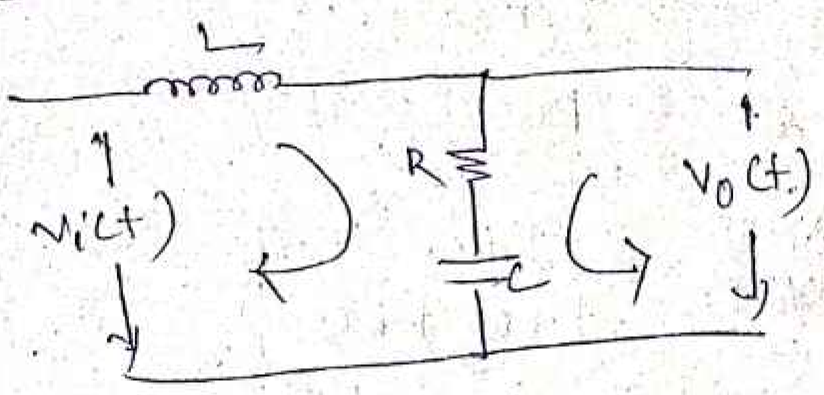


$$v(t) = L\frac{dI(t)}{dt} \rightarrow V(s) = LS\,I(s)$$



$$v(t) = \frac{1}{C}\int I(t)\,dt \rightarrow v(s) = \frac{1}{CS}I(s)$$

Q:) calculate the transfer function of given figure



Ans Applying KVL in input loop

$$v_i(t) = L\frac{dI(t)}{dt} + RI(t) + \frac{1}{C}\int I(t)\,dt \quad \text{---(1)}$$

Applying KVL in o/p loop

$$v_0(t) = RI(t) + \frac{1}{C}\int I(t)\,dt \quad \text{---(2)}$$

Taking L.T. of above eq we get

from eq^n (1)

$$\longrightarrow v_i(s) = LS\,I(s) + R\,I(s) + \frac{1}{CS}I(s)$$

$$= I(s)\left[R + LS + \frac{1}{CS}\right] \quad \text{---(3)}$$

from eq^n (2)

$$V_0(S) = RI(s) + \frac{1}{CS}I(s)$$

$$= I(s)\left[R + \frac{1}{CS}\right] \quad \text{---(4)}$$

Transfer function $G(s) = \frac{V_o(s)}{V_i(s)}$

$$\frac{V_o(s)}{V_i(s)} = \frac{I(s)\left[R + \frac{1}{Cs}\right]}{I(s)\left[R + Ls + \frac{1}{Cs}\right]}$$

$$= \frac{\frac{RCs+1}{Cs}}{\frac{RCs + LCs^2 + 1}{Cs}}$$

$$\boxed{G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1 + RCs}{1 + RCs + LCs^2}}$$

Initial value theorem

→ Here we can find out the initial value of time funct. $f(t)$.

$$\boxed{f(0^+) = \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} s \cdot F(s)}$$

→ The only restriction is that $f(t)$ must be continuous or the most, a step ~~function~~ discontinuity $t = 0$.

Final value theorem

$$F(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} s \cdot F(s)$$

→ The only restriction is that the root of the denominator polynomial of $F(s)$, i.e. pole of $F(s)$ have $-$ve or '0' value.

Poles and zero

$$\frac{C(s)}{R(s)} = G(s) = \frac{k(s+z_1)(s+z_2)(as^2+bs+c)}{(s+P_1)(s+P_2)(As^2+Bs+C)} \quad (1)$$

$K = \frac{bm}{an}$ is known as the gain factor 's' is the complex frequency.

Poles → The poles of $G(s)$ are those values of 's' which make $G(s)$ tend to infinity. For e.g. in eqn (1) we have poles at $S = -P_1$, $S = -P_2$ and pairs of poles at $S = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$.

Zeros The zeros of $G(s)$ are those values of 's' which make $G(s)$ tends to zero. For e.g. in eqn (1) we have zeros $S_1 = -Z_1$, $S_2 = -Z_2$ and pairs of zero's at

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

* If either poles and zero coincide then such type of poles and zero are called multiple pole or multiple zero, otherwise they are known as simple pole or simple zero.

