IMPACT OF JETS

A jet of water issuing from a nozzle has a velocity and hence it possesses a kinetic energy. If this jet strikes a plate then it is said to have an impact on the plate. The jet will exert a force on the plate which it strikes. This force is called a dynamic force exerted by the jet. This force is due to the change in the momentum of the jet as a consequence of the impact. This force is equal to the rate of change of momentum i.e., the force is equal to (mass striking the plate per second) x (change in velocity).

We will consider some particular cases of impact of a jet on a plate or vane.

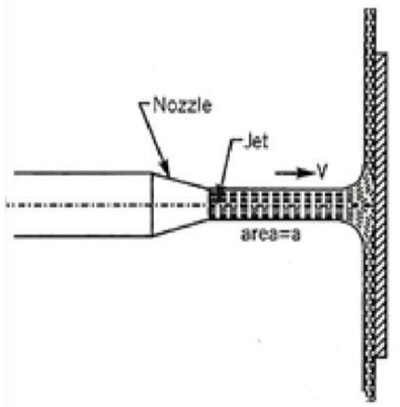


Fig. 18.1.

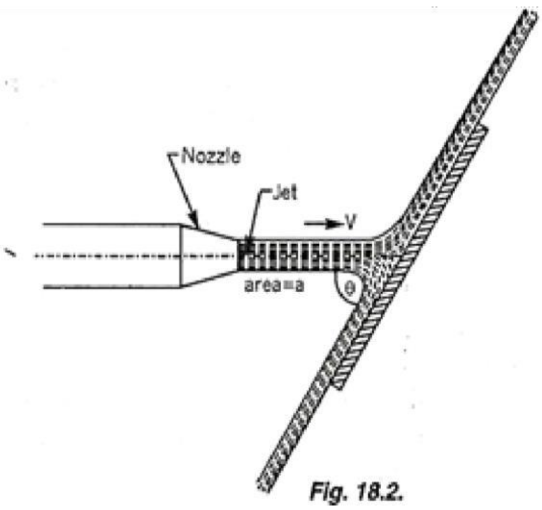


Fig. 18.2.

Direct Impact of a Jet on a Stationary Flat Plate:

Consider a jet of water impinging normally on a flat plate at rest.

Let, a = Cross-sectional area of the jet in metre^2.:

V = Velocity of the jet in metres per second.

M = Mass of water striking the plate per second.

∴ $M = \rho a V$ kg/sec where ρ = density of water in kg/cum Force exerted by the jet on the plate- P = Change of momentum per second = (Mass striking the plate per second) x (Change in velocity)

$= M (V - 0) = MV = \rho aV.V.$ ∴
$P = \rho aV^2$ Newton

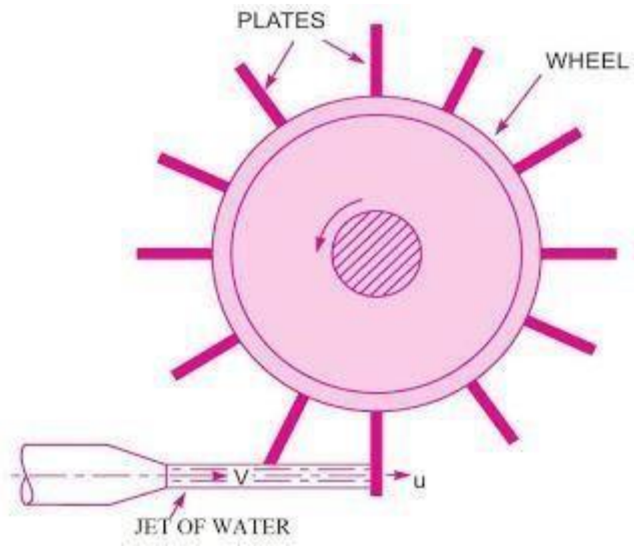
Direct Impact of a Jet on a Moving Plate:

Let,
V = Velocity of the jet v
= Velocity of the plate.
Velocity of the jet relative to the plate = (V – v)
We may consider as though the plate is at rest and that the jet is moving with a velocity (V –v) relative to the plate. ∴ Force exerted by the jet on the plate
$= P = \rho a (V - v)^2$ Newton
In this case, since the point of application of the force moves, work is done by the jet.
Work done by the jet on the plate per second
$= Pv = \rho a (V - v)^2 v$ Nm/s or Joule/sec

Force exerted by a jet of water on a series of vanes

If we see practically, force exerted by a jet of water on a single moving plate will not be feasible. Therefore, we will see the practical case where large number of plates will be mounted on the circumference of a wheel at a fixed distance apart as displayed here in following figure.

