

LECTURE NOTES
ON
DATA STRUCTURE
3RD SEMESTER
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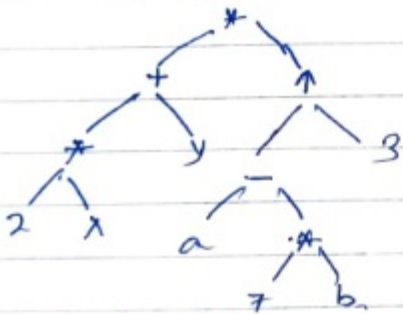
Approved by AICTE, New Delhi



Data are Simple Values or Set of values.
Meaningful or processed data is Information.
(data with given attribute)

The logical or mathematical model of a particular organization of data is called datastructure. The datastructure must be rich enough in structure to mirror the actual relationships of the data in real world. On the other hand, the structure should be simple enough that one can effectively process the data when necessary.

One dimensional array.	Stack.	Graph.
Two " "	Queue	when data
linked list	Tree	Airline flies only bet ⁿ cities connected by lines.
	$(2x+y)(a-7b)$	



Primitive Vs Non primitive ⇒

Linear Vs Nonlinear ⇒

Static Vs Dynamic Datastructure ⇒

Arrays.

A linear array is a list of a finite number 'n' of homogeneous data elements.

$$\text{Loc}(\text{LA}[k]) = \text{Base}(\text{LA}) + w(k - \text{Lower bound}),$$

Operations on Array

Traversal. Deletion.

Search. Sorting.

Insertion. Merging.

Linear Search.

Algo: (Linear Search) $\text{LINEAR}(\text{DATA}, N, \text{ITEM}, \text{LOC})$

Here DATA is a linear array with 'N' elements, and ITEM is a given item of information. This algorithm finds the location LOC of ITEM in DATA or sets $\text{LOC} := 0$ if the search is unsuccessful.

1. [INSERT ITEM at the end of DATA.] ~~Set~~

Set $\text{DATA}[N+1] := \text{ITEM}$.

2. [Initialize Counter] Set $\text{LOC} := 1$.

3. [Search for ITEM]

Repeat while $\text{DATA}[\text{LOC}] \neq \text{ITEM}$:

$\text{LOC} \leq N$

Set $\text{LOC} := \text{LOC} + 1$.

[End of Loop.]

4. [Successful?] If $\text{LOC} = N+1$, then: Set $\text{LOC} := 0$.

5. Exit.

Binary Search.

Algo: (Binary Search) BINARY (DATA, LB, UB, ITEM, LOC)
Here DATA is a sorted array with lower bound LB and upper bound UB, and ITEM is a given item of information. The variables BEG, END and MID denote respectively, the beginning, end and middle location of a segment of elements of DATA. This algorithm finds the location LOC of ITEM in DATA or sets $LOC = NULL$.

1. [Initialize segment variables.]
Set $BEG := LB$, $END := UB$ and $MID = INT((BEG + END) / 2)$.
2. Repeat steps 3 and 4 while $BEG \leq END$ and $DATA[MID] \neq ITEM$.
3. If $ITEM < DATA[MID]$, then:
Set $END := MID - 1$.
Else:
Set $BEG := MID + 1$.
[End of If structure.]
4. Set $MID := INT((BEG + END) / 2)$.
[End of Step 2 loop]
5. If $DATA[MID] = ITEM$, then:
Set $LOC := MID$.
Else:
Set $LOC := NULL$.
[End of If structure.]
6. Exit.

11, 22, 30, 33, 40, 44, 55, 60, 66, 77, 80, 88, 99

Representation of Linear Arrays in Memory.

$$\Rightarrow \text{LOC}(\text{LA}[k]) = \text{Base}(\text{LA}) + w(k - \text{lower bound})$$

Multidimensional Array:

Two dimensional array.

\Rightarrow

Column-major Order :- If the 2-dimensional array is represented in memory column by column, known as column-major order.

Row-major Order :- If 2-dimensional array is arranged row by row in memory, known as row-major order.

	Column major	row major
0	11	11
1	21	12
2	31	13
3	12	21
4	22	22
5	32	23
6	31	31
7	32	32
8	33	33



Column-major order

$$\text{LOC}(A[J, k]) = \text{Base}(A) + w[M(k-1) + (J-1)]$$

Row-major order

$$\text{LOC}(A[J, k]) = \text{Base}(A) + w[N(J-1) + (k-1)]$$

M \rightarrow NO. of rows.

N \rightarrow NO. of columns.

3x3.

00	01	02
10	11	12
20	21	22

$$\text{C.M.O} = (2, 1) = 1000 + 2[3(0) + 0]$$

UPPER TRIANGULAR

$$L = \left(\frac{n(n+1)}{2} \right) - \left(\frac{(n-j)(n-j+1)}{2} + (n-k) \right)$$

SPARSE MATRICES

Matrices with a relatively high proportion of zero entries called sparse matrices.

→ Triangular matrix → Tridiagonal matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 2 & & \\ \cancel{3} & \cancel{4} & 5 & \\ & 6 & 7 & 8 \\ & & 9 & 10 \end{pmatrix}$$

Lower Upper

B will contain $1+2+3+\dots+n = \frac{n(n+1)}{2}$ elements

$$B[L] = a_{jk}$$

no. of elements upto $j-1$ rows
 $= 1+2+3+\dots+j-1 = \frac{(j-1)j}{2}$

$$L = \frac{j(j-1)}{2} + k$$

Tridiagonal Matrix:

There are $3(j-2)+2$ elements above $A[j,k]$ and $k-j+1$ elements to the left of $A[j,k]$.

$$\text{Hence } L = [3(j-2)+2] + [k-j+1] + 1 \\ = 2j+k-2.$$

Primitive Data Structures :-

These are basic structures and are directly operated upon by the machine instructions. Integer, Float, character, pointer etc. are primitive data structures.