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Q. Determine pole and zero of given function

$$F(s) = \frac{s(s+1)}{(s+2)(s+4)}$$

Ans zero - $s=0$, $s=-1$

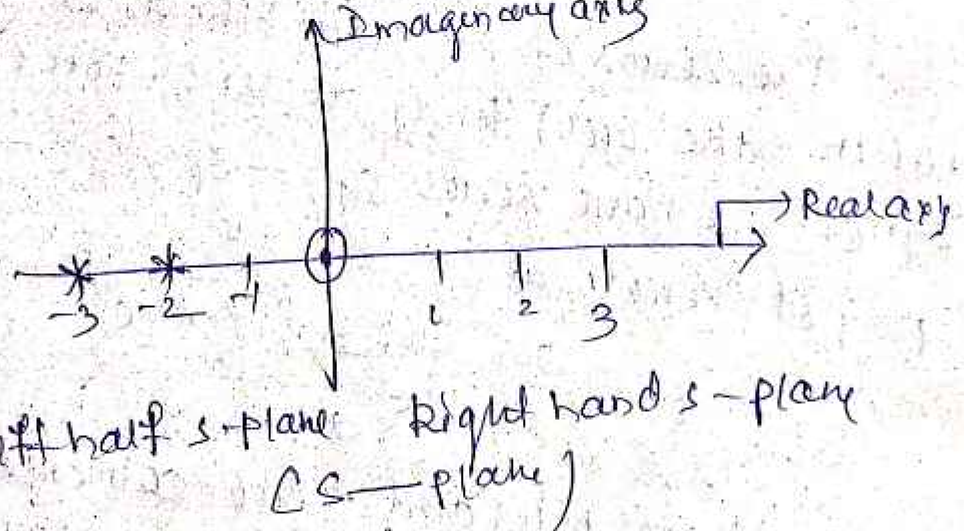
poles $s=-2$ $s=-4$.

Q. Draw the pole and zero of $F(s) = \frac{s}{(s+2)(s+3)}$

Soln zeros $s=0$

pole $s=-2, -3$

Plot



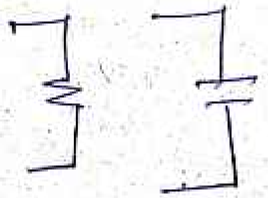
Pole denoted as '$*$'

zero denoted as $\odot$

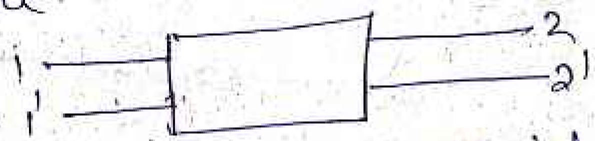
Two port Network

1) Port → A pair of terminal is known as port

2) Single port Network → If a n/w consists of one pair of terminal or 2 pair of terminal is known as single phase port N/W.



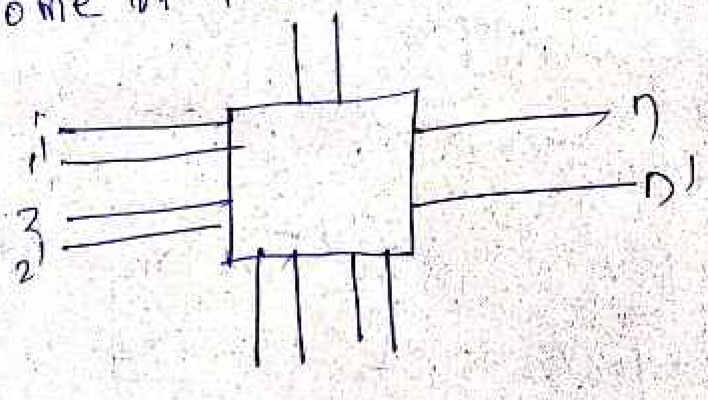
3) 2 port Network: — The n/w having 2 pair of terminal or four terminal is known as 2 port network.