Jet will strike a plate and due to the force exerted by the jet on plate, wheel will be started to move and therefore second plate mounted on the circumference of wheel will be appeared before the jet and jet will again exert the force to the second plate.



Therefore, each plate will be appeared successively before the jet and jet will strike each plate or jet will exert force to each plate. Therefore, wheel will be rotated with a constant speed.

Let us consider the following terms as mentioned here

V = Velocity of jet d = Diameter of jet

a = Cross-sectional area of jet = $(\pi/4) \times d^2$ u

= Velocity of vane

Mass of water striking the series of plate per second = ρaV

Jet strikes the plate with a velocity = V-u

After striking, jet will move tangential to the plate and therefore velocity component in the direction of motion of plate will be zero.

Force exerted by the jet in the direction of motion of plate

F_X = Mass striking the series of plate per second x [Initial velocity – final velocity]

$F_X = \rho aV [(V-u)-0] = \rho aV (V-u)$

Work done by the jet on the series of plate per second = Force x Distance per second in the direction of force

Work done by the jet on the series of plate per second = F_X x u = $\rho aV (V-u) \times u$

Kinetic energy of the jet per second = $(1/2) \times mV^2$

Kinetic energy of the jet per second = $(1/2) \times \rho aV\, V^2$

Kinetic energy of the jet per second = $(1/2) \times \rho aV^3$

Efficiency = Work done per second / Kinetic energy per second

Efficiency = $\rho aV (V-u) \times u / (1/2) \times \rho aV^3$

Efficiency = $2u(V-u)/V^2$

$$\eta = \frac{2u[V-u]}{V^2}$$

Maximum efficiency will be 50 % and it will be when u = V/2

WORKDONE Of Jet Impinging On A Moving Curved Vane:

Consider a jet of water entering and leaving a moving curved vane as shown in fig.

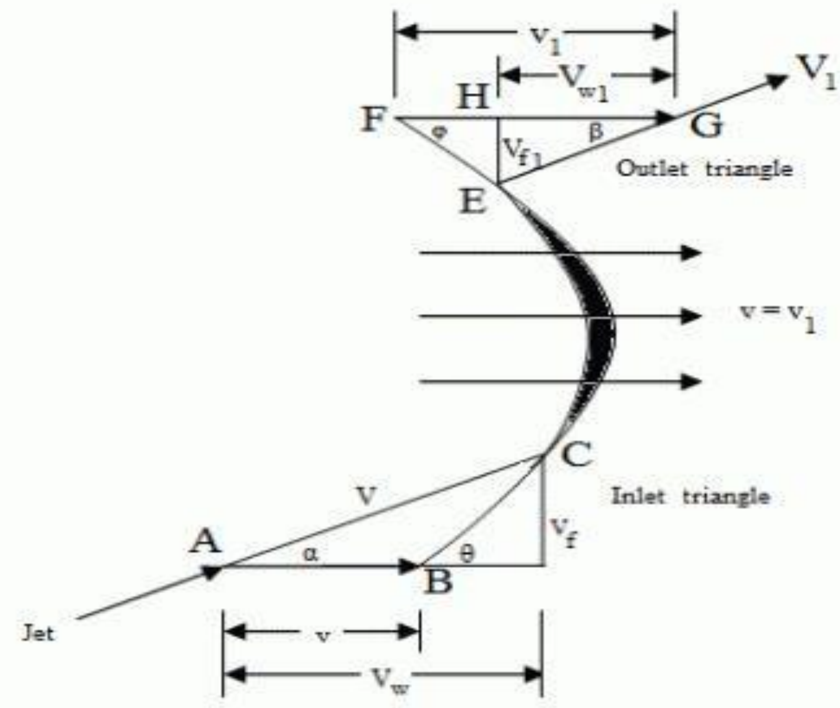


Fig-4 : Jet impinging on a moving curved vane

Let,

V = Velocity of the jet (AC), while entering the vane,

V1 = Velocity of the jet (EG), while leaving the vane,

v1, v2 = Velocity of the vane (AB, FG)