Non-primitive Data Structures :- These are more sophisticated data structures. These are derived from the primitive data structures. The non-primitive data structures emphasize on structuring of a group of homogeneous (same type) or heterogeneous (diff. type) data items. eg: Arrays, Lists, Stacks, Queue, Tree, Graph.

Algo: (Bubble Sort) BUBBLE (DATA, N)

Here DATA is an array with N elements. This algorithm sorts the elements in DATA.

1. Repeat steps 2 and 3 for $k=1$ to $N-1$
2. Set $PTR := 1$. [Initializes pass pointer PTR.]
3. Repeat while $PTR \leq N-k$: [Executes pass.]
 - (a) If $DATA[PTR] > DATA[PTR+1]$, then:
Interchange $DATA[PTR]$ and $DATA[PTR+1]$.
[End of If structure.]
 - (b) Set $PTR := PTR + 1$.
[End of inner loop.][End of step 1 outer loop.]
4. Exit.

Algo: (Insertion Sort) INSERTION(A, N)

This algorithm sorts the array A with N elements.

1. Set $A[0] := -\infty$. [Initializes sentinel element.]
2. Repeat steps 3 to 4 for $k=2, 3, \dots, N$:
 2. Set $TEMP := A[k]$ and $PTR := k-1$.
 3. Repeat while $TEMP < A[PTR]$ and $PTR > 0$:
 - a) Set $A[PTR+1] := A[PTR]$. [Moves element forward.]
 - b) Set $PTR := PTR - 1$.[End of loop.]
 4. Set $A[PTR+1] := TEMP$. [Insert element in proper place.]
[End of step 2 loop.]
5. Return/Exit

77, 33, 44, 11, 88, 22, 66, 55

33, 77, 44, 11, 88, 22, 66, 55 $k=2$

83, 44, 77, 11, 88, 22, 66, 55 $k=3$

11, 33, 44, 77, 88, 22, 66, 55 $k=4$

The insertion sort algorithm scans the array A from $A[1]$ to $A[N]$, inserting each element $A[k]$ into its proper position in the previously sorted subarray $A[1], A[2], \dots, A[k-1]$.

Storage of Sparse Matrix.

$$\begin{bmatrix} 7 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{bmatrix}$$

No. Rows	No. Col	No. non-zero
6	6	8
0	0	7
0	3	1
0	5	2
1	1	1
1	2	9
2	3	7
4	0	8
5	2	3

Transpose of a Sparse Matrix.

Algo: (Transpose Matrix) TRANSMAT[MAT1[][3], MAT2[][3]

6	6	8
0	0	7
0	4	8
1	1	1
2	1	9

$$m = \text{MAT1}[1][1]$$

$$n = \text{MAT1}[1][2]$$

$$t = \text{MAT1}[1][3]$$

$$\text{MAT2}[1][1] = n$$

$$\text{MAT2}[1][2] = m$$

$$\text{MAT2}[1][3] = t;$$

$$k = 2;$$

for (j=1; j<=n; j++)

for (i=2; i<=t+1; i++)

if (MAT1[i][2] == j)

{

$$\text{MAT2}[k][1] = \text{MAT1}[i][2]$$

$$\text{MAT2}[k][2] = \text{MAT1}[i][1]$$

$$\text{MAT2}[k][3] = \text{MAT1}[i][3]$$

k++;

}

Storage of Sparse Matrix:

$$\begin{bmatrix} 7 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{bmatrix}$$

	No. Rows	No. Col	No. non-zero
6	6	8	
0	0	7	
0	3	1	
0	5	2	
1	1	1	
1	2	9	
2	3	7	
4	0	8	
5	2	3	

Transpose of a Sparse Matrix.

↑ Algo: (Transpose Matrix) TRANSMAT (MAT1 [] [3], MAT2 [] [3]

6	6	8
0	0	7
0	4	8
1	1	1
2	1	9

$$m = \text{MAT1}[1][1]g.$$

$$n = \text{MAT1}[1][2]$$

$$t = \text{MAT1}[1][3]$$

$$\text{MAT2}[1][1] = n.$$

$$\text{MAT2}[1][2] = m$$

$$\text{MAT2}[1][3] = t;$$

k = 2;

for (j = 1; j <= n; j++)

for (i = 2; i <= t+1; i++)

if (MAT1[i][2] == j)

{

$$\text{MAT2}[k][1] = \text{MAT1}[i][3]$$

$$\text{MAT2}[k][2] = \text{MAT1}[i][1]$$

$$\text{MAT2}[k][3] = \text{MAT1}[i][3]$$

k++;

}

Selection Sort

~~Procedure~~ MIN(A, K, N, LOC)

Algorithm: (Selection Sort) SELECTION(A, N)

This algorithm sorts an array A with N element. It first find the smallest element in the array and put it in it's place.

1. Repeat steps 2 to 4 for $k = 1, 2, \dots, N-1$
2. Set $MIN := A[k]$ and $LOC := k$
3. ~~Repeat~~ Repeat for $J = k+1, k+2, \dots, N$:
 - If $MIN > A[J]$, then: Set $MIN := A[J]$ and $LOC := A[J]$ and $LOC := J$.

[End of Loop]
4. [Interchange $A[k]$ and $A[LOC]$.]
 - Set $TEMP := A[k]$, $A[k] := A[LOC]$ and $A[LOC] := TEMP$.