→ In case of 2 port Network if one pair act as input (1-1') and the other pair act as output port (2-2')

N-port Network → If a network having n- pair of terminals or 2n terminal is known as n-port n/w.

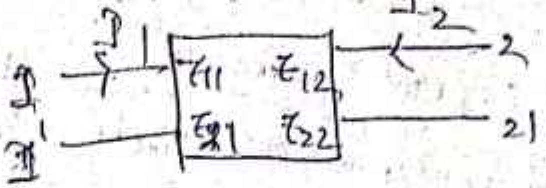
→ In n port network some of them act as i/p port and some of them act as o/p port.



Parameter Representing of 2 port network

The relationship between V_1, I_1 & V_2, I_2 can be represented in different parameter from i.e Z-parameter, Y-parameter, h-parameter, ABCD (transmission) parameter.

Z-parameter (open ckt parameter)



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

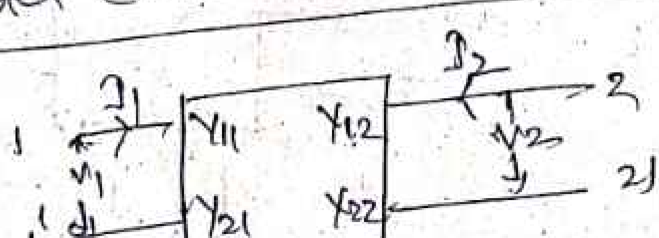
The parameter can be obtained as.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

→ The Z-parameter are also known as open circuit parameter because all the parameter are obtained by opening input port ($I_1 = 0$) or opening the output port ($I_2 = 0$)

Y-parameter (short-circuit parameter)



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

The parameter can be obtained as

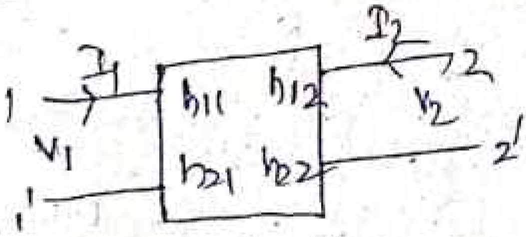
$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \qquad Y_{22} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

The above parameter are also known as short circuit parameter because we can obtaining all the parameter by short circuit the i/p port ($V_1=0$) or by short circuit the o/p port ($V_2=0$)

Representation of h-parameter (Hybrid parameter)

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$



The parameter can be obtained as

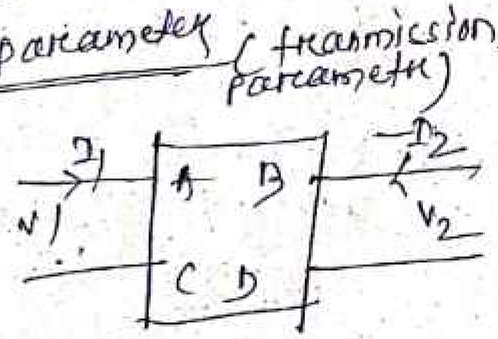
$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \qquad h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} \qquad h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

The h-parameter are obtaining the combination and constant of Z-parameter and Y-parameter and for which, they are called as hybrid parameter.

Representation of ABCD parameter (transmission line parameter)

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$



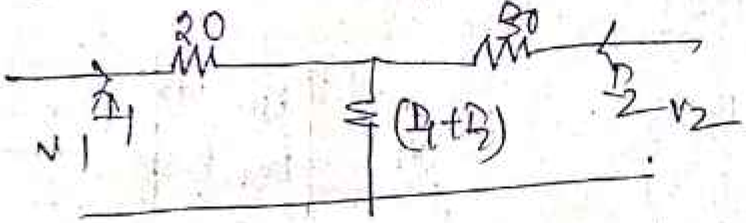
The parameter can be obtained as

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \qquad B = \frac{V_1}{-I_2}\bigg|_{V_2=0}$$

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} \qquad D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$

The above parameter calculated are used in transmission line for which they are known as transmission parameter.

Q Determine Z-parameter for the network.



