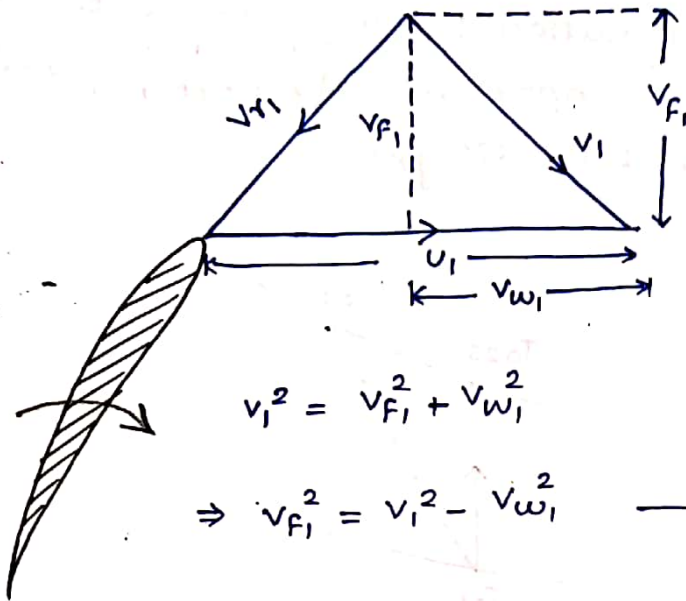


# Turbo Machinery

Generalised equation for any rotodynamic machine:



$$v_1^2 = v_{f1}^2 + v_{w1}^2$$

$$\Rightarrow v_{f1}^2 = v_1^2 - v_{w1}^2 \quad \text{--- (1)}$$

$$v_{r1}^2 = v_{f1}^2 + (U_1 - v_{w1})^2$$

$$\dots v_{f1}^2 = v_{r1}^2 - (U_1 - v_{w1})^2 \quad \text{--- (2)}$$

From equation (1) & (2)

$$v_1^2 - v_{w1}^2 = v_{r1}^2 - (U_1 - v_{w1})^2$$

$$v_1^2 - v_{w1}^2 = v_{r1}^2 - U_1^2 + v_{w1}^2 + 2v_{w1}U_1$$

$$\Rightarrow v_{w1}U_1 = \frac{v_1^2 + U_1^2 - v_{r1}^2}{2}$$

Similarly

$$v_{w2}U_2 = \frac{v_2^2 + U_2^2 - v_{r2}^2}{2}$$

$$\text{Work} = v_{w1}U_1 - v_{w2}U_2 \Rightarrow \frac{v_1^2 - v_2^2}{2} + \frac{U_1^2 - U_2^2}{2} + \frac{v_{r2}^2 - v_{r1}^2}{2}$$

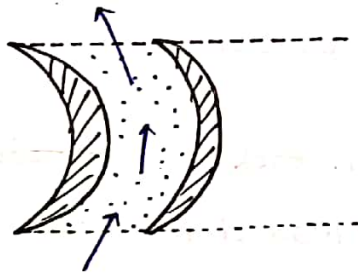
$$W = \underbrace{\frac{V_1^2 - V_2^2}{2}}_{\text{Impulse effect}} + \underbrace{\frac{U_1^2 - U_2^2}{2}}_{\text{centrifugal effect}} + \underbrace{\frac{V_{r2}^2 - V_{r1}^2}{2}}_{\text{Reaction effect}}$$

### Impulse Turbine:

[1] In pure impulse turbine there is no reaction effect ( $V_{r1} = V_{r2}$ ).

[2] No centrifugal effect ( $U_1 = U_2$ )

In impulse turbines as the fluid flows through blade passages static pressure remains constant, it is the kinetic energy which is responsible for rotation or developing power.



Flow passage area remains constant

Impulse blading.

Work developed by impulse turbine

$$W = \frac{V_1^2 - V_2^2}{2}$$

### Pelton wheel:

static pr.

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + z_2$$

$$z_1 = z_2$$

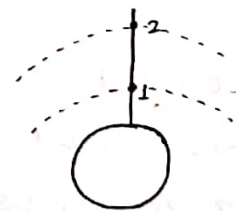
$$V_1 > V_2$$

total pressure at head

exit ↓ but

static pressure remains constant.

b/c of area const.  
blw the two blades  
relative velocity  
at inlet &  
exit  
remains  
same.  
i.e. No  
Reaction effect



$$D_2 > D_1$$

$$U = \frac{\pi D N}{60}$$

$$\text{so } U_2 > U_1$$

but we want

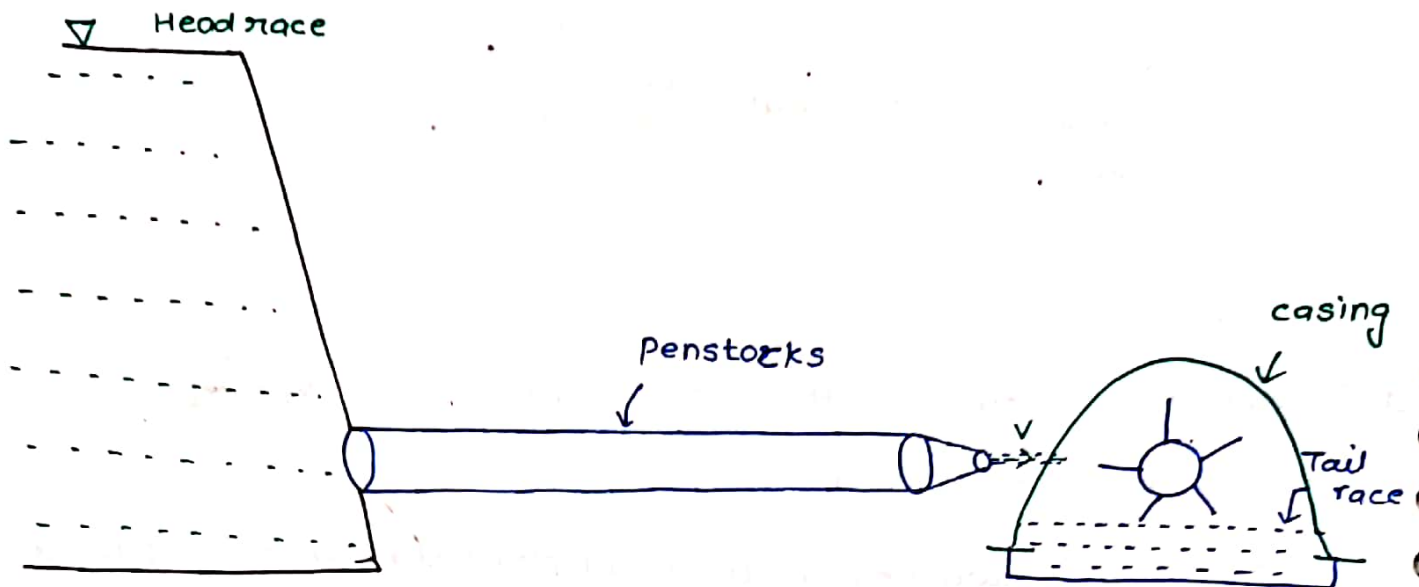
$U_1 = U_2$  to get

pure impulse

so entry & leaving

should be at

same dia. c/s.



The level of water in the storage reservoir is called Head Race the water leaving the runner blade flow through a channel leading to downstream is called tail race. the difference between Head race & tail race when no water is flowing is known as Gross head or Total head or static head.  $[H_1]$

As the fluid flows through penstocks there are friction losses and hence the available head is less than Gross head.

$$H = H_1 - h_L$$

$H$  = Net head or available head

$H_1$  = Gross head

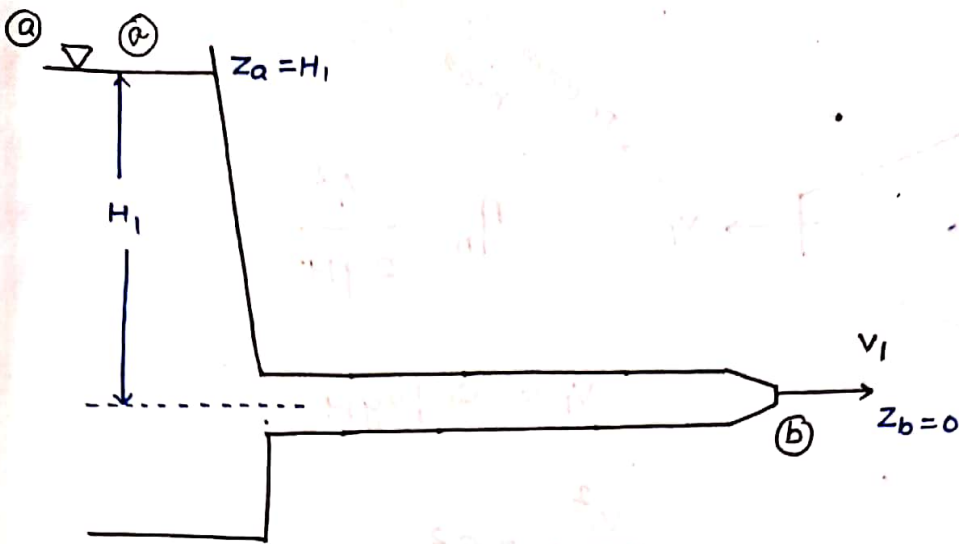
$h_L$  =  $\phi$  Head loss due to friction in penstocks.

Based on the available head turbines are classified as-

(a) High head ( $H > 250\text{m}$ )

(b) Medium head ( $60 < H < 250\text{m}$ )

(c) low head ( $H < 60\text{m}$ )



Apply Bernoulli's equation between a & b

$$\frac{P_a}{\rho} + \frac{V_a^2}{2g} + Z_a = \frac{P_b}{\rho} + \frac{V_b^2}{2g} + Z_b + h_L$$

$$(\because P_a = P_b = P_{atm})$$

$$V_b = V_1$$

$$a_a V_a = V_b a_b$$

$$V_a = \frac{V_b a_b}{a_a \text{ (very large)}}$$

$$V_a = 0$$

$$\Rightarrow H_1 = \frac{V_b^2}{2g} + h_L$$

$$\frac{V_1^2}{2g} = H_1 - h_L = H$$

$$V_1 = \sqrt{2gH}$$

$$V_1 = C_v \sqrt{2gH}$$

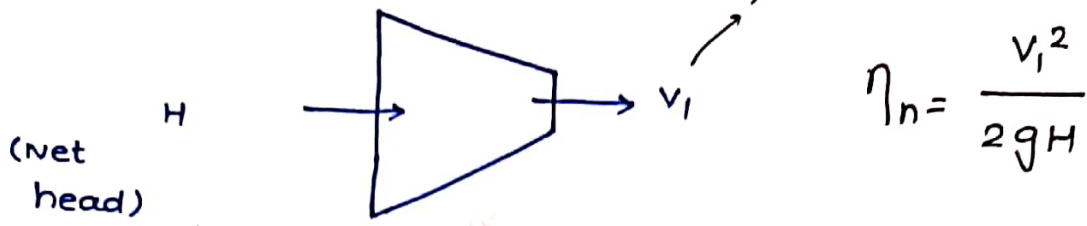
$V_1$  = Jet speed or spouting velocity

Generally  $C_v$  is 0.96 to 0.99

Note: In impulse turbine the fluid enters and leaves the turbine blades at atm. pressure. therefore casing in impulse turbine has no hydraulic function. but it is used to prevent splashing of water.