α = Angle with the direction of motion of the vane, at which the jet enters the vane,

β = Angle with the direction of motion of the vane, at which the jet leaves the vane,

Vr = Relative velocity of the jet and the vane (BC) at entrance (it is the vertical difference between V and v)

Vr1 = Relative velocity of the jet and the vane (EF) at exit (it is the vertical difference between v1 and v2)

ϴ = Angle, which Vr makes with the direction of motion of the vane at inlet (known as vane angle at inlet),

β = Angle, which Vr1 makes with the direction of motion of the vane at outlet (known as vane angle at outlet),

Vw = Horizontal component of V (AD, equal to). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at inlet),

Vw1 = Horizontal component of V1 (HG, equal to). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at outlet),

Vf = Vertical component of V (DC, equal to). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at inlet),

Vf1 = Vertical component of V1 (EH, equal to). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at outlet),

a = Cross sectional area of the jet. As the jet of water enters and leaves the vanes tangentially, therefore shape of the vanes will be such that Vr and Vr1 will be a long with tangents to the vanes at inlet and outlet.

The relations between the inlet and outlet triangles (until and unless given) are: (i) V=v1 , and

(ii) Vr=Vr1 We know that the force of jet, in the direction of motion of the vane,

$$F_x = Mass\ of\ water\ flowing\ per\ second \times Change\ of\ velocity\ of\ whirl$$

$$\Rightarrow F_x = \frac{waV}{g}(V_w - V_{w1})$$

$$= \frac{W}{g}[V_w - V_{w1}]\ Newton$$

Work done per second $= \frac{W}{g}[V_w - V_{w1}]\, v\ \ Nm/sec.$

Work done per second per N of water

$$= \frac{1}{g}[V_w - V_{w1}]v\ \ Nm/sec/N\ \text{of water}$$

te. *If the direction of velocity of whirl at outlet is opposite to that at inlet then the work done per second per N of water*

$$= \frac{1}{g}[V_w + V_{w1}]v\ \ Nm/sec/N\ \text{of water}$$

<u>Unit-5</u> <u>FLOW THROUGH PIPES</u>

5.1Friction Losses of Head in Pipes	*5-2 Secondary Losses of Head in Pipes*
5.3 Flow through Pipe Systems	

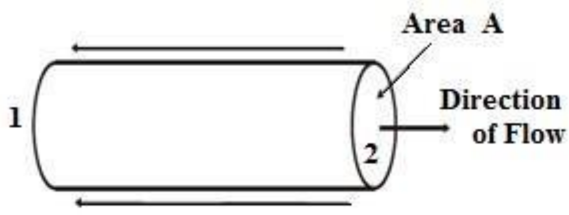
<u>Friction Losses of Head in Pipes:</u>

There are many types of losses of head for flowing liquids such as friction, inlet and outlet losses. The major loss is that due to frictional resistance of the pipe, which depends on the inside roughness of the pipe. The common formula for calculating the loss of head due to friction is Darcy's one.

<u>*Darcy's formula for friction loss of head:*</u>

For a flowing liquid, water in general, through a pipe, the horizontal forces on water between two sections (1) and (2) are:

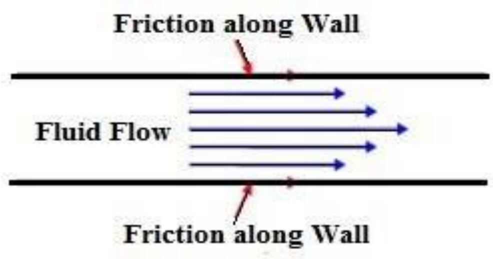
P1 A = P2 A + FR



P1= Pressure intensity at (1).

A = Cross sectional area of pipe. P2= Pressure intensity at (2).

FR= Frictional Resistance at (2).



FR / □ A = (P1 / □) - (P2 / □) = hf

Where, hf = Loss of pressure head due to friction.

□ = Specific gravity of water.

It is found experimentally that:

FR = Factor x Wetted Area x Velocity ^{2}

FR = (□ f / 2g) x (□ d L) x v ^{2}

Where, f = Friction coefficient.

d = Diameter of pipe.