[End of Step 1 loop.]
5. Exit.

77, 33, 44, 11, 88, 22, 66, 55

Selection Sort first finds the smallest element in the list and put it in first place then finds the second smallest element in the list and put it in the second position and so on.

```

n = 7;
for (i = 0; i < row; i++)
    for (j = 0; j < col; j++)
        scanf("%d", &mat[i][j]);
        if (i % 4 == 0)
            { spa[i][0] = i;
              spa[i][1] = j;
              spa[i][2] = mat[i][j];
              n++; }
        }
    spa[0][0] = row;
    spa[0][1] = col;
    spa[0][2] = n-1;
    
```

```

void transpose (int mat1[][3], mat2[][3])
{
    int row = mat1[0][0];
    int col = mat1[0][1];
    int n = mat1[0][2];
    for (j = 0; j < col; j++)
        for (i = 0; i < row; i++)
            if (mat1[i][j] != 0)
                { mat2[j][0] = mat1[i][0];
                  mat2[j][1] = mat1[i][1];
                  mat2[j][2] = mat1[i][2];
                  k++; }
    }
    
```


STACKS

A Stack is a List of elements in which an element may be inserted or deleted only at one end, called the top of the stack. This means, in particular, that elements are removed from a stack in the reverse order of that in which they were inserted into the stack.

Operations on STACK.

- a) Push : Inserts elements into stack.
- b) Pop : Delete " " "

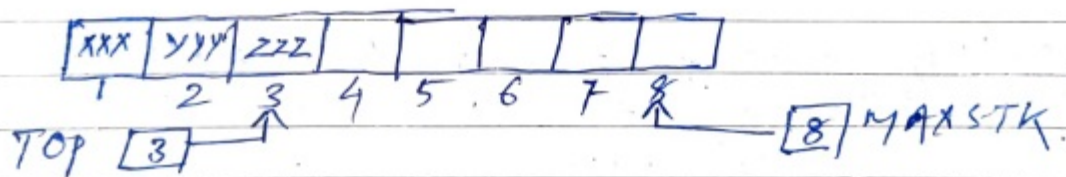
Array Representation of STACK:-

TOP:- a pointer variable which points to the top element of stack.

MAXSTK :- Maximum no. of elements can be held by the stack.

if $TOP := 0 \leftarrow NULL \rightarrow$ stack is empty.

$TOP := MAXSTK \rightarrow$ stack is full.



procedure PUSH (STACK, TOP, MAXSTK, ITEM)

This procedure pushes an ITEM onto a stack

1. [Stack already filled?]

If $TOP = MAXSTK$, then: Print: OVERFLOW, and Return

2. Set $TOP := TOP + 1$. [Increases TOP by 1.]

3. Set $STACK[TOP] := ITEM$. [Inserts ITEM in new TOP position]

4. Return.

Procedure: POP(STACK, TOP, ITEM)

This procedure deletes the top element of STACK and assigns it to the variable ITEM.

1. [Stack has an item to be removed?]

If $TOP = 0$, then: Print: UNDERFLOW, and Return

2. Set $ITEM := STACK[TOP]$. [Assigns TOP element to ITEM.]

3. Set $TOP := TOP - 1$

4. Return.

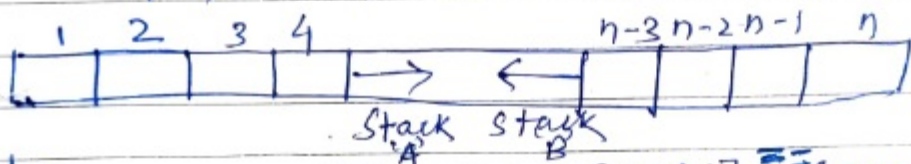
Minimizing Overflow

overflow \rightarrow ~~less~~ small stack size

underflow \rightarrow More stack size.

To minimize the ~~of~~ overflow, we can take 2 stack in a single array.

$A \rightarrow n_1$, $B \rightarrow n_2$ $n = n_1 + n_2$.



push if ~~TOP < 0~~ $STACK[TOP_A] \neq STACK[TOP_B] - 1$ then full.

pop if ~~STACK~~ $TOP_A = 0$ and $TOP_B > n$ then underflow.

As the push operation push the new element on top of old element, stack also known as pushdown List.

Arithmetic Expression [Infix; Prefix; Postfix]

Infix \rightarrow operand operator operand.

Prefix \rightarrow polish notation \rightarrow operator operand operand.

Postfix \rightarrow Reverse polish \rightarrow operand operand operator.

Infix.	Polish. Prefix	Reverse Polish. Postfix
A+B	+AB	AB+
C-D	-CD	CD-
E*F	*EF	EF*
(A+B)*C	*+ABC	AB+C*
A+(B*C)	+A*BC	A(BC*)+
(A+B)/(C-D)	+AB-CD/	AB+CD-/

Advantage:- One never need paranthesis to evaluate Reverse Polish Expr.

For our purpose we only assume the below three levels of precedence.