Nozzle efficiency: ( $\eta_n$ ):



$$\eta_n = \frac{v_1^2}{2gH}$$

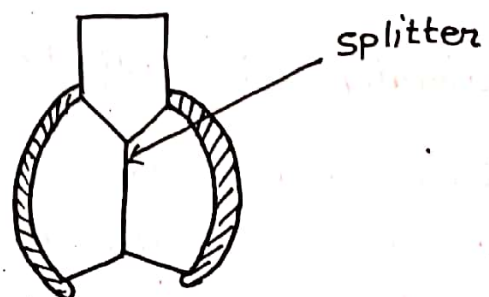
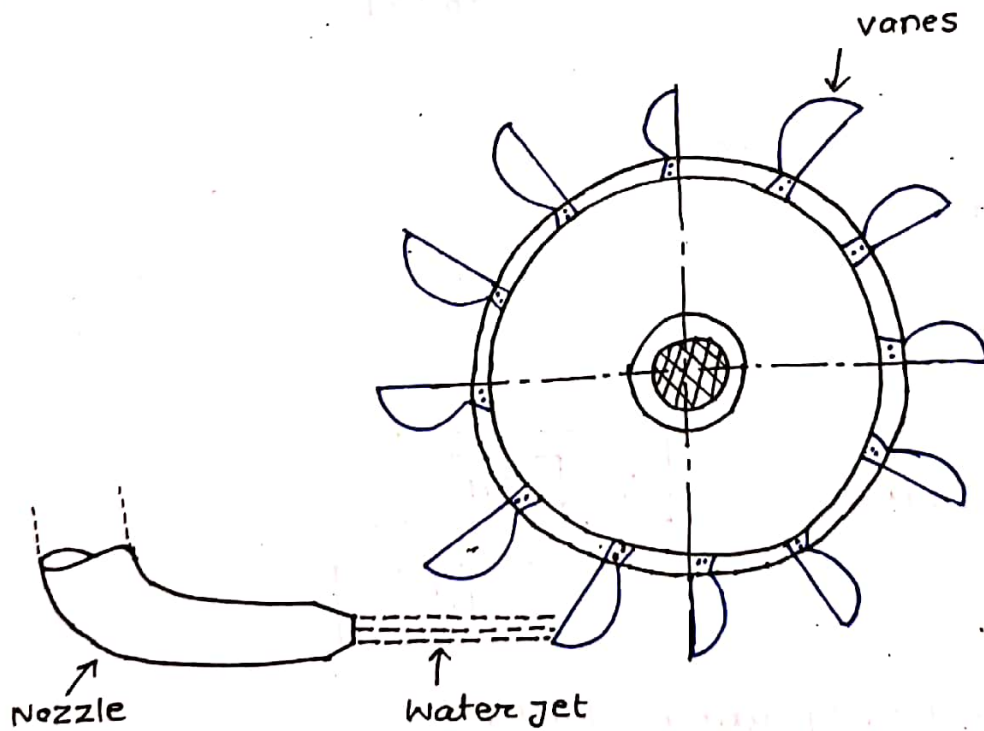
$$v_1 = C_v \sqrt{2gH}$$

$$\frac{v_1^2}{2gH} = C_v^2$$

$$\eta_n = C_v^2$$

①

Figure: Pelton turbine



side view of bucket

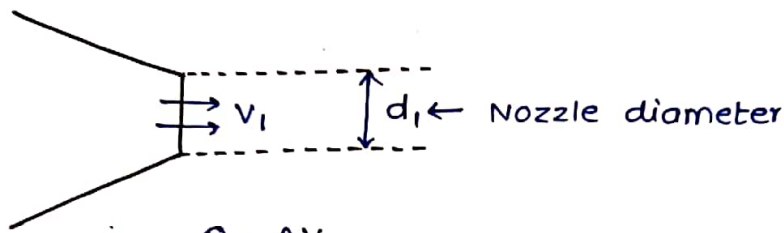
## Pelton wheel is

- (1) Impulse
- (2) Tangential flow
- (3) High head
- (4) Low discharge
- (5) Low specific speed hydraulic turbine.

$$P = \omega Q H = \text{const.}$$

$$QH = \text{const.}$$

## Discharge of pelton wheel:



$$Q = AV_1$$

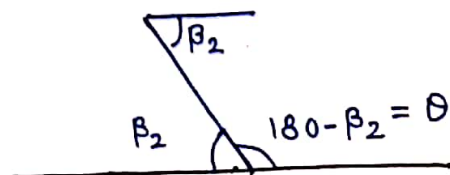
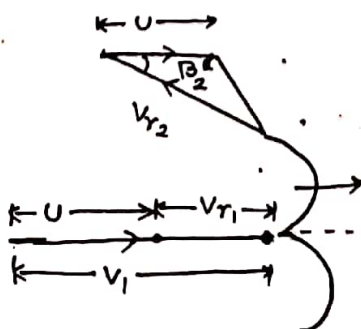
$$Q = \frac{\pi}{4} d_1^2 v_1$$

If it is a multijet pelton wheel then the total discharge  $Q$  is equal to  $N \cdot$

$$Q = \frac{\pi}{4} d_1^2 v_1 N \quad N = \text{NO. OF JETS.}$$

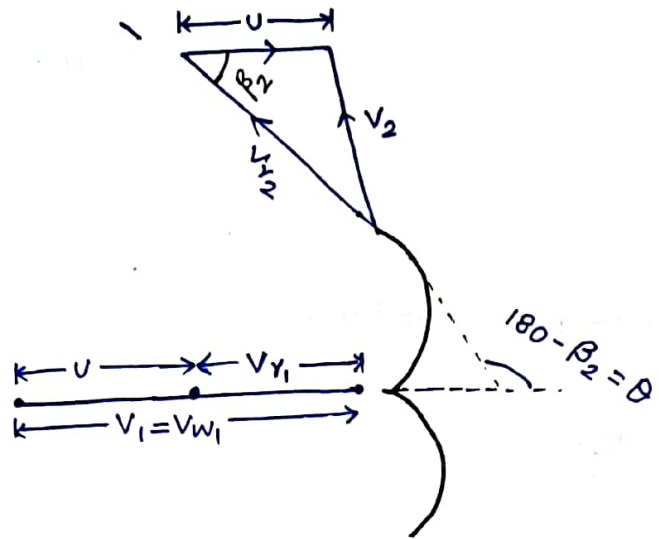
Note: the max. no. of jets that can be used on pelton wheel is 6.  
 If the max. no. jet or nozzles are more than 6, the fluid coming from one jet may interfere with the fluid coming from the other jet. therefore it is restricted to 6.

## Velocity triangle's for pelton wheel:



$\theta =$  jet deflection angle

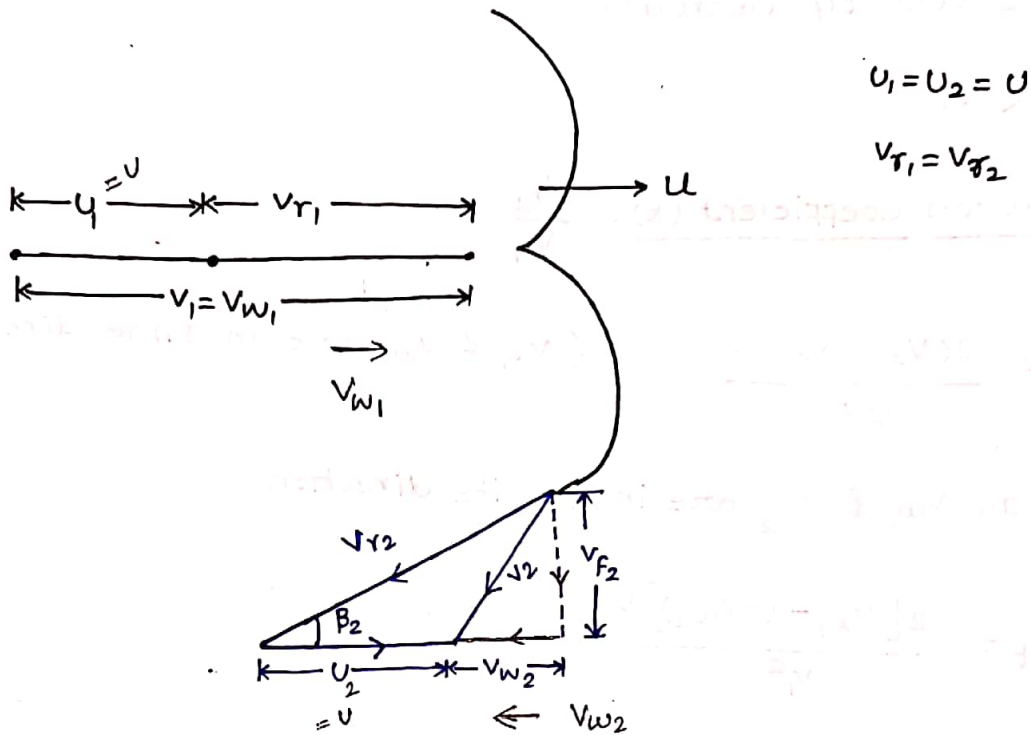
$$\beta_2 = 180 - \theta$$



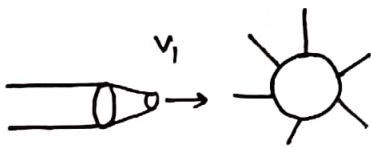
$\beta_2$  = blade angle at exit

- The blades are of double semi-ellipsoidal cups. Double cups are taken to in order to neutralise axial thrust on bearings.
- The deflection angle is generally kept at  $165^\circ$  but it's never kept at  $180^\circ$  because if the deflection angle is  $180^\circ$ , the jet leaving the vane will exert a retarding force on the succeeding vane, therefore the blade angle at the exit is generally kept at  $15^\circ$ . In case of pelton wheel, the inlet velocity triangle is straight line.