Sol^n As per formula

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$Z_{11} = \frac{20 I_1 + 10 I_1}{I_1} = \frac{30 I_1}{I_1}$$

$$Z_{11} = 30\Omega$$

$$Z_{21} = \frac{V_2}{I_2}\bigg|_{I_2=0}$$

$$= \frac{10 I_1}{I_1} = 10\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$Z_{22} = \frac{10 I_2 + 30 I_2}{I_1}$$

$$Z_{22} = 40 I_2 / I_2 = 40\Omega$$

$$\boxed{Z_{22} = 40\Omega}$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_2=0}$$

$$= \frac{10 I_2}{I_2}$$

$$\boxed{Z_{12} = 10\Omega}$$

Condition for symmetry

→ A 2 port network is said to be symmetrical if the port can be interchanged without changing port voltage or current.

(a) In term of Z parameter ⇒ $Z_{11} = Z_{22}$

(b) In " Y " ⇒ $Y_{11} = Y_{22}$

(c) In " h " ⇒ $\Delta h = 1$

or

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

d) " " ABCD " ⇒ $\boxed{A = D}$

Condition for reciprocity

→ A network is said to be reciprocal if the ratio of the responce to be excitation remains same. to interchange of the position of the excitation and responce in the network.

(a) In term of Z parameter ⇒ $Z_{12} = Z_{21}$

(b) " " Y " ⇒ $Y_{12} = Y_{21}$

(c) " " h " ⇒ $h_{12} = -h_{21}$

(d) " " ABCD " ⇒ $\Delta T = 1$

or $AD - BC = 1$

or $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$.

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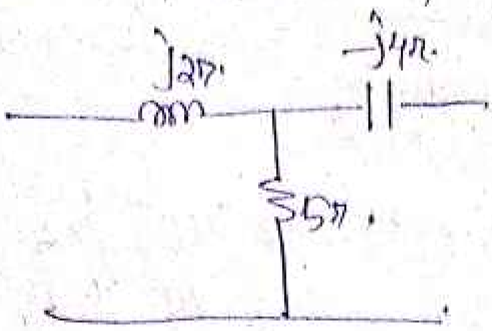
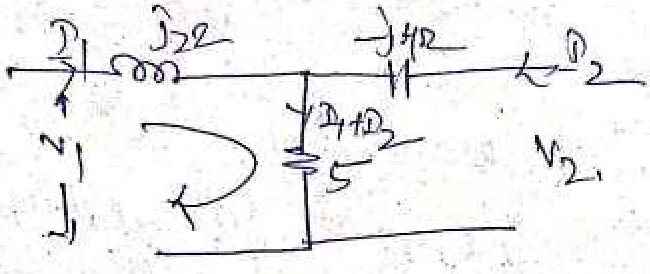
Q: For the network shown in fig calculate T-parameters



Soln T-parameter eqn

$$V_1 = AV_2 - BI_2 \quad \text{—(1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—(2)}$$



By KVL in loop-(1) $-j2 \times I_1 - 5(I_1+I_2) + V_1 = 0$

$$\Rightarrow V_1 = \cancel{5(\ldots)}\ (5+j2)I_1 + 5I_2 \quad \text{—(3)}$$

By KVL in loop-(2)

$$j4I_2 - 5(I_1+I_2) + V_2 = 0$$

$$\Rightarrow V_2 = 5I_1 + (5-j4)I_2 \quad \text{—(4)}$$

putting I_1 from eqn (4) in eqn (3) we get

$$V_1 = (5+j2)\left[\frac{V_2 - I_2(5-j4)}{5}\right] + 5I_2$$

$$\Rightarrow V_1 = \left(\frac{5+j2}{5}\right)V_2 - \left[\frac{(5+j2)(5-j4)}{5} - 5\right]I_2$$

—(5)

For making as eqn(2) rearrange eqn(4) we get.