Highest:- Exponentiation (^)

Next highest:- Multiplication (*) and division (/)

Lowest:- Addition (+) and Subtraction (-).
 ordinary operators and evaluation as from left to right even for exponentiation.

$$5 * (6 + 2) - 12 / 4 \quad \text{---, *5, +6, 2, /, 12, 4}$$

Postfix: ~~5(6+2)* - 12/4/~~
 Infix: 5, 6, 2 + * 12 / 4 / -

$$A + (B * C - (D / E \wedge F) * G) * H$$

$$A + (BC * - (D / E F \wedge) * G) * H$$

$$A + (BC * - (DEF \wedge /) * G) * H$$

$$A + (BC * DEF \wedge / G * -) * H$$

$$A + (BC * DEF \wedge / G * - H *)$$

$$A BC * DEF \wedge / G * - H * +$$

$$12, 7, 3, -, /, 2, 1, 5, +, *, +$$

$$12, [7-3], /, 2, 1, 5, +, *, +$$

$$(12 / [7-3]), 2, [1+5] * +$$

$$(12 / (7-3)) \neq 2 * (1+5)$$

Evaluation of a Postfix Expression

Suppose P is an arithmetic expression written in postfix notation. The following algorithm, which uses a STACK to hold operands, evaluates P .

Algorithm: This algorithm finds the VALUE of an arithmetic expression P written in postfix notation.

1. Add a right parenthesis ")" at the end of P .
[This acts as Sentinel]
 2. Scan P from left to right and repeat steps 3 and 4 for element of P until the sentinel ")" is encountered.
 3. If an operand is encountered, put it on STACK.
 4. If an operator \odot is encountered, then:
 - a) Remove the two top elements of STACK, where A is the top element and B is the next-to-top element.
 - b) Evaluate $B \odot A$.
 - c) Place the result of (b) back on STACK.[End of If Structure.]
- [End of Step 2 Loop.]
5. Set VALUE equal to the top element on STACK.
 6. Exit.

$P: 5, 6, 2, +, *, 12, 4, /, -$

$$5 * (6 + 2) - (12 / 4)$$

Symbol Scanned	STACK
1) 5	5
2) 6	5, 6
3) 2	5, 6, 2
4) +	5, 8
5) *	40
6) 12	40, 12
7) 4	40, 12, 4
8) /	40, 3
9) -	37
10))	End

Transforming Infix Expressions into Postfix Exp.

Algorithm POLISH(Q, P)

Suppose Q is an arithmetic expression written in infix notation. This algorithm finds the equivalent postfix expression P.

1. Push "(" onto STACK, and add ")" to the end of Q.
2. Scan Q from left to right and repeat steps 3 to 6 for each element of Q until the STACK is empty.
3. If an operand is encountered, add it to P.
4. If a left parenthesis is encountered, push it onto STACK.
5. If an operator \otimes is encountered, then:
 - a) Repeatedly pop from STACK and add to P each operator (on the top of STACK) which has the same precedence as or higher precedence than \otimes .
 - b) Add \otimes to STACK.
6. If a right parenthesis is encountered, then:
 - a) Repeatedly pop from STACK and add to P each operator (on the top of STACK) until a left parenthesis is encountered, ~~then~~.
 - b) ~~Repeatedly pop from~~ Remove the left parenthesis. [Do not add the left parenthesis to P.]

[End of If Structure.]

[End of Step 2 Loop.]
7. Exit.

← added in algorithm.

Symbol Scanned	Stack	Symbol Scanned	Stack	Symbol Scanned	Stack
A	C	C	C+C-C	G	C+C-*
+	C+	D	C+C-C)	C+ * - added to P
C	C+C	/	C+C-C/	*	C+*
B	C+C	E	C+C-C/	H	C+* added to P
*	C+C*	↑	C+C-C/↑)	Empty * + added to P
C	C+C*	↓	C+C- ↑, / to P		
-	C+C-	*	C+C-*		

P := A, B, C, *, D, E, /, G, *, -, H + *

QuickSort, An Application of STACKS.

(QuickSort) This algorithm sorts an array A with N element.

1. [Initialize.] $TOP := NULL$.
2. [Push boundary values of A onto stacks when A has 2 or more elements.]

If $N > 1$, then: $TOP := TOP + 1$, $LOWER[TOP] := 1$, $UPPER[TOP] := N$.

3. Repeat steps 4 to 7 while $TOP \neq NULL$.

4. [Pop sublist from stacks.]

Set $BEG := LOWER[TOP]$, $END := UPPER[TOP]$,
 $TOP := TOP - 1$.

5. Call $QUICK(A, N, BEG, END, LOC)$.

6. [Push left sublist onto stacks when it has 2 or more elements.]

If $BEG < LOC - 1$, then:

$TOP := TOP + 1$, $LOWER[TOP] := BEG$,

$UPPER[TOP] := LOC - 1$,

[End of IF structure.]

7. [Push right sublist onto stacks when it has 2 or more elements.]

If $LOC + 1 < END$, then:

$TOP := TOP + 1$, $LOWER[TOP] := LOC + 1$,

$UPPER[TOP] := END$.

[End of IF structure.]

[End of step 3 loop.]

8. Exit.

$QUICK(A, N, BEG, END, LOC)$

[Here A is an array with N elements, parameters BEG and END contain the boundary values of the sublist of A to which this procedure applies. LOC keeps track of the position of the first element $A[BEG]$ of the sublist during the procedure. The local variables $LEFT$ and $RIGHT$ will contain the boundary values of the list of elements that have not been scanned.]

1. [Initialize.] Set $LEFT := BEG$, $RIGHT := END$ and $LOC := BEG$.

2. [Scan from right to left.]

- a) Repeat while $A[LOC] \leq A[RIGHT]$ and $LOC \neq RIGHT$
 $RIGHT := RIGHT - 1$
 [End of loop]
- b) If $LOC = RIGHT$, then: Return.
- c) If $A[LOC] > A[RIGHT]$, then:
 - i) [Interchange $A[LOC]$ and $A[RIGHT]$]
 $TEMP := A[LOC]$, $A[LOC] := A[RIGHT]$
 $A[RIGHT] := TEMP$.
 - ii) ~~$A[RIGHT] := TEMP$~~ set $LOC := RIGHT$.
 - iii) Go to step 3.