L = Length of pipe.

hf = (□ f / 2g) x (□ d L) x v2 = 4 f * L * v[1]

□ (□ d2 /4) d * 2 g

$$\mathbf{hf = \frac{4\ f\ L\ v\ 2}{2\ g\ d}}$$

It may be substituted for [v = Q / (□ d2 /4)] in the last equation to get the head loss for a known discharge. Thus,

$$\mathbf{hf = \frac{32\ f\ L\ Q\ 2}{□ 2\ g\ d\ 5}}$$

[1] 2 x 9.81 x 1

Note: In American practice and references, $\lambda = f_{American} = 4f$

Example 1:

A pipe 1 m diameter and 15 km long transmits water of velocity of 1 m/sec. The friction coefficient of pipe is 0.005.

Calculate the head loss due to friction?

Solution

$$hf = \frac{4\, f\, L\, v^2}{2\, g\, d}$$

hf = 4x0.005x15000x 12 = 15.29 m

The Darcy – Weisbach equation relates the head loss (or pressure loss) due to friction along a given length of a pipe to the average velocity of the fluid flow for an incompressible fluid.

The friction coefficient f (or $\lambda = 4f$) is not a constant and depends on the parameters of the pipe and the velocity of the fluid flow, but it is known to high accuracy within certain flow regimes.

For given conditions, it may be evaluated using various empirical or theoretical relations, or it may be obtained from published charts.

R_e (Reynolds Number) is a dimensionless number. $\mathbf{R_e = \frac{\rho\, v\, d}{\mu}}$

For pipes, Laminar flow, $R_e < 2000$

Transitional flow, $2000 < R_e < 4000$
Turbulent flow, $R_e > 4000$

For laminar flow,

Poiseuille law, ($f = 64/Re$) where Re is the Reynolds number .

For turbulent flow,

Methods for finding the friction coefficient f include using a diagram such as the Moody chart, or solving equations such as the Colebrook–White equation.

Also, a variety of empirical equations valid only for certain flow regimes such as the Hazen – Williams equation, which is significantly easier to use in calculations. However, the generality of Darcy – Weisbach equation has made it the preferred one.

The only difference of (hf) between laminar and turbulent flows is the empirical value of (f).

Introducing the concept of smooth and rough pipes, as shown in Moody chart, *we find:*

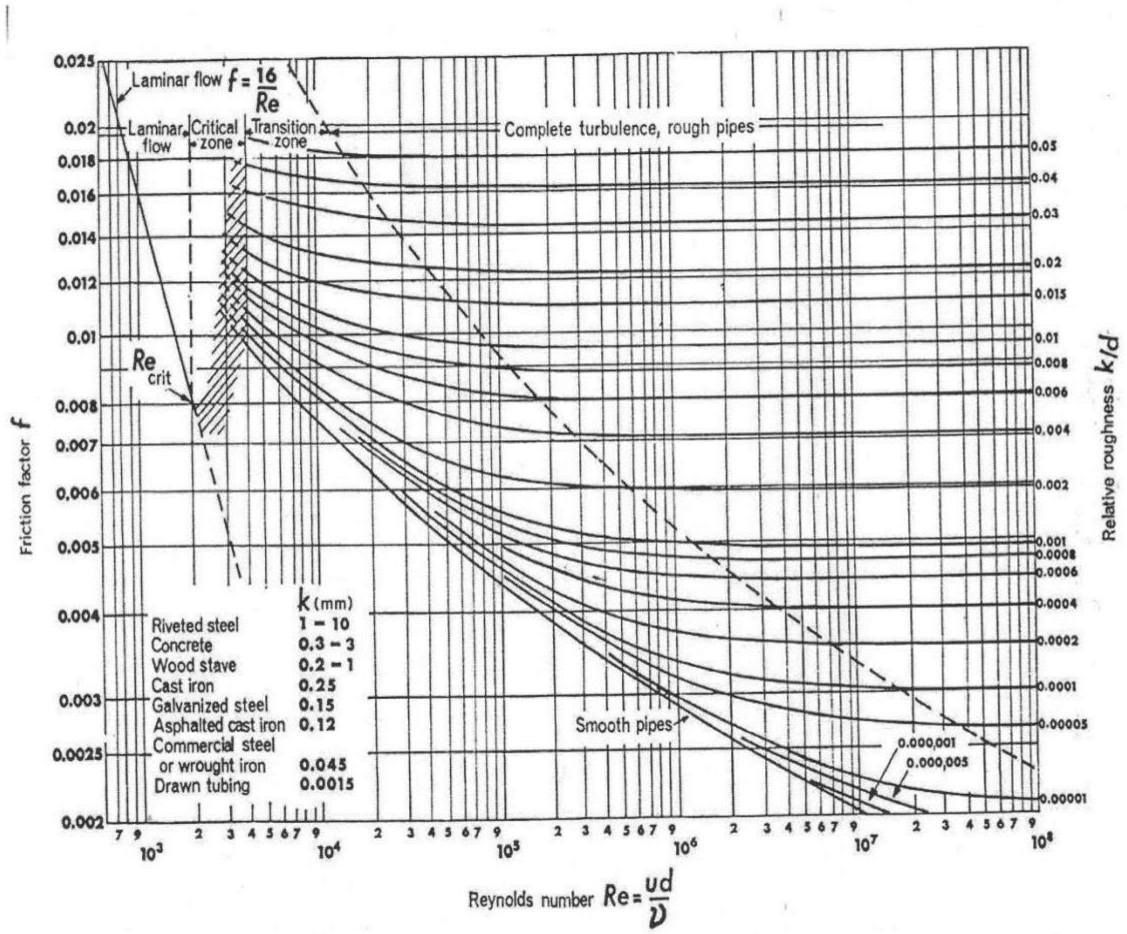
1) For laminar flow, $f = 16 / R_e$
2) For transitional flow, pipes' flow lies outside this region.
3) For smooth turbulent (a limiting line of turbulent flow), all values of relative roughness (k_s/d) tend toward this line as R decreases. Blasius equation: $f = 0.079 / R_e^{0.25}$