# Diagram or blading efficiency ( $\eta_d$ )



Input energy =  $\frac{1}{2} \dot{m} v_1^2$



$W = v_{w1} u_1 - v_{w2} u_2$

$P = \dot{m} W$

$P = \dot{m} [v_{w1} u_1 - v_{w2} u_2]$

$P = \dot{m} [v_{w1} - v_{w2}] u \quad \{u_1 = u_2\}$

$$\eta_b = \frac{\dot{m} [v_{w1} - v_{w2}] u}{\frac{1}{2} \dot{m} v_1^2}$$

$$\eta_b = \frac{2u [v_{w1} - v_{w2}]}{v_1^2}$$



### Condition For max. blading efficiency:

3

In case of an ideal impulse turbine  $V_{r1} = V_{r2}$  but in actual case as the fluid flows through blade passages to overcome friction the relative velocity decreases.

i.e.  $V_{r2} < V_{r1}$

blade friction coefficient (k):  $\frac{V_{r2}}{V_{r1}}$

$$\eta_b = \frac{2(V_{w1} - V_{w2})U}{V_1^2} \quad (V_{w1} \& V_{w2} \text{ are in same direction})$$

as  $V_{w1}$  &  $V_{w2}$  are in opposite direction

$$\eta_b = \frac{2[V_{w1} - (-V_{w2})]U}{V_1^2}$$

$$\eta_b = \frac{2[V_{w1} + V_{w2}]U}{V_1^2} \quad (\text{when } V_{w1} \& V_{w2} \text{ are in opposite dirn})$$

$$V_{w1} = V_1$$

$$V_{w2} = V_{r2} \cos \beta_2 - U$$

$$V_{w2} = kV_{r1} \cos \beta_2 - U$$

$$V_{w2} = k(V_1 - U) \cos \beta_2 - U$$

$$\eta_b = \frac{2[V_1 + k[V_1 - U] \cos \beta_2 - U]U}{V_1^2}$$

$$\eta_b = \frac{2(V_1 - U)[1 + k \cos \beta_2]U}{V_1^2}$$

$$\eta_b = 2 \left[ \frac{U}{V_1} - \frac{U^2}{V_1^2} \right] (1 + K \cos \beta_2)$$

$\frac{U}{V_1}$  = blade speed ratio ( $\rho$ )

$$\eta_b = 2(\rho - \rho^2)(1 + K \cos \beta_2)$$

d For max.  $\eta_b$  ;  $\frac{d\eta_b}{d\rho} = 0$

$K = \frac{V_{r2}}{V_{r1}}$  = blading friction coefficient

$$\frac{d\eta_b}{d\rho} = 2[1 - 2\rho][1 + K \cos \beta_2] = 0$$

$$1 - 2\rho = 0$$

$$\rho = \frac{1}{2}$$

$$\frac{U}{V_1} = \frac{1}{2} \Rightarrow$$

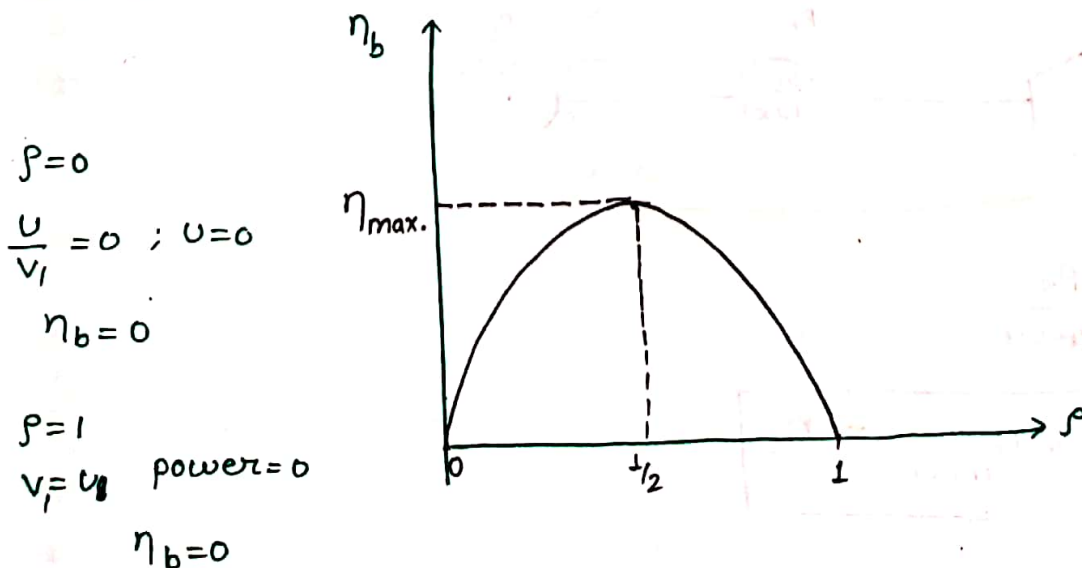
$$U = \frac{V_1}{2} \quad **$$

(Remember-1)

$$\eta_{\max, b} = 2 \left[ \frac{1}{2} - \frac{1}{4} \right] [1 + K \cos \beta_2]$$

$$\eta_{\max, b} \Rightarrow \frac{1 + K \cos \beta_2}{2}$$

-(R2).



⇒ For max. blading efficiency the blade speed must be half of the jet speed. (4)

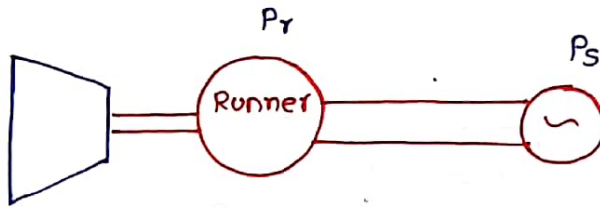
Various efficiencies in hydraulic turbine:

Hydraulic efficiency: ( $\eta_{hyd}$ )

$$\eta_{hyd} = \frac{\text{Runner power } (P_r)}{\omega Q H}$$

$$P_r = \dot{m} [v_{w1} u_1 - v_{w2} u_2]$$

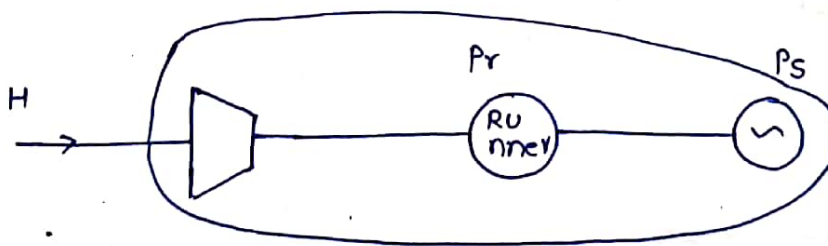
Mechanical efficiency ( $\eta_m$ ):



$$\eta_m = \frac{P_s}{P_r} = \frac{\text{shaft power}}{\text{Runner power}}$$

$$P_r = \dot{m} [v_{w1} u_1 - v_{w2} u_2]$$

Overall efficiency ( $\eta_o$ )



$$\eta_o = \frac{P_s}{\omega Q H}$$

$$\eta_{hyd} \times \eta_{mech} = \eta_o$$

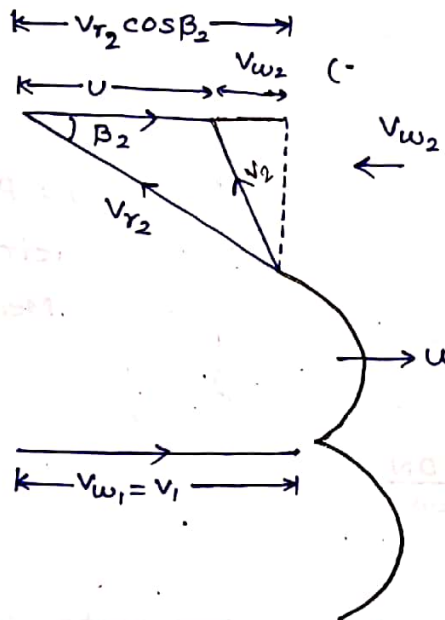
Limitation of pelton wheel:

(1) pelton wheel is efficient when operating under high heads. to generate given power (WQH), the discharge must be high if the head is low. this results in increase in jet diameter. and subsequently increase in runner diameter therefore the system becomes bulky under low heads and hence pelton wheel is suitable for high heads.

Note: if the power developed by with single jet 'P' then the power developed with 'n' number of jets is "np"

Exit velocity triangles

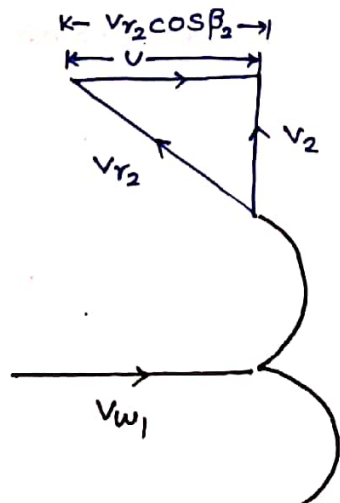
(1)



$$P_r = \dot{m} [V_{w1} + V_{w2}] U$$

$$V_{r2} \cos \beta_2 > U$$

(2)



$$V_{w2} = 0$$

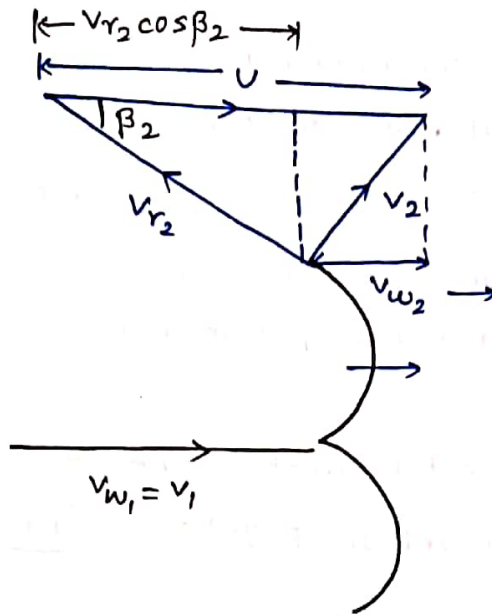
$$P_r = \dot{m} [V_{w1} U]$$

$$\Rightarrow V_{r2} \cos \beta_2 = U$$



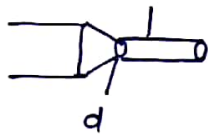
(3)

$v_{r2} \cos \beta_2 < U$

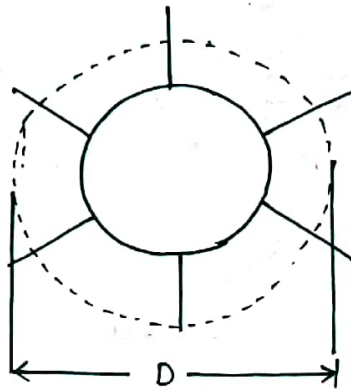


$P_x = \dot{m} [v_{w1} - v_{w2}] U$

Question:



dia. of nozzle = d



$U = \frac{\pi D N}{60}$

D = PCD (pitch circle diameter or Mean dia.)

Q. The mean bucket speed is 15 m/s, the rate of flow of water under a head of 42 m is 1 m<sup>3</sup>/sec. the jet deflection angle is 165°. the coefficient of velocity 0.985 Find

- (1) Runner power
- (2) hydraulic efficiency.