3. [Scan from left to right.]

- a) Repeat while $A[LEFT] \leq A[LOC]$ and $LEFT \neq LOC$
 $LEFT := LEFT + 1$.
- b) If $LOC = LEFT$, then: Return.
- c) If $A[LEFT] > A[LOC]$, then
 - i) [Interchange $A[LEFT]$ and $A[LOC]$.]
 $TEMP := A[LOC]$, $A[LOC] := A[LEFT]$
 $A[LEFT] := TEMP$.
 - ii) Set $LOC := LEFT$.
 - iii) Go to step 2.

Quick Sort is an algorithm of the divide and conquer type.

44, 33, 11, 55, 77, 90, 40, 60, 99, 22, 88, 66
 22, 33, 11, 55, 77, 90, 40, 60, 99, 44, 88, 66
 22, 33, 11, 44, 77, 90, 40, 60, 99, 55, 88, 66

void quick_sort (int a[], int l, int h)

```
{
  int temp, key, low, high;
  low = l; high = h;
  key = a[(low+high)/2];
  do
  {
    while (key > a[low])
      low++;
    while (key < a[high])
      high--;
    if (low <= high)
    {
      temp = a[low];
```

```
      a[low++] = a[high];
      a[high--] = temp;
    }
  } while (low <= high);
  if (low < high)
    quick_sort(a, l, high);
  if (low < h)
    quick_sort(a, low, h);
}
```

Circular QUEUES

A Queue is a linear list of elements in which deletion can take place only at one end called front, and insertion can take place only at the other end, called the rear. Queues are also called first-in first-out (FIFO) lists, since the first element in a queue will be the first element out of the queue.

Representations of Queues

Unless until specified queues are represented by linear array in computer, with two pointer variable FRONT & REAR.

1	2	3	4	
A	B	C	D	F = 1
	B	C	D	R = 4
				F = 2, R = 4

Item Inserted

$$\text{REAR} = \text{REAR} + 1$$

Item Deleted

$$\text{FRONT} = \text{FRONT} + 1$$

QINSERT (QUEUE, N, FRONT, REAR, ITEM)

This procedure inserts an element ITEM into a queue.

1. [Queue already full?]

If $\text{FRONT} = 1$ and $\text{REAR} = N$, or if $\text{FRONT} = \text{REAR} + 1$, then: write OVERFLOW, and return.

2. [Find new value of REAR].

If $\text{FRONT} := \text{NULL}$, then

Set $\text{FRONT} := 1$ and $\text{REAR} := 1$

Else if $\text{REAR} = N$, then:

Set $\text{REAR} := 1$

Else: set $\text{REAR} := \text{REAR} + 1$

3. Set $\text{QUEUE}[\text{REAR}] := \text{ITEM}$

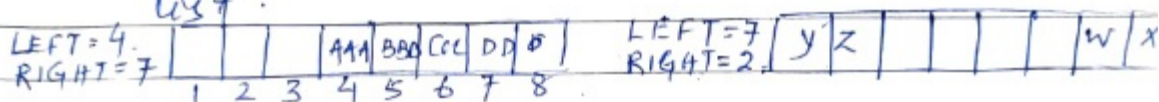
4. Return.

DEQUES

A deque ~~or Bronon~~ is a linear list in which elements can be added or removed at either end but not in the middle.

There are two variations of a deque.

- ✓ Input Restricted deque: a deque which allows insertions at only one end of the list but allows deletions at both ends of the list.
- ✓ Output Restricted deque: a deque which allows deletions at only one end of the list but allows insertions at both end of the list.



Priority Queues

A priority Queue is a collection of elements such that each element has been assigned a priority and such that the order in which elements are deleted and processed comes from the following rule.

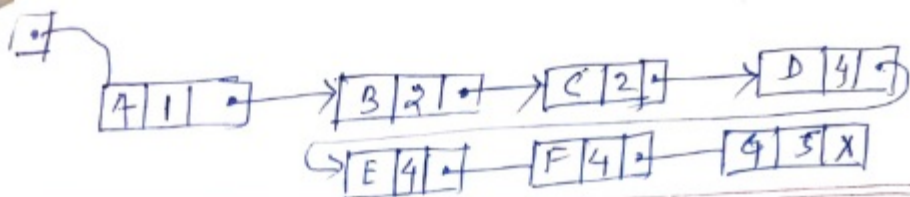
- 1) An element of higher priority is processed before any element of lower priority.
- 2) Two elements with the same priority are processed according to the order in which they were added to the queue.

One-way List Representation of a priority Queue

- ✓ an information field INFO } Each node
- a priority number PRN. }
- a link number LINK }

- ✓ A node X precedes a node Y in the list
- 1) when X has higher priority than Y or
- 2) when both have the same priority but X was added to the list before Y.

START



	INFO	PRN	LINK
	BBB	2	6
AVAIL → 2			7
	DDD	4	4
	EEE	4	9
START → 5	AAA	1	1
	CCC	2	3
			10
	GGG	5	0
	FFF	4	8
			11
			12
			0

Array Representation of a Priority Queue

Use a Separate Queue for each level of priority. Each queue will appear in it's own circular array and must have it's own pair of pointers. If each queue

having the same size then can be represented by matrix.