4) For transitional turbulent, it is the region where (f) varies with both (k_s/d) & (R_e). Most pipes lie in this region.
5) For rough turbulent, (f) is constant for given (k_s/d) and is independent of (R_e).

Doing a large number of experiments for the turbulent region for commercial pipes, Colebrook-White established the equation:

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left(\frac{k_s}{3.71d} + \frac{1.26}{\text{Re}\sqrt{f}}\right)$$

This equation is easily solved employing Moody chart.



<u>Moody Chart</u>

<u>$\lambda = 4f$</u> & <u>values of k_s are provided by pipe manufactures.</u>

Pipe Material	**K, mm**
Brass, Copper, Glass	0.003
Asbestos Cement	0.03

Iron	0.06
Galvanised Iron	0.15
Plastic	0.03
Bitumen-lined Ductile Iron	0.03
Concrete-lined Ductile Iron	0.03

Example 2:

Water flows in a steel pipe ($d = 40$ mm, $k = 0.045x10^{-3}$ m, $\mu = 0.001$ k/ms) with a rate of 1 lit/s.

Determine the friction coefficient and the head loss due to friction per meter length of the pipe using:

1- Moody chart? 2- Smooth pipe formula?

Solution

$v = Q / A = 0.001 / (\pi (0.04)^2/4) = 0.796$ m/s

$R_e = \rho\, v\, d / \mu = (1000x0.796x0.04) / 0.001 = 31840 > 4000$

Turbulent flow.

1. Moody chart:

$k/d = 0.045x10^{-3} / 0.04 = 0.0011$ & $Re = 31840$

from the chart, f = 0.0065

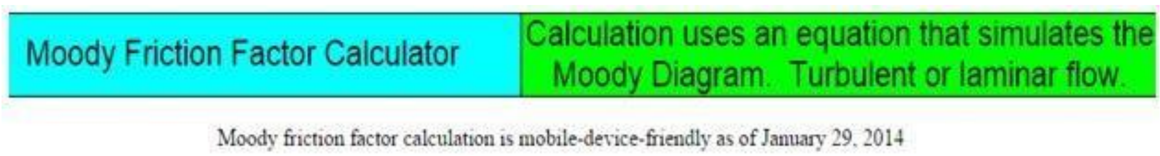
$$hf = \frac{4\,f\,L\,v^2}{2\,g\,d} = \frac{4x0.0065x1x(0.796)2}{2x9.81x0.04} = 0.0209 \text{ m / m of pipe}$$