Solution:

Given that  $U = 15 \text{ m/s}$

$H = 42 \text{ m}$

$Q = 1 \text{ m}^3/\text{s}$

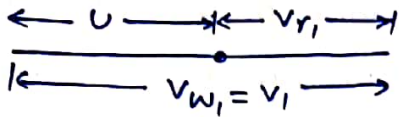
$\beta_2 = 15^\circ$

$$C_v = 0.985$$

$$v_1 = C_v \sqrt{2gH}$$

$$v_1 = 0.985 \times \sqrt{2 \times 9.81 \times 42}$$

$$v_1 = 28.27 \text{ m/s}$$

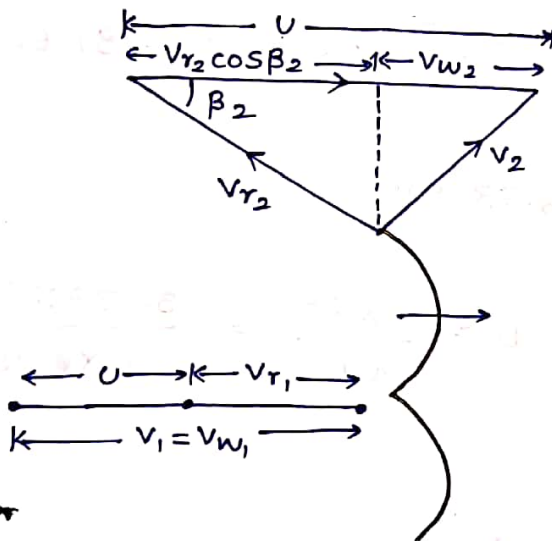


$$v_{r1} = v_1 - U = 28.27 - 15 = 13.27 \text{ m/s}$$

$$v_{r2} = v_{r1} = 13.27 \text{ m/s}$$

$$v_{r2} \cos \beta_2 = 13.27 \cos 15^\circ = 12.81 \text{ m/s}$$

$$v_{r2} \cos \beta_2 < U$$



$$v_{w2} = U - v_{r2} \cos \beta_2$$

$$= 15 - 12.87$$

$$\Rightarrow 2.176 \text{ m/s}$$

$$P_T = \dot{m} [v_{w1} - v_{w2}] U$$

$$P_T \Rightarrow \frac{10^3 [28.27 - 2.176] \times 15}{1000}$$

$$P_T = 391.2 \text{ kW}$$

$$\rho = \frac{\text{mass}}{\text{vol.}}$$

$$\frac{m}{t} = \rho \cdot \frac{\text{vol.}}{t}$$

$$\dot{m} = \rho Q = 10^3 \times 1 = 10^3 \text{ kg/s}$$

$$\text{Hydraulic efficiency } (\eta_{\text{hyd.}}) = \frac{P_r}{\omega Q H}$$

$$\Rightarrow \frac{391.2 \times 10^3}{9810 \times 10^3 \times 42} = 0.949$$

$$\Rightarrow 94.9\%$$

Que: 2. A pelton wheel is to be design for a shaft power of 9560 kW under a head of 350 m, the overall efficiency is 85%. the jet diameter (d) is  $\frac{1}{6}$ th of wheel diameter (D) the speed is 750 rpm,  $c_v$  is 0.985 blade speed ratio ( $U/V_1$ ) 0.45 Find

(1) Wheel diameter

(2) Jet diameter

(3) No. of jets.

$$V_1 = c_v \sqrt{2gH} = 0.985 \times \sqrt{2 \times 9.81 \times 350} = 81.62 \text{ m/s}$$

$$U = 0.45 V_1 = 36.73 \text{ m/s}$$

$$U = \frac{\pi D N}{60}$$

$$D = \frac{U \times 60}{\pi N} = \frac{36.73 \times 60}{\pi \times 750} = 0.935 \text{ m}$$

$$\text{Jet diameter} = D/6$$

$$d = 0.155 \text{ m.}$$

$$\text{S.P} = 9560.$$

$$\eta_{\text{hyd}} = \frac{P_r}{\omega Q H}$$

$$\eta_o = 0.85 = \frac{P_s}{\omega Q H}$$

$$\omega Q H = \frac{9560}{0.85} = 11247.05 \text{ kW.}$$

$$\text{discharge} = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} (0.155)^2 \times 81.62$$

$$\Rightarrow 1.54 \text{ m}^3/\text{sec.}$$

$$Q = \frac{11247.05 \times 10^3}{9810 \times 350} = 3.2756 \text{ m}^3/\text{sec.}$$

$$\text{No. of Jet} = \frac{\text{total discharge}}{\text{discharge from a single nozzle/jet}}$$

$$= \frac{3.2756}{1.54} = 2.127 \approx \underline{\underline{2}} \text{ Nos.}$$

Q.3. Water having a density of  $1000 \text{ kg/m}^3$  comes out from nozzle with a velocity of  $10 \text{ m/s}$  [ $v_1$ ] and jet strikes the bucket mounted on pelton wheel the wheel rotates at  $10 \text{ rad/sec}$ . the wheel diameter is  $1 \text{ m}$  [ $D$ ] the deflection angle ( $\theta$ ) is  $120^\circ$  if the mass flow rate is  $1 \text{ kg/sec}$ . then find the torque exerted on the wheel.

Solution:

$$\rho = 1000 \text{ kg/m}^3$$

$$v_1 = 10 \text{ m/s}$$

$$\omega = 10 \text{ rad/sec}$$

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{\omega \times 60}{2\pi} = 95.49 \text{ rpm}$$

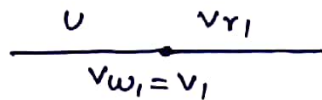
$$D = 1 \text{ m}$$

$$\theta = 120^\circ \quad \beta_2 = 60^\circ$$

$$\dot{m} = 1 \text{ kg/s}$$

$$\tau = ?$$

$$v_1 = 10 \text{ m/s}$$



$$P = \dot{m} [v_{w1}]$$

$$U = \frac{\pi D N}{60} = \frac{\pi \times 1 \times 95.49}{60} = 5 \text{ m/s}$$

$$P = \text{Torque} \times \omega$$

$$v_{r1} = v_1 - U = 10 - 5 = 5 \text{ m/s}$$

$$P_r = \dot{m} [v_{w1} + v_{w2}] U$$

$$v_{r1} = v_{r2} \Rightarrow 5 \text{ m/s}$$

$$v_{r2} \cos \beta_2 = 5 \times \cos 60^\circ = 2.5 < U$$

$$v_{w2} = U - v_{r2} \cos \beta_2 = U - 2.5 = 2.5 \text{ m/s}$$

$$\underline{v_{r2} \cos \beta_2 < U} \quad (\text{3rd case triangle})$$

$$P_r = 1 [10 + 2.5] \times 5$$

$$P_r = 37.5 \text{ W}$$



$$\text{Power} = F \times V$$

$$F = \frac{75}{10} = 7.5 \text{ N}$$

$$\text{Torque} = E \times r = 7.5 \times 0.5 = 3.75 \text{ N-m. Ans.}$$

$$\text{Power} = \text{Torque} \times \omega$$

$$\text{Torque} = \frac{37.5}{10} = 3.75 \text{ N-m. Ans.}$$

Que:3. A single jet pelton wheel is required to drive a generator of 10 MW power. the available head is 760 m, Mechanical efficiency is 0.95, hydraulic efficiency is 0.87, coefficient of velocity is 0.97 the bucket speed is 0.46 times the jet speed. the relative velocity of water at inlet leaving the bucket is 0.85 times the relative velocity at inlet Find.

(1) Discharge.

(2) tangential force exerted on the wheel.

(3) Best synchronous speed for generation at 50 Hz frequency.

& the corresponding wheel diameter if the ratio of wheel dia. to jet diameter is not to be less than 10.

Solution:

Given that

$$P_s = 10 \text{ MW}$$

$$H = 760 \text{ m}$$

$$\eta_m = 0.95 ; \eta_{hyd} = 0.87, \quad C_v = 0.97$$

$$U = 0.46 V_1$$

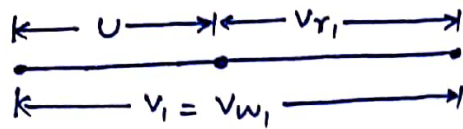
$$V_{r2} = 0.85 V_{r1}$$

$$\frac{D}{d} \geq 10$$

$$f = 50 \text{ Hz.}$$

$$V_1 = C_v \sqrt{2gH} \Rightarrow 0.97 \times \sqrt{2 \times 9.81 \times 760} = 118.45 \text{ m/s}$$

$$U = 0.46 V_1 = 0.46 \times 118.45 \Rightarrow 54.48 \text{ m/s}$$



$$v_{r2} = v_1 - U = 118.45 - 54.48 \Rightarrow 63.97 \text{ m/s}$$

$$v_{r2} = 0.85 v_{r1} = 0.85 \times 63.97 = 54.37 \text{ m/s.}$$

(1)

$$\eta_o = \frac{P_s}{WQH}$$

$$\eta_o = \eta_m \times \eta_h$$

↓

$$= 0.95 \times 0.87 = \frac{10 \times 10^6}{9810 \times Q \times 760}$$

$$Q = 1.62283 \text{ m}^3/\text{sec.}$$

$$Q = \frac{\pi}{4} d^2 v_1$$

$$= 1.622 = \frac{\pi}{4} (d^2) \times 118.45$$

$$d = 0.13207 \text{ m}$$

$$\frac{D}{d} \geq 10 \Rightarrow \frac{D}{d} = 10$$

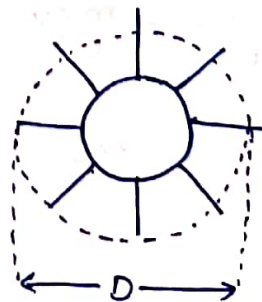
$$D = 10d$$

$$D = 1.3207 \text{ m}$$

$$U = \frac{\pi D N}{60}$$

$$N = \frac{U \times 60}{\pi D} = \frac{54.48 \times 60}{\pi \times 1.3207}$$

$$N = 788.25 \text{ rpm.}$$



$$N = \frac{120f}{P} \rightarrow \text{always even.}$$

$$788.25 = \frac{120 \times 50}{P}$$

$$P = 7.6 = \underline{P = 8}$$

$$\text{Synchronous Speed (N)} = \frac{120 \times 50}{8} = 750 \text{ rpm.}$$

$$U = \frac{\pi D N}{60}$$

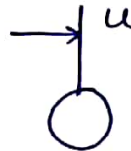
$$= \frac{54.48 \times 60}{\pi \times 750} = D \Rightarrow D = 1.387 \text{ m.}$$

$$\frac{D}{d} = \frac{1.387}{0.132} = 10.51 > 10$$

(3) tangential force:

$$\eta_m = \frac{P_s}{P_r} \quad P_r = \frac{10}{0.95} = 10.51 \text{ MW}$$

$$P_r = F_t \times u$$



$$F_t = \frac{10.51 \times 10^6}{54.48}$$

$$F_t = 193.214 \text{ kN}$$

II<sup>nd</sup> Method

$$F = m \left( \frac{v-u}{t} \right) = \dot{m} (v-u)$$

$$F_t = \dot{m} (v_{w1} - v_{w2})$$

## Reactions turbine

Francis turbine:

the turbine

In reaction turbines the fluid enters at high pressure & high velocity, as it flows through runner vanes, pressure decreases and relative velocity increases because of this rotation is possible therefore in reaction turbine as the fluid flows through runner vanes both pressure and k.E decreases and hence power is obtained.