			1	2	3	4	5	6
F	R		1	4A				
1	2	2	2	BB	CC	XX		
2	1	3	3					
3	0	0	4	FF			DD	EE
4	5	1	5			GG		
5	4	4						

No. of Elements in a Queue

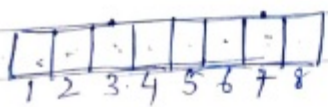
$$F \leq R \Rightarrow NE = R - F + 1$$

$$F > R \Rightarrow NE = N - (F - R - 1) = N + R - F + 1$$

$$NE = (R - F + 1) \bmod N$$

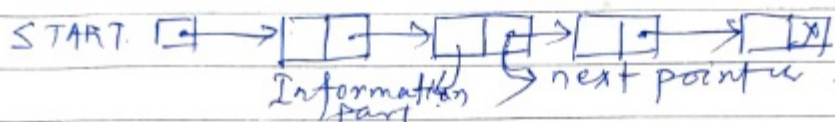
$$F=3, R=7 \Rightarrow 7 - 3 + 1 \bmod 8 = 5 \bmod 8 = 5$$

$$R=3, F=7 \Rightarrow 3 - 7 + 1 \bmod 8 = -3 \bmod 8$$



LinkedList

A linked list, or one-way list, is a linear collection of data elements, called nodes, where the linear order is given by means of pointers. Each node is divided into two parts: the first part contains the information of the element, and the second part, called the link field or next pointer field, contains the address of the next node in the list.



Next pointer of last node will contain NULL.

INFO LINK

	INFO	LINK
1		
2		
3	0	6
4	T	0
5		
6	□	11
7	X	10
8		
9	N	3
10	I	4
11	E	7
12		

START = [9] INFO[9] = N.
 LINK[9] = 3 INFO[3] = 0
 LINK[3] = 6 INFO[6] = □
 LINK[6] = 11 INFO[11] = E.
 LINK[11] = 7 INFO[7] = X
 LINK[7] = 10 INFO[10] = I
 LINK[10] = 4 INFO[4] = T
~~LINK~~ LINK[4] = 0 / NULL.

Algo: (Traversing a linked list)

1. Set PTR := START. [Initializes pointer PTR.]
2. Repeat steps 3 and 4 while PTR ≠ NULL.
3. APPLY PROCESS to INFO[PTR].
4. Set PTR := LINK[PTR].
5. EXIT.

Algorithm 5.2 SEARCH(INFO, LINK, START, ITEM, LOC)
 LIST is a linked list in memory. This algorithm finds the location LOC of the node where ITEM first appears in LIST, or sets LOC = NULL.

1. Set PTR := START.
2. Repeat step 3 while PTR ≠ NULL:
3. If ITEM = INFO[PTR], then:
 Set LOC := PTR, and EXIT.
 Else
 Set PTR := LINK[PTR].
4. Set LOC := NULL.
5. EXIT.

Self Referential Structure

A structure ^{which} contains a pointer variable of its own type known as Self Referential Structure.

```
#include <stdio.h>
```

```
typedef struct node
```

```
{ char info;
```

```
  struct node *next-node;
```

```
};
```

```
typedef struct node List-node;
```

```
main()
```

```
{ List-node *list-of-char, n1, n2, n3;
```

```
  n1.info = 'X';
```

```
  n1.next-node = NULL;
```

```
  n2.info = 'Y';
```

```
  n2.next-node = &n1;
```

```
  n3.info = 'Z';
```

```
  n3.next-node = &n2;
```

```
  list-of-char = &n3;
```

```
  printf("node 3 = %c %d, node 2 = %c %d\n", n3.info, n3.next-node, n2.info, n2.next-node);
```

```
  printf("node 1 = %c %d, header addr = %d\n", n1.info, n1.next-node, list-of-char);
```

```
}
```

Dynamic Memory allocation.

ptr = (Cast-type*) malloc (byte-size)

ptr = (Cast-type*) calloc (n, elem-size)

Contiguous Space for n blocks of ^{size} elem-size bytes is allocated

```
#include <stdio.h>
```

```
#include <malloc.h>
```

```
struct link
```

```
{ int info;
```

```
  struct link *next;
```

```
};
```

```
void createlist (struct link*);
```

```
void display (struct link*);
```

```
void main()
```

```
{ struct link *node;
```

```
  clrscr();
```

```
  node = (struct link*) malloc (sizeof (struct link));
```

```
  if (node == NULL)
```

```
  { printf ("Out of memory space"); exit(0); }
```

```
  createlist (node); display (node);
```

```
}
```

```
void createlist (struct link *node)
```

```
{ char ch;
```

```
  int i = 1;
```

```
  printf ("Enter the value for %d node:", i);
```

```
  scanf ("%d", &node->info);
```

```
  node->next = NULL;
```

```
  i++;
```

```
  printf ("Press 'n' to quit, any other to continue");
```

```
  fflush (stdin);
```

```
  ch = getchar();
```

```
  while (ch != 'n')
```

```
  { node->next = (struct link*) malloc (sizeof (struct link));
```

```
    if (node->next == NULL)
```

```
    { printf ("Out of memory space");
```

```
      exit(0);
```

```
    }
```



```

node = node → next;
printf("In Enter the value for %d node:", i);
scanf("%d", &node → info);
node → next = NULL;
i++;
printf("In Press 'n' to quit any other key to continue.");
flush(stdin);
ch = getchar();
}
}

```

```

void display(struct link *node)
{
    struct link *ptr;
    printf("In Value of nodes in the list are  

    as follows: \n\n");
    while(ptr != NULL)
    {
        printf("%d", node → info);
        node = node → next;
    }
}
}

```

✓ Advantages of linked lists.

- Linked list are dynamic in nature, i.e. they can grow or shrink during execution of a program.
- Efficient memory utilization: Here memory is not pre-allocated. Memory is allocated whenever it is required and it is deallocated when it is no longer needed.
- Insertion and deletions are easier and efficient: Linked lists provide flexibility in inserting a data item at a specified position and deletion of a data item from the given position.
- Many complex applications can be easily carried out with linked lists.