$$I_1 = \frac{1}{5}V_2 - \left(\frac{5-j4}{5}\right)I_2 \quad \text{——(6)}$$

To obtain T-parameter comparing eqn(1) & eqn(2)

$$A = \left(\frac{5+j2}{5}\right) \quad B = \frac{(5+j2)(5-j4)-5}{5}$$

$$= \frac{8-j10}{5}$$

comparing eqn(2) & (6) we get

$$C = \left(\frac{1}{5}\right) \quad D = \left(\frac{5-j4}{5}\right)$$

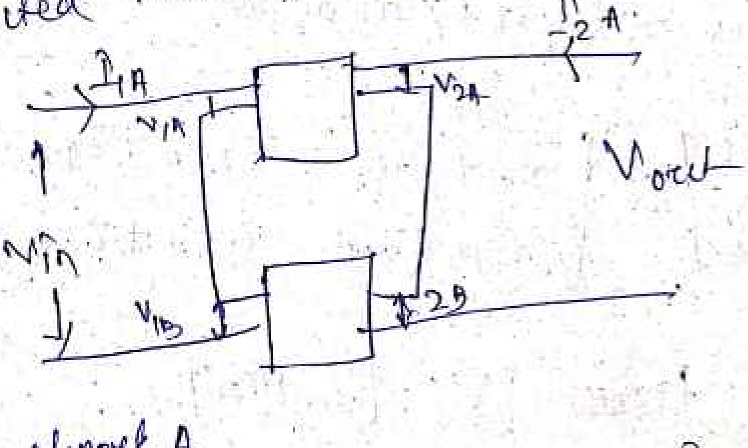
Interconnection of 2-port network

(1) series connection (2) cascade connection

(3) parallel connection

(1) series connection

Let network A & B be the 2 port network connected in series shown in fig.



For network A

$$V_{1A} = Z_{11A}I_{1A} + Z_{12A}I_{2A}$$

$$V_{2A} = Z_{21A}I_{1A} + Z_{22A}I_{2A}$$

For network B

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}$$

The interconnection Results

$I_1 = I_{1A} = I_{1B}$ | $V_1 = V_{1A} + V_{1B}$

$I_2 = I_{2A} = I_{2B}$ | $V_2 = V_{2A} + V_{2B}$

$$V_1 = V_{1A} + V_{1B}$$

$$\Rightarrow V_1 = (Z_{11A} I_{1A} + Z_{12A} I_{2A}) + (Z_{11B} I_{1B} + Z_{12B} I_{2B})$$

$$\Rightarrow V_1 = I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B}) \quad -(1)$$

And $V_2 = V_{2A} + V_{2B}$

$$V_2 = (Z_{21A} I_{1A} + Z_{22A} I_{2A}) + (Z_{21B} I_{1B} + Z_{22B} I_{2B})$$

$$V_2 = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22A} + Z_{22B}) \quad -(2)$$

Thus we get eqn (1) & (2)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

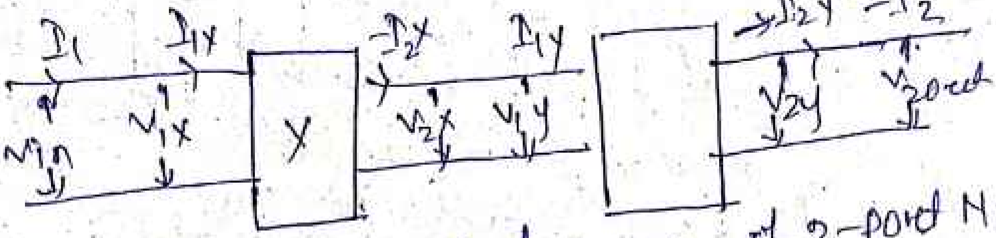
Hence $[Z] = [Z_A] + [Z_B]$

Thus it has been observed that the overall Z parameter matrix for series connected two port network is simply the sum of Z matrices of each individual network.

(2) cascade connection

→ ABCD parameter are highly used in characteristic cascade of 2-port network.