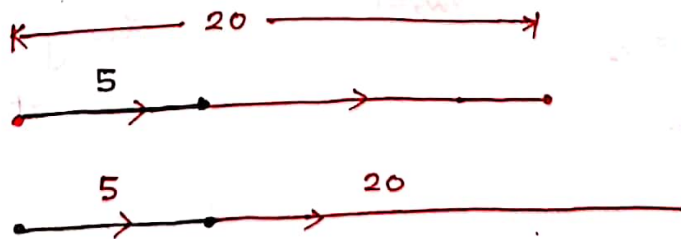
[In reaction turbine ( $V_{r2} > V_{r1}$ )

"It is a mixed flow, medium head, medium discharge, medium specific speed reaction turbine.

$$V_{r2} > V_{r1}$$

In Francis turbine the fluid from penstocks enter the spiral casing (logarithmic), from casing it enters stay vanes which resist load and transmit to the foundation. From stay vanes the fluid enters guide vanes, the purpose of guide vanes is to allow the fluid on the runner blades at correct angle, from guide vanes the fluid enters the runner vanes from runner exit, the fluid then enters draft tube, **Draft tube is used in reaction turbines.**

Velocity triangles for Francis turbine:



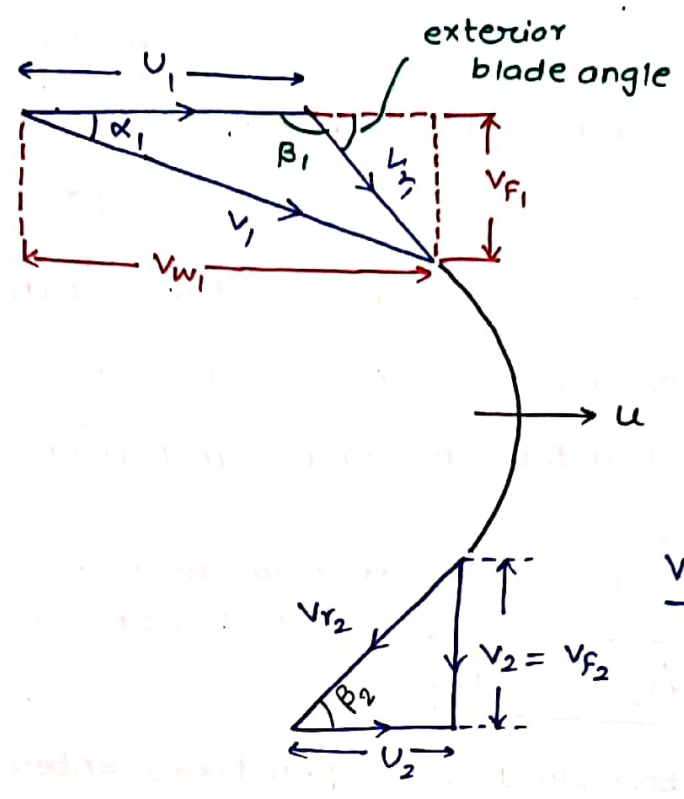
$$\text{Relative} = 5 + 20 = 25$$



$D_1 > D_2$   
 $U_1 > U_2$   
 $V_{r2} > V_{r1}$  ] \*

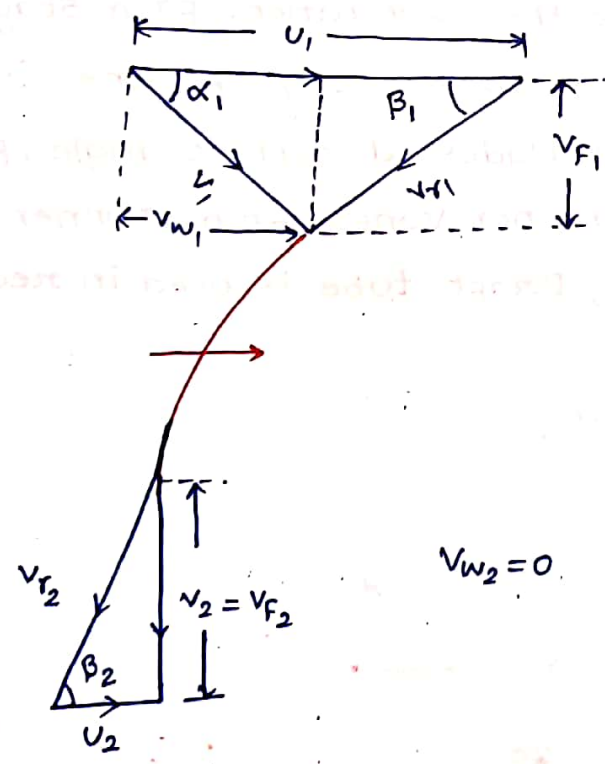
$\beta_1 =$  interior blade angle.

(1)  
 $\beta_1 > 90$



$V_{w2} = 0$  (b/c axial exit)

(2)  
 $\beta_1 < 90$



$\beta_1 \& \beta_2 \rightarrow$  Blade angles at inlet & outlet respectively

$\alpha_1 =$  guide vane angle at inlet.

$V_{w2} = 0$

$$P_r \text{ (runner power)} = \dot{m} [v_{w1} u_1 - v_{w2} u_2]$$

$$P_r = \dot{m} [v_{w1} u_1]$$

$$\therefore v_{w2} = 0$$

axial exit

$$\eta_h = \frac{P_r}{\rho g Q H}$$

$$\eta_h = \frac{\dot{m} v_{w1} u_1}{\rho g Q H}$$

$$\rho = \frac{m}{\text{vol.}}$$

$$\frac{m}{t} = \rho \times \frac{\text{vol.}}{t}$$

$$\dot{m} = \rho Q$$

$$\boxed{\eta_h = \frac{v_{w1} u_1}{g H}}$$

(very important eq. in Francis turbine)

$$\eta_{\text{mech.}} = \frac{P_s}{P_r}$$

$$\eta_o = \frac{P_s}{\omega Q H}$$

$$\boxed{\eta_o = \eta_m \times \eta_h}$$

Speed ratio: ( $K_u$ )

$$K_u = \frac{u_1}{\sqrt{2gH}}$$

$$K_u = 0.70 \text{ to } 0.85$$

Flow ratio ( $\psi$ )

$$\psi = \frac{v_{F1}}{\sqrt{2gH}}$$

$$\psi = 0.15 \text{ to } 0.35$$

## Discharge of Francis turbine:

blades in  
No. of ~~stator~~  
Runner  $\rightarrow$  16 to 24.

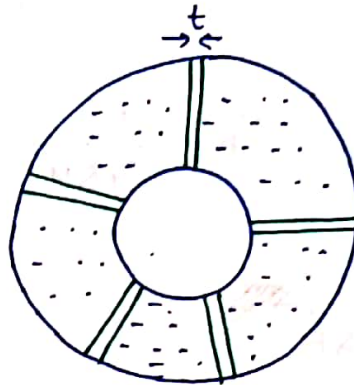
10

Area.

$$A = (\pi D - zt) B$$

$$Q = A_1 V_{F1} = A_2 V_{F2}$$

$$Q = (\pi D - zt) B V_F$$



$$Q = k \pi D B V_F$$

$Z =$  No. of vanes

$t =$  thickness of vanes

$k =$  Factor which accounts for thickness of vanes as the flow is steady  $\pi D_1 B_1 V_{F1} = \pi D_2 B_2 V_{F2}$

$B_1$  &  $B_2$  are width of runner at inlet & outlet respectively.

Q. An inward flow reaction turbine works under a head of 110 m. the outer diameter 1.5m and the inner diameter is 1m. the width of the runner is constant throughout is equal to 150 mm. outlet blade angle is  $15^\circ$ ;  $\eta_h = 90\%$ , the axial discharge velocity at the exit is 6 m/s. find speed of the turbine in rpm., blade angles and runner power.

Given that

$$H = 110 \text{ m}$$

$$D_1 = 1.5 \text{ m}$$

$$D_2 = 1 \text{ m}$$

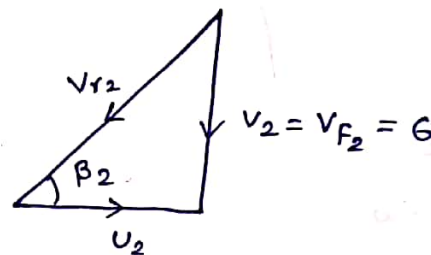
$$B_1 = B_2 = 0.15 \text{ m}$$

$$\beta_2 = 15^\circ$$

$$\eta_h = 90\%$$

$$V_2 = V_{F2} = 6 \text{ m/s}$$

outlet velocity  $\Delta$  (b/c Max. information given for that)



$$\tan \beta_2 = \frac{V_2}{U_2}$$

$$U_2 = \frac{6}{\tan 15}$$

$$U_2 = 22.39 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60}$$

$$22.39 = \frac{\pi \times 1 \times N}{60} \Rightarrow N = 427.87 \text{ rpm.}$$

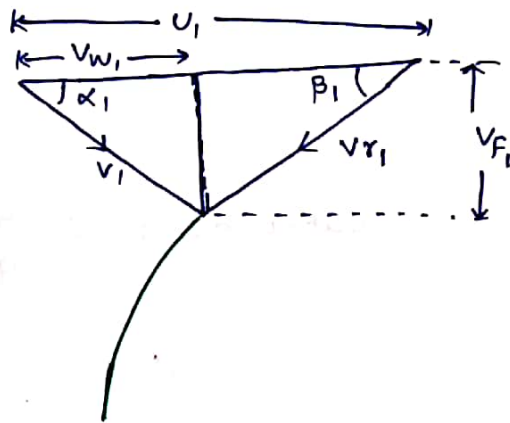
$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.5 \times 427.87}{60} = 33.59 \text{ m/s}$$



$$\eta_h = \frac{v_{w1} u_1}{gH}$$

$$\Rightarrow 0.9 = \frac{v_{w1} \times 33.59}{9.81 \times 110} \quad ; \quad v_{w1} = 28.9 \text{ m/s.}$$

$v_{w1} < u_1$  so 2<sup>nd</sup> velocity triangle.



$$\cancel{D_1} \beta_1 v_{f1} = \cancel{D_2} \beta_2 v_{f2}$$

$$D_1 v_{f1} = D_2 v_{f2}$$

$$v_{f1} = \frac{6}{1.5} = 4 \text{ m/s}$$

$$\tan \beta_1 = \frac{v_{f1}}{u_1 - v_{w1}}$$

$$\tan \beta_1 = \frac{4}{33.59 - 28.9} \Rightarrow \boxed{\beta_1 = 40.52^\circ}$$

$$\tan \alpha_1 = \frac{v_{f1}}{v_{w1}} = \frac{4}{28.9}$$

$$\boxed{\alpha_1 = 7.88^\circ}$$

$$\frac{P_r}{\omega Q H} = \eta_h$$

$$\eta_h \times \omega Q H = P_r \Rightarrow 0.9 \times 9810 \times \pi \times 1.5 \times 0.15 \times 4 \times 110 = 2.745 \text{ MW.}$$

$$\text{or } P_r = \dot{m} v_{w1} u_1 = \rho Q v_{w1} u_1 = \rho \times \pi D_1 \beta_1 v_{f1} \times v_{w1} u_1$$

Que: 2. Estimate the main dimensions of a Francis turbine to suit the following conditions.  
 Head = 100 m, power = 2.5 MW, speed ( $\omega$ ) = 500 rpm,  $\eta_h = 0.9$ ,  $\eta_o = 0.85$   
 Flow ratio ( $\psi$ ) = 0.15, the ratio of width of the wheel to the diameter at inlet ( $B_1/D_1$ ) = 0.1, the outer diameter of the runner is twice the inner dia. ( $D_1 = 2D_2$ ), the flow velocity is constant throughout.