2. Smooth pipe (Blasius equation):

$f = 0.079 / R_e^{0.25} = 0.079 / (31840) = 0.0059$

$$hf = \frac{4\,f\,L\,v^2}{2\,g\,d} = \frac{4x0.0059x1x(0.796)2}{2x9.81x0.04} = 0.02 \text{ m / m of pipe}$$

Another Solution:



Select Calculation:

- (●) Circular Duct: Enter D and Q
- ( ) Circular Duct: Enter D and V
- ( ) Circular Duct: Enter D and Re
- ( ) Non-circular Duct: Enter A, P, and Q
- ( ) Non-circular Duct: Enter A, P, and V
- ( ) Non-circular Duct: Enter A, P, and Re

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[Initial Values]

[Click to Calculate]

Kinematic viscosity, ν (L^2/T):	1.0E-6
Surface Roughness, e (L):	4.5E-5
Duct Diameter, D (L):	0.04
Duct Area, A (L^2):	0.0012566371
Duct Perimeter, P (L):	0.12566371
Velocity, V (L/T):	0.79577472
Discharge, Q (L^3/T):	0.001
Reynolds Number:	31830.989
e/D:	0.001125
Moody Friction Factor, f:	0.026171935

$$f = \frac{64}{\mathrm{Re}} \text{ for } \mathrm{Re} \leq 2100 \text{ (laminar flow)} \qquad \mathrm{Re} = \frac{VD}{\nu}$$

$$f = \frac{1.325}{\left[\ln\left(\frac{e}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \text{ for } 5000 \leq \mathrm{Re} \leq 10^8 \text{ (turbulent flow) and } 10^{-6} \leq \frac{e}{D} \leq 10^{-2}$$

D = Diameter of a circular duct. If duct is non-circular, then D is computed as the hydraulic diameter of a rectangular duct, where D = 4A / P per our non-circular duct page.
Re = Reynolds Number. The symbol Re is not the product (R)(e).

Kinematic viscosity (ν) depends on the fluid (water, air, etc.). Click for table.
Surface roughness depends on the duct material (steel, plastic, iron, etc.). Click for table.

The equations used in this program represent the Moody diagram which is the old-fashioned way of finding f. You may enter numbers in any units, so long as you are consistent. (L) means that the variable has units of length (e.g. meters). (L^3/T) means that the variable has units of cubic length per time (e.g. m^3/s). The Moody friction factor (f) is used in the Darcy-Weisbach major loss equation. Note that for laminar flow, f is independent of e. However, you must still enter an e for the program to run even though e is not used to compute f. Equations can be found in Discussion and References for Closed Conduit Flow.

A more complicated equation which represents a slightly larger range of Reynolds numbers and e/D's is used in Design of Circular Liquid or Gas Pipes.

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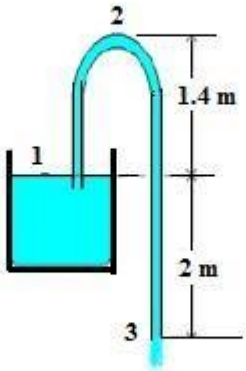
August 25, 2015: Made text fields show 8 significant figures rather than 16. Calculation still uses double precision internally.

Example 3:



The pipe of a syphon has 75 mm diameter and discharges water to the atmosphere, as shown in figure.
Neglect all possible losses.

a. Determine the velocity of flow?
b. Find the discharge?
c. What is the absolute pressure at the point 2?

Solution

(a) *Applying Bernoulli's equation between (1) and (3),* $2 + 0 + 0 = 0 + 0 + (v^2_3/2g)$

$$v_3 = 6.26 \text{ m/s}$$

(b) $Q = v_3 \text{ x } A = 6.26 \text{ x } (\pi\ (0.075)^2/4) = 0.028 \text{ m}^3/\text{s}$

(c) *Applying Bernoulli's equation between (1) and (2),*

$$2 + 0 + 0 = 3.4 + P_2/\rho g + (6.26^2/2g)$$

$$P_2 = -\ 3.397 \text{ x } (1000 \text{ x } 9.81) = -\ 33327.8 \text{ N/m}^2 = -\ 33.33 \text{ kPa}$$