→ Let X & Y be 2 network connected in cascade



(cascade connected 2 no. of 2-port Network)

For network X

$$V_{1x} = A_x V_{2x} - B_x I_{2x}$$

$$I_{1x} = C_x V_{2x} - D_x I_{2x}$$

For network Y

$$V_{1y} = A_y \cdot V_{2y} - B_y I_{2y}$$

$$I_{1y} = C_y V_{2y} - D_y I_{2y}$$

For the cascade connection

$I_1 = I_{1x}$; $-I_{2x} = I_{1y}$; $I_2 = I_{2y}$

$V_1 = V_{1x}$; $V_{2x} = V_{1y}$; $V_2 = V_{2y}$

The overall transmission parameter for the combined network as shown in fig (above) in matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1x} \\ I_{1x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

$$= \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix}$$

$$= \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

$$= \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

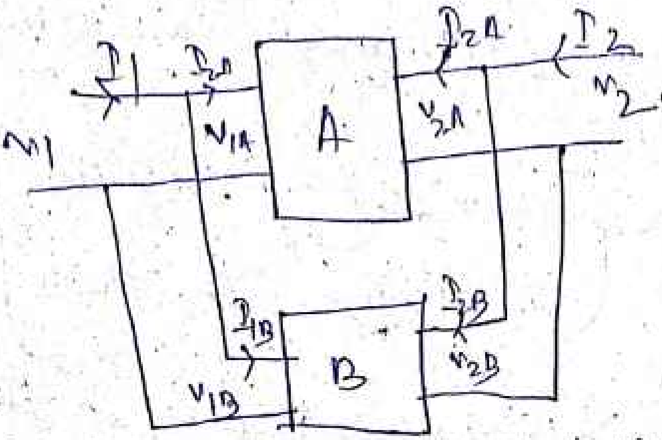
where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix}$

The overall ABCD parameter network matrix for cascaded n/w is then the matrix product of ABCD matrices of individual network.

(3) Parallel connection

→ let A & B network be connected in $||^{el}$ as shown in fig.

→ Y-parameter representation is very much useful.



(Parallel connection of two 2-port network)

<u>For network A</u>

$$I_{1A} = Y_{11A} V_{1A} + Y_{12A} V_{2A}$$

$$I_{2A} = Y_{21A} V_{1A} + Y_{22A} V_{2A}$$

<u>network B</u>

$$I_{1B} = Y_{11B} \cdot V_{1B} + Y_{12B} V_{2B}$$

$$I_{2B} = Y_{21B} V_{1B} + Y_{22B} V_{2B}$$

<u>For the parallel connection</u>

$V_1 = V_{1A} = V_{1B}$ $\quad$ $V_2 = V_{2A} = V_{2B}$

$I_1 = I_{1A} + I_{1B}$ $\quad$ $I_2 = I_{2A} + I_{2B}$

Thus $I_1 = I_{1A} + I_{1B}$

$\Rightarrow I_1 = (Y_{11A} V_{1A} + Y_{12A} V_{2A}) + (Y_{11B} V_{1B} + Y_{12B} V_{2B})$

$\Rightarrow I_1 = (Y_{11A} + Y_{11B}) V_1 + (Y_{12A} + Y_{12B}) V_2$ — (1)

And $I_2 = I_{2A} + I_{2B}$.

$\Rightarrow I_2 = (Y_{21A} V_{1A} + Y_{22A} V_{2A}) + (Y_{21B} V_{1B} + Y_{22B} V_{2B})$

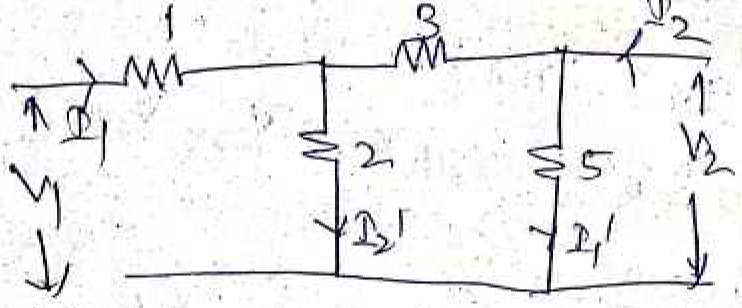
$\Rightarrow I_2 = (Y_{21A} + Y_{21B}) V_1 + (Y_{22A} + Y_{22B}) V_2$ — (2)

Finally from eqn (1) and (2) in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11A}+Y_{11B} & Y_{12A}+Y_{12B} \\ Y_{21A}+Y_{21B} & Y_{22A}+Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

→ The overall Y parameter matrix is then simply the summation of Y matrices of each individual 2 port network.