Solution:

Given that

$$H = 100 \text{ m}$$

$$P_s = 2.5 \text{ MW}$$

$$N = 500 \text{ rpm}; \quad \eta_h = 0.9, \quad \eta_o = 0.85, \quad \psi = 0.15, \quad B_1/D_1 = 0.1$$

$$v_{f1} = v_{f2}$$

$$\pi D_1 B_1 v_{f1} = \pi D_2 B_2 v_{f2}$$

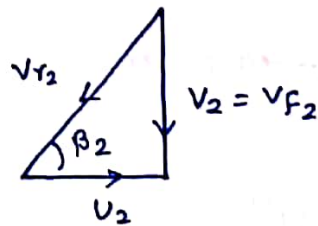
$$\Rightarrow 2D_2 \cdot D_1 \times 0.1 D_1 = \frac{D_1}{2} \times B_2$$

$\Rightarrow$

$$v_{f1} = \psi \sqrt{2gH} = 0.15 \sqrt{2 \times 9.81 \times 100} = 6.64 \text{ m/s}$$

$$\eta_o = \frac{\text{S.P.}}{\omega Q H}$$

$$= 0.85 = \frac{2.5 \times 10^6 \text{ W}}{9810 \times Q \times 100}$$



$$Q = 2.9981 \text{ m}^3/\text{s}$$

$$Q = \pi D_1 B_1 v_{f1}$$

$$= 2.99 = \pi \times D_1 \times 0.1 D_1 \times 6.64$$

$$D_1 = 1.1988 \text{ m.}$$

$$B_1 = 0.1 D_1$$

$$B_1 = 0.1198 \text{ m}$$

$$D_1 = 2D_2$$

$$\Rightarrow D_2 = \frac{D_1}{2} = 0.599 \text{ m}$$

$$Q = \pi D_2 B_2 V_{f2}$$

$$= 2.99 = \pi \times 0.599 \times B_2 \times 6.64$$

$$B_2 = 0.239 \text{ m}$$

Blade angles:

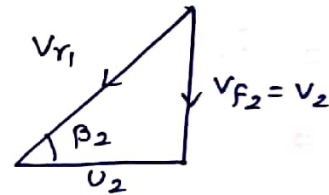
$$U_1 = \frac{\pi D_1 N_1}{60} = \frac{\pi \times 1.198 \times 500}{60} = 31.38 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N_2}{60} = \frac{\pi \times 0.599 \times 500}{60} = 15.68 \text{ m/s}$$

$$\tan \beta_2 = \frac{V_2 = V_{f2}}{U_2}$$

$$\tan \beta_2 = \frac{6.64}{15.68}$$

$$\beta_2 = 22.94^\circ$$



$$P_r = \dot{m} v_{w1} U_1$$

$$\frac{P_s}{P_r} = \eta_m$$

$$\eta_h = \frac{v_{w1} U_1}{gH}$$

$$= 0.9 = \frac{v_{w1} \times 31.38}{9.81 \times 100}$$

$$v_{w1} = 28.13 \text{ m/s}$$

$$v_{w1} < U_1$$

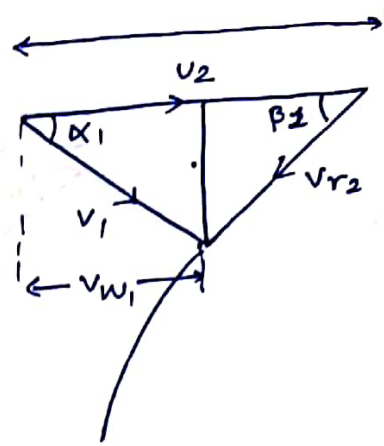
$$\tan \beta_2 = \frac{V_{f2}}{U_2 - V_{w1}}$$

$$\tan \beta_2 = \frac{6.64}{31.38 - 28.13}$$

$$\beta_2 = 63.96^\circ$$

$$\tan \alpha_1 = \frac{V_{f1}}{V_{w1}} = \frac{6.64}{28.13}$$

$$\alpha_1 = 13.28^\circ$$



Q. An inward flow reaction turbine of Francis type has a runner diameter of 60 cm and width 5 cm at the outer ring, the inner diameter is 39 cm the blade inlet angle is  $85^\circ$  (interior) and the outlet blade angle is  $14^\circ$ , the velocity of flow is uniform throughout and 8% of circumferential area is occupied by blade thickness, Head = 54 m,  $\eta_h = 0.88$ ,  $\eta_o = 0.81$  for axial exit calculate speed in rpm and shaft power.

Solution:

Given that

$$D_1 = 0.6 \text{ m}$$

$$B_1 = 0.05 \text{ m}$$

$$D_2 = 0.39 \text{ m}$$

$$\beta_1 = 85^\circ \text{ (interior)}$$

$$\beta_2 = 14^\circ$$

$$V_{f1} = V_{f2}$$

$$k = 0.92$$

$$H = 54 \text{ m}$$

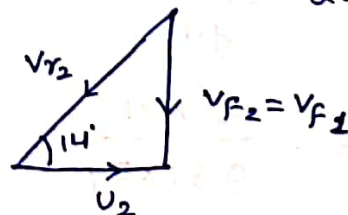
$$\eta_h = 0.88 \quad \eta_o = 0.81$$

$$U_1 = \frac{\pi D_1 N_1}{60}$$

$$U_2 = \frac{\pi D_2 N_2}{60}$$

$$= \frac{19.58 \times 60}{\pi \times 0.6}$$

$$623 \text{ rpm}$$



$$Q = A \times V_{f1}$$

$$V_{f2} = \psi \sqrt{2gH}$$

$$\eta_h = \frac{V_{w1} U_1}{gH}$$

$$Q = k \pi D_1 B_1 V_{f1} = \pi \times 0.6$$

$$\eta_o =$$

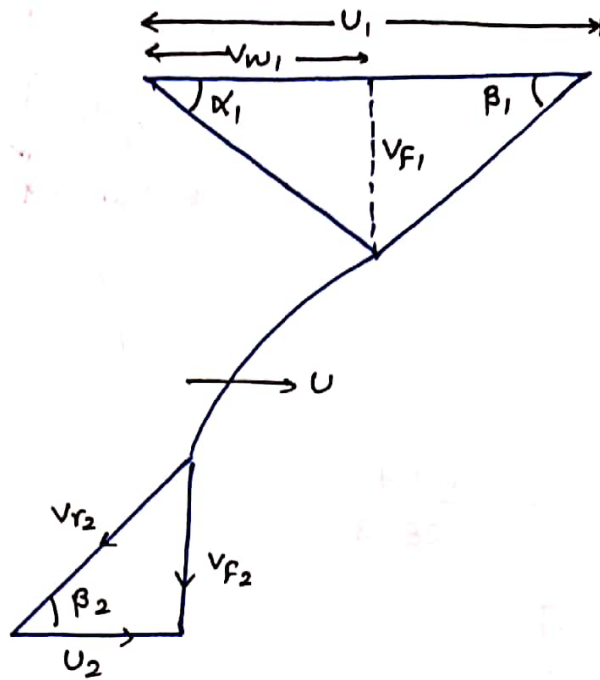
$$V_{f1} = 0.15 \sqrt{2 \times 9.8 \times 54} = 4.88$$

$$\tan 14^\circ = \frac{V_{f1}}{U_2}$$



$$\beta_1 < 90^\circ$$

so we draw this



(13)

$$\tan \beta_2 = \frac{V_{f2}}{U_2}$$

$$V_{f2} = U_2 \tan 14^\circ \Rightarrow 0.249 U_2 \quad \text{--- (1)}$$

$$\tan \beta_1 = \frac{V_{f1}}{U_1 - V_{w1}}$$

$$V_{f1} = 11.43 (U_1 - V_{w1}) \quad \text{--- (2)}$$

From (eq 1) & (2)  $V_{f1} = V_{f2}$

$$= 0.249 U_2 = 11.43 (U_1 - V_{w1})$$

$$\Rightarrow \eta_n = \frac{V_{w1} U_1}{gH}$$

$$0.88 = \frac{V_{w1} \times U_1}{9.81 \times 54} \Rightarrow V_{w1} U_1 = 466.17 \quad \left[ V_{w1} = \frac{466.17}{U_1} \right]$$

$$\Rightarrow \frac{0.249 \cdot \pi (0.6)^3 N}{60} = 11.43 \left[ \frac{\pi \times 0.6 N}{60} - \frac{466.17 \times 60}{\pi (0.6) N} \right]$$

$$\Rightarrow 5.82 \times 10^{-3} \text{ N} = 11.43 \left[ \frac{0.0204 \text{ N}}{0.0314 \text{ N}} - \frac{14838.65}{\text{N}} \right]$$

$$= 7.82 \times 10^{-3} \text{ N} = 11.43 \times 0.0204 \text{ N} - \frac{169605.7}{\text{N}}$$

$$N = 692.8 \text{ rpm.}$$

$$\eta_0 = \frac{P_s}{\omega Q H} = Q = k \pi D_1 B_1 V_{f1}$$

$$U_2 = \frac{\pi D_2 N}{60} = 14.15 \text{ m/s}$$

$$Q = 0.92 \times \pi \times 0.6 \times 0.05 \times 3.52$$

$$= 0.305 \text{ m}^3/\text{sec}$$

$$U_2 \times \tan \beta_2 = V_{f2}$$

$$V_{f1} = V_{f2} = 3.5247 \text{ m/s}$$

$$\eta_0 = \frac{P_s}{\omega Q H}$$

$$P_s = 131.3 \text{ kW.}$$

Assignment-

A Francis turbine has to be design to develop 367.5 kW. Under a head of 70 m, while running at 750 rpm the ratio of width of the runner to the diameter at inlet is 0.1 the inner diameter is  $\frac{1}{2}$  the outer diameter the flow ratio is 0.15,  $\eta_h = 0.95$   
 $\eta_{\text{Mech.}} = 0.84$ , 4% of circumferential area is occupied by thickness of vanes, the velocity of flow is constant and fluid leaves axially at the exit, calculate

[1] Diameter of the wheel ( $D_1$  &  $D_2$ ) [ $D_1 = 0.6325 \text{ m}$  ;  $D_2 = 0.31625 \text{ m}$ ]

[2] Quantity of water supplied. [ $0.67063 \text{ m}^3/\text{sec}$ ]

[3] Guide vane angle at inlet ( $\beta_1 = 104.35^\circ$  (interior))

[4] Runner vane angle at inlet and exit. ( $\alpha_1 = 11.95^\circ$ )

### Specific Speed : ( $N_s$ )

It is the speed of Geometrically similar turbine working under unit head (1m) and developing unit power (1kW).

$$Q = K \pi D B V_f$$

$$Q \propto D B V_f \quad \text{--- (I)}$$

$$\psi = \frac{V_{f1}}{\sqrt{2gH}} \quad V_{f1} = \psi \sqrt{2gH} \quad \Rightarrow \quad V_{f1} \propto \sqrt{H} \quad \text{--- (1)}$$

$$K_u = \frac{U}{\sqrt{2gH}} \quad U = K_u \sqrt{2gH} \quad \Rightarrow \quad \frac{\pi D N}{60} = K_u \sqrt{2gH}$$
$$D \propto \frac{\sqrt{H}}{N} \quad \text{--- (2)}$$

$$D/B = 0.1$$

$$D \propto B$$

$$B \propto \frac{\sqrt{H}}{N} \quad \text{--- (3)}$$

Put the value from (1) (2) (3) into (I)

$$Q \propto \frac{\sqrt{H}}{N} \cdot \frac{\sqrt{H}}{N} \cdot \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2} \quad \text{--- (II)} \quad Q = C \frac{H^{3/2}}{N^2}$$

if  $H = 1 \text{ m}$   $Q = 1 \text{ m}^3/\text{s}$  then  $N = N_s$

$$\Rightarrow 1 = \frac{C (1)}{N_s^2} \quad \Rightarrow \quad N_s^2 = C$$

$$\Rightarrow Q = N_s^2 \cdot \frac{H^{3/2}}{N^2}$$

$$N_s^2 = \frac{N^2 Q}{H^{3/2}}$$

$$N_s = \frac{N \sqrt{Q}}{H^{3/4}}$$

Imp.

specific speed for pump.