Disadvantages

- For a node we need extra memory spaces for pointers.
- Access to an arbitrary data item is little bit cumbersome and also time-consuming.

Types of linked list

- ✓ Singly-link list
- ✓ Doubly link list
- ✓ Circular link list
- ✓ Circular doubly linked list

Operation on linked list

- Creation
- Insertion
- Deletion
- Traversing
- Searching
- Concatenation
- Display

INSERTION.

Insertion at beginning.

```
void insertatbegin (struct link *node, int item)
{
    struct link *ptr;
    ptr = (struct link *) malloc (sizeof (struct link));
    ptr->info = item;
    ptr->next = startnode;
    node startnode = ptr;
}
```

Insertion at end.

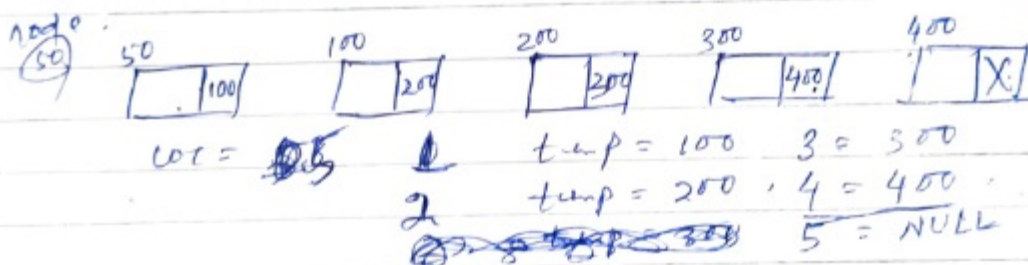
```
void insertatend (struct link *node, int item)
{
    struct link *ptr, *loc;
    ptr = (struct link *) malloc (sizeof (struct link));
    ptr->info = item;
    ptr->next = NULL;
    if (start startnode == NULL)
    {
        start startnode = ptr;
    }
    else {
        loc = start startnode;
        while (loc->next != NULL)
        {
            loc = loc->next;
        }
        loc->next = ptr;
    }
}
```


Insert after specified location

```

void insert_loc(struct link * start next, int info, int loc)
{
    struct link * ptr, * temp;
    int k;
    for (k = 1, temp = startnode; k < loc; k++)
    {
        temp = temp -> next;
        if (temp == NULL or k == loc)
        {
            printf("There are less no. of nodes than '%d'", loc); return;
        }
        temp = temp -> next;
    }
    ptr = (struct link *) malloc(sizeof(struct link));
    ptr -> info = info; ptr -> next = temp -> next;
    temp -> next = ptr;
}

```



Void Rev-list()

```

{
    node * p, * c, * n;
    if (start == NULL)
        return;
    if (start -> next == NULL)
        return;
    p = start; c = start -> next;
    p -> next = NULL;
    while (c -> next != NULL)
    {
        n = c -> next;
        c -> next = p;
        p = c;
    }
    c -> next = p;
    start = c;
}

```

Deletion of nodes in Single linked list

Deleting the first node of the linked list

```
void delete_beg (struct link *node)
{
    struct link *ptr;
    if (node == NULL)
        return;
    else
        ptr = node;
    node = node->next;
    free(ptr);
}
```

Deleting the Last Node

```
void delete_end (struct link *node)
{
    struct link *ptr, *temp;
    if (node == NULL)
        return;
    else if (node->next == NULL)
    {
        ptr = node;
        node = NULL;
        free(ptr);
    }
    else
    {
        ptr = node;
        while (ptr->next != NULL)
        {
            temp = ptr;
            ptr = ptr->next;
        }
        temp->next = NULL;
        free(ptr);
    }
}
```

Deleting the node from Specified Position

void delete_spec_loc (struct link *node, int loc)

```
{
    struct link *ptr, *temp;
    int i;
    if (node == NULL)
    {
        printf ("empty list");
        return;
    }
    else
    {
        ptr = node;
        for (i = 1; i <= loc; i++)
        {
            ptr = ptr->next;
            if (ptr == NULL)
            {
                printf (" ");
                return;
            }
        }
        temp = ptr->next;
        ptr->next = ptr->next->next;
        free(temp);
    }
}
```


Circular Linklist

The ~~last node~~ link part of last node will contain the first nodes address rather than NULL;

It will be better if we maintain two pointers FIRST & LAST to point to the first node and last node.

```
typedef struct Node
{
    int num;
    struct node *next;
} node;

node *start = NULL;
node *last = NULL;
```

void main()

```
{
    node *temp;
    temp = (node *) malloc (sizeof (node));
start = temp;
last = temp;
start->next = start;
}
```

~~edges~~
~~temp~~
~~temp~~
~~temp~~
Inserting a node at the beginning

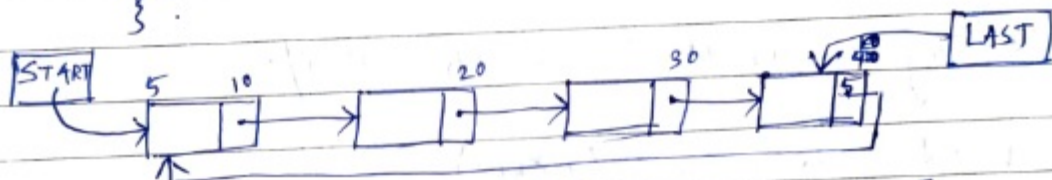
```
void insertfirst (node)
{
    node *ptr;
    ptr = (node *) malloc (size of (node));
    printf ("Enter the no");
    scanf ("%d", &ptr->num);
    if (start == NULL)
    {
        ptr->next = ptr;
        start = ptr;
        last = ptr;
    }
    else
    {
        ptr->next = start;
        start = ptr;
        last->next = ptr;
    }
}
```