Q:



(Find Z-parameter)

Soln case-I ($I_2=0$)

$Z_{11} = (8\|2)+1 = 2.6\Omega$. $I_1' = \frac{2}{10} \times I_1 = I_1/5$

$V_2 = 5 I_1' = 5 \times \frac{I_1}{5} = I_1$

$\Rightarrow Z_{21} = \frac{V_2}{I_1} = 1\Omega$

case-2 ($I_1 = 0$)

$Z_{22} = 5\|5 = \frac{5\times5}{5+5} = 2.5\Omega$

$I_2' = \frac{5}{10} I_2 = \frac{I_2}{2}$

$V_1 = 2\times I_2' = 2 \times \frac{I_2}{2} = I_2$.

$Z_{12} = \frac{V_1}{I_2} = 1\Omega$.

(Ans)

Extra

A series R-L-C ckt with value $R = 10\Omega$, $L = 0.2H$ & $C = 100\mu F$ is excited from a D.C source of 50V by sudden switching ON of a key at time $t = 0$ s. Find the expression for the resulting current.

Ans $R = 10\Omega$ $L = 0.2H$ $C = 100\mu F$ $V = 50V$

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Taking Laplace transform.

$$\frac{V}{s} = RI(s) + sL(I(s) - I(0)) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0)}{s}\right]$$

There is no intial current in the inductor & intial charge at capacitor

$$\frac{V}{s} = RI(s) + sLI(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right]$$

$$[\because I(0) = 0,\ q(0) = 0]$$

$$\frac{V}{s} = \left[R + sL + \frac{1}{Cs}\right]I(s)$$

$$VC = [RCs + s^2LC + 1]\, I(s)$$

$$VC = LC\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]I(s)$$

$$\Rightarrow \frac{\frac{V}{L}}{s^2 + (R/L)s + 1} = I(s)$$

$$\Rightarrow I(s) = \frac{50/0.2}{s^2 + \frac{10}{0.2}s + \frac{1}{0.2 \times 100 \times 10^{-6}}}$$

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$$= \frac{250}{s^2+50s+5\times10^4}$$

Taking inverse laplace transform

$$I(s) = \frac{250}{s^2+2.s.25+(25)^2+5\times10^4-(25)^2}$$

$$= \frac{250}{(s+25)^2+(222.1)^2}$$

$$= \frac{250}{(s+25)^2+(222.1)^2} \times \frac{222.2}{222.2}$$

$$\boxed{I(t) = 1.25\, e^{-25t} \sin 222.2t.}$$

Q: A 50μF capacitor when connected in series with a coil having 40 Ω resistance, resonates at 1000 Hz. Find the inductance of the coil. Also obtain the circuit current if the applied voltage is 100V. Also calculate the voltage across the capacitor and the capacitor and the coil at resonance.

Sol^n At ~~[illegible]~~ resonance:

$$X_L = X_C$$

$$\Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

$$L = \frac{1}{4\pi^2 f_0^2 C}$$

$$= \frac{1}{4\pi^2\times1000^2\times50\times10^{-6}} = 0.5\,mH$$

$I_0 = \frac{V}{Z} = \frac{V}{R}$ at resonance.

$= \frac{100}{40} = 2.5\ A.$

power loss of the coil $= I_0^2 R = 2.5^2 \times 40 = 250\ W.$

Again $V_C = I_0 X_C = 2.5 \times \frac{1}{2\pi f_0 C}$

$= 2.5 \times \frac{1}{2\pi \times 1000 \times 50 \times 10^{-6}}$

$= 7.96\ V$

$X_L = \omega L = 2\pi \times 1000 \times 0.5 \times 10^{-3}$

$= 3.14\ \Omega.$

$V_{coil} = I_0 Z_{coil}$ (At resonance)

$= 2.5 \sqrt{40^2 + 3.14^2}$

$= 100.31\ V.$

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