From equation (II)

$$Q \propto \frac{H^{3/2}}{N^2}$$

$$P = \omega Q H$$

$$P \propto Q H$$

$$P \propto \frac{H^{3/2} \cdot H}{N^2}$$

$$P \propto \frac{H^{5/2}}{N^2} \Rightarrow P = C_1 \frac{H^{5/2}}{N^2}$$

$$N = N_s \text{ if } H = 1 \text{ m ; } P = 1 \text{ kW}$$

$$\Rightarrow 1 = \frac{C_1 (1)}{N_s^2}$$

$$C_1 = N_s^2$$

$$\Rightarrow P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$$

Imp.

$$\Rightarrow N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

Specific speed for turbine

(Dimensional specific speed)

P = shaft power in kW

H = head in meter



Note: All Geometrically Similar turbines have Same Specific Speed.

Specific Speed of Various Turbines:

S. No.	Type of turbine	Specific speed (Dimensional)
1	Pelton Wheel with Single Jet	10-30
2	Pelton Wheel with Multi Jet	30-60
3	Francis turbine	60-300
4	Kaplan turbine	300-600
5	propeller turbine	600-1000

Dimensionless  
Specific Speed

0.03 - 0.3

0.3 - 2

2 - 5

Similitude & Modelling of turbine:

From dimensional analysis three number i.e. Head Number, discharge number and power number can be obtained as-

$$\text{Head Number} = \frac{gH}{N^2 D^2} \quad (\text{all three dimensionless})$$

$$\text{Discharge Number} = \frac{Q}{ND^3}$$

$$\text{power Number} = \frac{P}{\rho N^3 D^5}$$

Unit Quantities: unit quantities are those quantities when the same turbine is allow to operate under unit head (1m)

Turbine

$$H \longrightarrow 1 \text{ m}$$

$$D \longrightarrow D \text{ (same turbine)}$$

$$N \longrightarrow N_u$$

$$Q \longrightarrow Q_u$$

$$P \longrightarrow P_u$$

unit speed ( $N_u$ )

$$\frac{gH}{N^2 D^2} = \frac{g(1)}{N_u^2 D^2}$$

$$N_u = \frac{N}{\sqrt{H}}$$

unit discharge ( $Q_u$ ):

$$\frac{Q}{N D^3} = \frac{Q_u}{N_u D^3}$$

$$Q_u = Q \left( \frac{N_u}{N} \right) \Rightarrow Q_u = \frac{Q}{\sqrt{H}}$$

Unit power ( $P_u$ ):

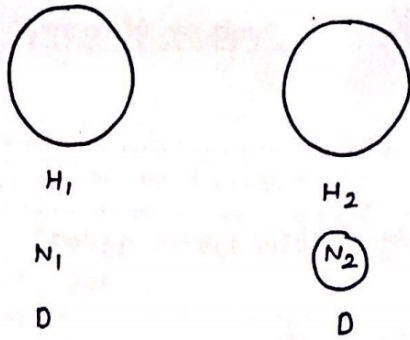
$$\frac{P}{N^3 D^5} = \frac{P_u}{N_u^3 D^5}$$

$$P_u = P \left( \frac{N_u}{N} \right)^3$$

$$P_u = \frac{P}{H^{3/2}}$$



Note:



$$\left( \frac{gH}{N^2 D^2} \right)_1 = \left( \frac{gH}{N^2 D^2} \right)_2$$

$$\frac{H_1}{N_1^2} = \frac{H_2}{N_2^2}$$

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} = N_{U1} = N_{U2}$$

When the same turbine is operating under different conditions, unit quantities are equated. therefore

$N \propto H^{1/2}$
$Q \propto H^{3/2}$
$P \propto H^{3/2}$

Q. A turbine develops 8000 kW when running at 1000 rpm. The head on the turbine is 30 m, if the head is reduced to 18 m. Determine the speed and power.

Solution. turbine is same so we equate unit quantities.

$$\begin{aligned} \frac{N_1}{\sqrt{H_1}} &= \frac{N_2}{\sqrt{H_2}} \\ &= \frac{1000}{\sqrt{30}} = \frac{N_2}{\sqrt{18}} \end{aligned} \quad N_2 = 774.6 \text{ rpm}$$

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\Rightarrow \frac{8000}{(30)^{3/2}} = \frac{P_2}{(18)^{3/2}} \quad \Rightarrow P_2 = 3718 \text{ kW}$$

Q. At a hydroelectric power plant the available head is 24.5m and the Flowrate is 10.1 m<sup>3</sup>/sec, if the speed is 4 rps (240 rpm) and  $\eta_o$  is 90%. based on specific speed what type of turbine is this.

Solution.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$N \rightarrow 4 \text{ rps} = 240 \text{ rpm}$$

P = shaft power

$$\eta_o = \frac{P_s}{\omega Q H}$$

$$P_s = 0.90 \times 9810 \times 10.1 \times 24.5 \\ = 2184.736 \text{ kW}$$

$$N_s = \frac{240 \sqrt{2184.736}}{(24.5)^{5/4}} = \frac{11217.87}{54.50} = 205.83$$

So  $N_s$  lies b/w 60 & 300, it is Francis turbine.

Q. A hydraulic turbine develops 100 kW power under a head of 40m. if the head is reduced to 20m then the power developed in kW.

Ans:

$$P_1 = 100 \text{ kW}$$

$$H_1 = 40 \text{ m}$$

$$H_2 = 20 \text{ m}$$

$$= \frac{100}{(40)^{3/2}} = \frac{P_2}{(20)^{3/2}} \quad P_2 = 35.35 \text{ kW}$$



Q. A large hydraulic turbine is to generate 300 kW. and 100 rpm Under a head of 40 m. For initial testing of  $1/4$  scale Model operating under a head of 10 m, power develop by the Model - in kW

Solution:

$$\left( \frac{P}{\rho N^3 D^5} \right)_m = \left( \frac{P}{\rho N^3 D^5} \right)_p \quad \left| \quad N_s = \left( \frac{N\sqrt{P}}{H^{5/4}} \right)_m = \left( \frac{N\sqrt{P}}{H^{5/4}} \right)_p$$

$$\left[ \frac{P}{\left( \frac{306.1862}{\sqrt{P}} \right)^3 \cdot \frac{D^5}{45}} \right]_m = \left[ \frac{300}{100^3 \times D^5} \right]_p \quad \left| \quad D_m = D/4 \right.$$

$$\Rightarrow \frac{P_m \times P_m^{3/2}}{28032.18} = \frac{300}{100^3} \quad \left| \quad \Rightarrow \frac{100}{(40)^{5/4}} \sqrt{300} = 17.2181 \right.$$

$$\Rightarrow N_m \sqrt{P_m} = \frac{306.1862}{\sqrt{P}} \quad \left| \quad N = \frac{306.1862}{\sqrt{P}} \right.$$

$$P_m^{5/2} = 8.409$$

$$P_m = 2.34 \text{ kW}$$

Tomorrow:

- Kaplan
- cavitation
- principle of draft tube.

Q. A model of a hydraulic turbine is tested at a head of  $1/4$  of prototype head, the diameter of the model of  $1/2$  of that of prototype if  $N$  is the speed of full scale turbine than what is the speed of prototype.

Sol.  $H_m = \text{model} = 1, \quad \text{prototype} = 2$

$$h_1 = \frac{h_2}{4}$$

$$D_1 = \frac{D_2}{2}$$

$$\left( \frac{gH}{N^2 D^2} \right)_1 = \left( \frac{gH}{N^2 D^2} \right)_2$$

$$= \frac{H_2}{N_1^2 \frac{D_2^2}{4}} = \frac{H_2}{N^2 D_2^2} \quad \left| \quad N_1^2 = N^2 \right.$$

$$\boxed{N_1 = N}$$

Q. A radial flow hydraulic turbine is required to be design to produce 20 MW power under a head of 16 m at a speed of 90 rpm a geometrically similar model with an output of 30 kW and a head of 4 m is to be tested under similar conditions at what speed the model must be run and what is the required diameter ratio between model & prototype & what is the volume flow rate through the model if the ~~max~~ efficiency is 90%.

Solution:

Turbine-1	Model-2
$P_1 = 20 \text{ MW}$	$P_2 = 30 \text{ kW}$
$H_1 = 16 \text{ m}$	$H_2 = 4 \text{ m}$
$N_1 = 90 \text{ rpm}$	$\frac{D_m}{D_p} = \frac{D_2}{D_1} = ?$

$$Q_m = Q_2 = ? , \eta_h = 90\%$$

$$(1) \left( \frac{P}{\rho N^3 D^5} \right)_1 = \left( \frac{P}{\rho N^3 D^5} \right)_2$$

$$(ii) \left( \frac{gH}{N^2 D^2} \right)_1 = \left( \frac{gH}{N^2 D^2} \right)_2$$

⇒ Geometrically similar Specific speed is same.

$$\left( \frac{N\sqrt{P}}{H^{5/4}} \right)_1 = \left( \frac{N\sqrt{P}}{H^{5/4}} \right)_2$$

$$= \frac{90\sqrt{20 \times 10^3}}{(16)^{5/4}} = \frac{N_2\sqrt{30}}{(4)^{5/4}} \Rightarrow N_2 = 410.79 \text{ rpm}$$

$$(2) \eta_o = \frac{P_m}{\omega Q H}$$

$$= Q = \frac{30 \times 10^3}{9810 \times 4 \times 0.9} = 0.849 \text{ m}^3/\text{s}$$



Q. A Francis turbine of 3m diameter develops 6750 kW at 300 rpm under a head of 45 m, a similar Model of scale ratio  $1/8$  is to be tested at a head of 9 m, estimate the diameter, speed, discharge and power developed by the model, assume overall efficiency of both model & prototype to be 82%.