Inserting node at the end.

```

void insertlast ( )
{
    node *ptr;
    ptr = (node *) malloc (sizeof (node));
    printf ("Enter the number");
    scanf ("%d", &p -> num);
    if (Start == NULL)
        {
            ptr -> next = ptr;
            Start = last = ptr;
        }
    else
        {
            last -> next = ptr;
            last = ptr;
            ptr -> next = Start; (last -> next = Start)
        }
}

```



Inserting in the middle.

```

for (i = 1; i < loc; i++)
{
    ptr = ptr -> next;
    if (ptr == Start)
        exit;
}
temp = Start;
if (ptr -> next == Start)
{
    ptr -> next = temp;
    temp -> next = Start;
    Start = temp;
}
else
{
    temp -> next = ptr -> next;
    ptr -> next = temp;
}

```

5
 1 - 4
 1 - ptr = 10
 2 - ptr = 20
 3 - ptr = 30
 4 - ptr = 5

Deleting node from Beginning.

```
void delete-first()
```

```
{ node *p;  
  if (start == NULL)  
    printf("List empty");  
  else if (start->next == startNULL) { startstart = lastNULL; }  
    p = start;  
    start = start->next;  
    last->next = start; free(p);  
}
```

Deleting a node from the end.

```
void del-last(node)
```

```
{ node *p, *q;  
  if (start == NULL)  
    printf("List empty");  
  else  
    p = start;  
    while (p->next != last)  
    if (p->next == NULL)  
      { start = NULL;  
        last = NULL;  
        free(p);  
      }  
    else  
      { while (p->next != last)  
          { p = p->next;  
            p = p->next;  
          }  
      }
```

```
printf(" ");
```

```
q = p->next;
```

```
p->next = start;
```

```
last = p;
```

```
free(q); }
```

Deleting node from Beginning.

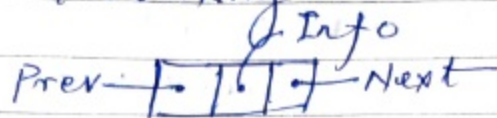
```
void delete-first()
{
    node *p;
    if (start == NULL)
        printf("List empty");
    else if (start->next == NULL) {
        start = last = NULL;
    }
    else {
        p = start;
        start = start->next;
        last->next = start;
        free(p);
    }
}
```

Deleting a node from the end.

```
void del-last()
{
    node *p, *q;
    if (start == NULL)
        printf("List empty");
    else {
        p = start;
        while (p->next != NULL)
        if (p->next == NULL)
            start = NULL;
            last = NULL;
            free(p);
        else {
            while (p->next != last)
                p = p->next;
            q = p->next;
            p->next = start;
            last = p;
            free(q);
        }
    }
    printf(" ");
    q = p->next;
    p->next = start;
    last = p;
    free(q);
}
```


Doubly linked list

A doubly linked list is one in which all nodes are linked together by two links which help in accessing both the successor node and predecessor node from the given node position. It provides bi-directional traversing.



struct node

```
{ int num;
```

```
  struct node *prev;
```

```
  struct node *next;
```

```
}; typedef struct node NODE;
```

start



```
start NODE *start = NULL;
```

Inserting a node at beginning

```
void insert_beg (int item)
```

```
{ NODE *ptr;
```

```
  ptr = (NODE *) malloc (sizeof(NODE));
```

```
  ptr->num = item;
```

```
  if (start == NULL)
```

```
  { ptr->prev = ptr->next = NULL;
```

```
    start = ptr; start = ptr;
```

```
  }
```

```
  else { ptr->prev = NULL;
```

```
        ptr->next = start;
```

```
        start->prev = ptr;
```

```
        start = ptr;
```

```
  }
```

```
}
```

Inserting a node at the end.

```
void insert_end (int item)
{
    NODE *ptr, temp;
    ptr = (NODE *) malloc (sizeof(NODE));
    ptr->data = item;
    if (tail == (ptr (NODE *) NULL))
    {
        ptr->prev = ptr->next = NULL;
        start = ptr;
    }
    else
    {
        ptr->next = NULL;
        temp = start;
        while (temp->next != NULL)
            temp = temp->next;
        ptr->prev = temp;
        temp->next = ptr;
    }
}
```

Deleting a node from the beginning

```
void delete_beg ()
{
    NODE *ptr;
    if (start == NULL)
    {
        // ... return();
    }
    else if (start->next == NULL)
    {
        ptr ptr = start;
        start = NULL;
        free(ptr);
    }
    else
    {
        ptr = start;
        start = start->next;
        ptr->prev = NULL;
        free(ptr);
    }
}
```


Deleting A Node from the End.

```
void delete_end ()
```

```
{ node *ptr; *ptr;
```

```
if (start == NULL)
```

```
return;
```

```
else if (to start->next == NULL)
```

```
{ ptr = start;
```

```
start = NULL;
```

```
free(ptr);
```

```
}  
else { ptr = start;
```

```
while (ptr->next != NULL)
```

```
① to ptr  
ptr = ptr->next;
```

```
ptr->prev->next = NULL
```

```
free(ptr)
```

```
}
```

```
}
```

Adv: Bidirectional Traversing is possible which helps in easy accessibility of nodes.

Disadv: Uses of more pointers leads to more memory requirement.