Solution:

Turbine-1,

Model-2

$$D_1 = 3\text{ m}$$

$$D_2 = D_1/8$$

$$P_1 = 6750\text{ kW}$$

$$H_2 = 9\text{ m}$$

$$N_1 = 300\text{ rpm}$$

$$H_1 = 45\text{ m}$$

$$(1) \left( \frac{\rho g H}{N^2 D^2} \right)_1 = \left( \frac{\rho g H}{N^2 D^2} \right)_2$$

$$\frac{H_1}{N_1^2 D_1^2} = \frac{H_2}{N_2^2 D_2^2}$$

$$\Rightarrow \frac{45}{300^2 \times 3^2} = \frac{9}{N_2^2 \cdot \frac{D_1^2}{8^2}} = \frac{45}{300^2} = \frac{72 \times 8}{N_2^2} = 1073.31\text{ rpm.}$$

$$(ii) D_2 = \frac{D_1}{8} = 0.375\text{ m}$$

$$(iii) \left( \frac{P}{\rho N^3 D^5} \right)_1 = \left( \frac{P}{\rho N^3 D^5} \right)_2$$

$$\Rightarrow \frac{6750}{300^3 \times 3^5} = \frac{P}{1073.31^3 \times 0.375^5} \Rightarrow \underline{P_2 = 9.43\text{ kW.}}$$

$$(iv) \eta_o = \frac{9.43 \times 10^3}{\omega Q H} \Rightarrow Q = \frac{9.43 \times 10^3}{9810 \times 9 \times 0.82}$$

$$\Rightarrow 0.13\text{ m}^3/\text{s}$$

$$(3) \frac{H_1}{N_1^2 D_1^2} = \frac{H_2}{N_2^2 D_2^2}$$

$$\Rightarrow \frac{16}{90^2 \times D_1^2} = \frac{4}{410.79^2 D_2^2}$$

$$= \frac{D_2^2}{D_1^2} = \frac{90^2 \times 4}{16 \times 410.79^2}$$

$$\Rightarrow \boxed{\frac{D_2}{D_1} = 0.109}$$

### Kaplan turbine:

It is an axial flow, low head, high discharge, high specific speed reaction turbine.

$$\Rightarrow Q = (\pi D - zt) B V_f \quad Q \text{ can be } \uparrow \text{ by } \uparrow D, \downarrow z \text{ or } \uparrow V_f$$

if  $D \uparrow$ , size is large, so we don't prefer

$V_f \uparrow$  - Flow velocity is more, losses is more at exit.

So  $\downarrow z$  (No. of blade) is best option to  $\uparrow$  the discharge.

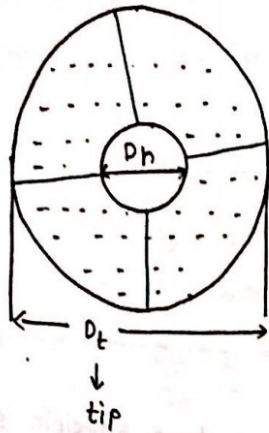
there are some places where the availability of head is low and at these places to develop a given power ( $P = WQH$ ),  $QH = \text{constant}$  and under low heads, discharge should be high we know that

$$Q = (\pi D - zt) B V_f$$

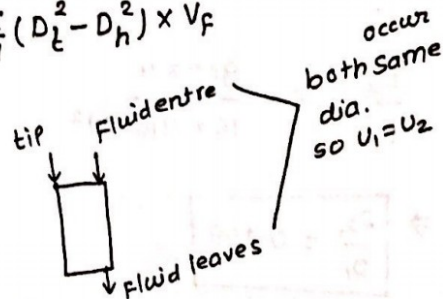
Discharge can be increased by increasing the flow velocity but this results in higher exit losses. there the suitable option is to decrease the number of vanes, the runner of a Kaplan turbine consist of 4-6 blades.



In Kaplan turbine fluids enters axially and leaves axially if,  $D_h / D_t$  is hub or boss diameter and  $D_t$  is tip diameter, then the discharge of a Kaplan turbine is



$$Q = \frac{\pi}{4} (D_t^2 - D_h^2) \times V_f$$



Note: As the fluid enters and leaves at the same diameter therefore

$U_1 = U_2$  in Kaplan turbine,

(2) Generally in Kaplan turbine  $V_{f1} = V_{f2}$

Q. A Kaplan turbine produces 25 MW power under a head of 40 m the blade tip diameter is 2.5 times hub diameter the overall efficiency is 90%, Speed ratio is 2, flow ratio is 0.6 calculate tip dia., hub meter, Speed of the runner.

Solution.

Given that

$$P = 25 \text{ MW}$$

$$H = 40 \text{ m}$$

$$D_t = 2.5 D_h$$

$$\eta_o = 0.9$$

$$K_u = 2$$

$$\psi = 2$$

$$K_u = \frac{U_1}{\sqrt{2gH}}$$

$$U_1 = K_u \sqrt{2gH}$$

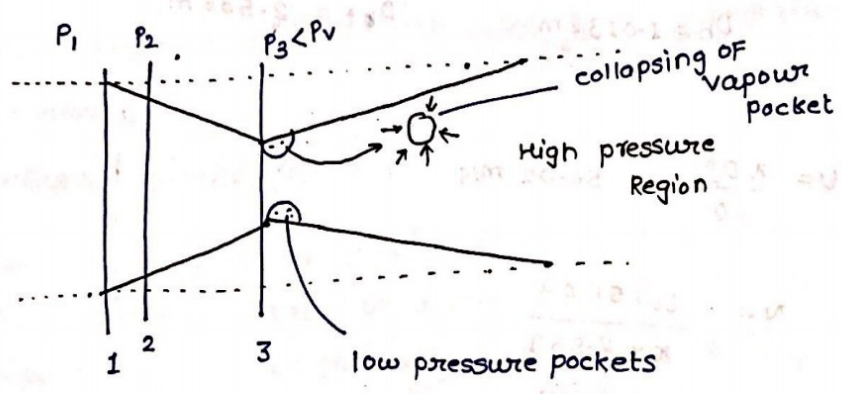
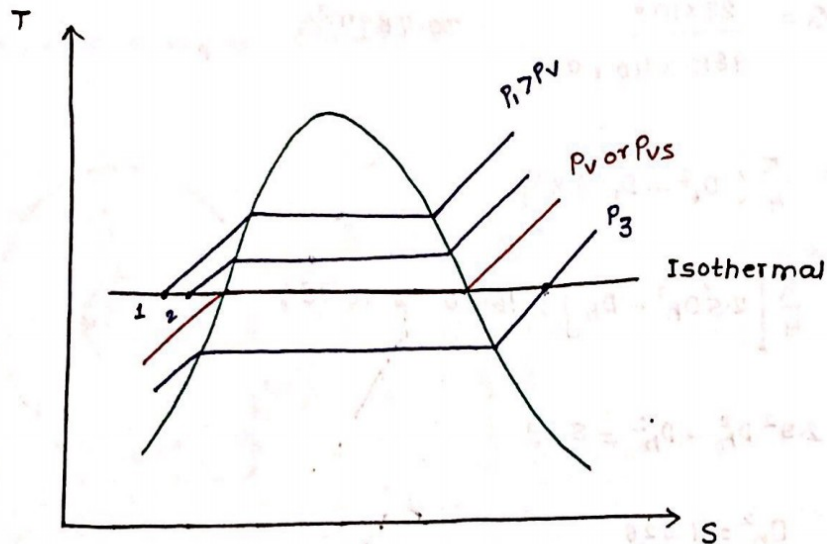
$$= 2 \sqrt{2 \times 9.81 \times 40} = 56.02 \text{ m/s}$$

$$\psi = \frac{V_f}{\sqrt{2gH}}$$

$$V_f = 0.6 \sqrt{2 \times 9.81 \times 40} = 16.80 \text{ m/s}$$



# Cavitation



$$a_1 v_1 = a_2 v_2$$

$$a_1 > a_2$$

So  $v_2 \uparrow$

$a \downarrow, v \uparrow, P \downarrow$  at point ③  $P_3 < P_v$ .

consider the a liquid flowing through a converging and diverging passage under isothermal conditions when the liquid flows through a converging passage pressure decrease, if the reduction in area is more, then the reduction in pressure



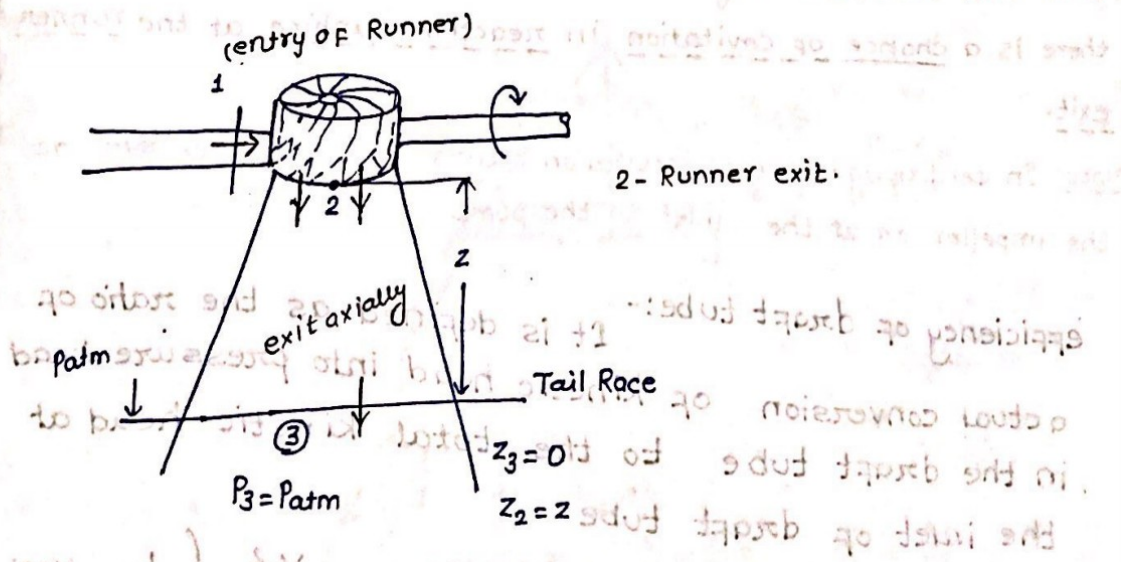
is high it may so happen that at some section the pressure is less than vapour pressure. When the pressure is less than vapour pressure, vapour pockets are formed, when these vapour pockets are carried to region of high pressure they collapse. and the surrounding high pressure liquid rushes suddenly in this region (as shown in figure) this results in huge noise and Mechanical damage this phenomenon is known as cavitation.

"Cavitation is a low pressure phenomenon." In order to avoid cavitation the liquid pressure must be greater than vapour pressure.

The thoma's cavitation factor is given by  $(\sigma) = \frac{P - P_v}{\frac{\rho V^2}{2}}$

$P$  = local pressure &  $P_v$  is vapour pressure.

Draft tube:



Apply Bernoulli's equation b/w 2 & 3

$$\frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2g} + z_3$$



$$\frac{P_2}{\omega} + \frac{V_2^2}{2g} + z = \frac{P_{atm}}{\omega} + \frac{V_3^2}{2g} + 0$$

$$\frac{P_2}{\omega} = \frac{P_{atm}}{\omega} - \left[ z + \frac{V_2^2 - V_3^2}{2g} \right] \quad V_2 > V_3$$

$$P_2 < P_{atm}$$

- Draft tube enables to install the turbine above the tail race without sacrificing head.
- With the installation of draft tube, pressure at the runner exit is low, thus it improves the efficiency.
- Draft tube is used in reaction turbine, in reaction turbine the pressure decreases from more than atmospheric to less than atmospheric.
- As the pressure at the runner exit is low it may so happen that this pressure may fall below vapour pressure and hence there is a chance of cavitation in reaction turbine at the runner exit.

Note: In centrifugal pumps cavitation occurs at the inlet of eye of the impeller or at the inlet to the pump.

efficiency of draft tube:- It is defined as the ratio of actual conversion of kinetic head into pressure head in the draft tube to the total kinetic head at the inlet of draft tube.

$$\eta_{d.T.} = \frac{\left( \frac{V_2^2}{2g} - \frac{V_3^2}{2g} \right)}{\frac{V_2^2}{2g}} = 1 - \frac{V_3^2}{V_2^2} \quad \left\{ \begin{array}{l} h_f - \text{Neglected} \\ \end{array} \right.$$

= 60 to 85%