$P_{2abs} = 64.77$ kPa *where,* ($P_{atm} = 98.1 \text{ kN/m}^2$)

Secondary Losses of Head in Pipes:

Any change in a pipe (in direction, in diameter, having a valve or other fitting) will cause a loss of energy due to the disturbance in the flow.

$$h_s = K (v^2 / 2g)$$

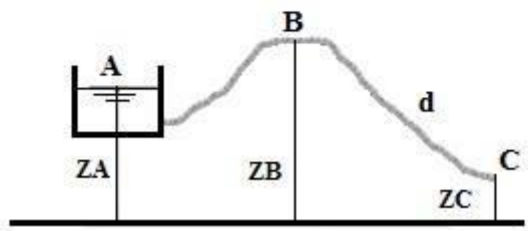
The velocity v is the velocity at the entry to the fitting. When the velocity changes upstream and downstream the section, the larger velocity is generally used.

Obstruction	**K**
Tank Exit	0.5
Tank Entry	1.0
Smooth Bend	0.3
90º Elbow	0.9
45º Elbow	0.4
Standard T	1.8
Strainer	2.0
Angle Valve, wide open	5.0
Gate Valve:	0.2
Wide Open	
3/4 open	1.2
1/2 open	5.6

1/4 open	24.0
Sudden Enlargement	0.1
Sudden Contraction:	
Area Ratio $(A_2/A_1) = 0.2$	0.4
Area Ratio $(A_2/A_1) = 0.4$	0.3
Area Ratio $(A_2/A_1) = 0.6$	0.2
Area Ratio $(A_2/A_1) = 0.7$	0.1

Example 4:

A pipe transmits water from a tank A to point C that is lower than water level in the tank by 4 m. The pipe is 100 mm diameter and 15 m long.



The highest point on the pipe B is 1.5 m above water level in the tank and 5 m long from the tank. The friction factor (4 f) is 0.08, with sharp inlet and outlet to the pipe.

a. Determine the velocity of water leaving the pipe at C?
b. Calculate the pressure in the pipe at the point B?

Solution

(a) *Applying Bernoulli's equation between A and C,*

Head loss due to entry (tank exit, *from table*) = $0.5\ (v^2_C/2g)$

Head loss due to exit into air without contraction = 0

$$Z_A + 0 + 0 = Z_C + 0 + (v^2_C/2g) + 0.5\,(v^2_C/2g) + 0 + \frac{4\,f\,L\;v^2_C}{2\,g\,d}$$

$$4 = (v^2_C/2g) \times \{1 + 0.5 + (4 \times 0.08 \times 15)/0.1\}$$

$\therefore v_C = 1.26$ m/s

(b) *Applying Bernoulli's equation between A and B,*

$$Z_A + 0 + 0 = Z_B + P_B/\rho g + (v^2_B/2g) + 0.5\,(v^2_B/2g) + \frac{4\,f\,L\;v^2_B}{2\,g\,d}$$

$$-1.5 = P_B/(1000 \times 9.81) + (1.26^2/2 \times 9.81) * \{1 + 0.5 + (4 \times 0.08 \times 5)/0.1\}$$

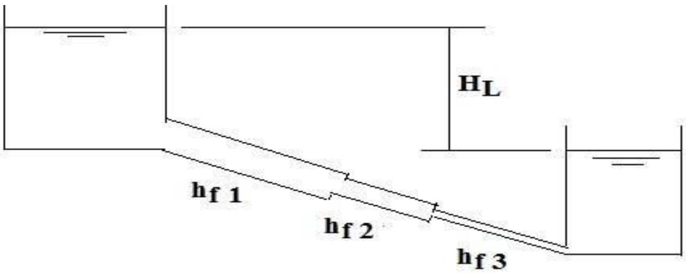
$\therefore P_B = -28.61$ kN/m^2

Flow through Pipe Systems:

Pipes in Series:

Pipes in series are pipes with different diameters and lengths connected together forming a pipe line. Consider pipes in series discharging water from a tank with higher water level to another with lower water level, as shown in the figure.

Neglecting secondary losses, it is obvious that the total head loss HL between the two tanks is the sum of the friction losses through the pipe line.



Friction losses through the pipe line are the sum of friction loss of each pipe.
HL = hf 1 + hf 2 + hf 3 + ...

$$HL = \frac{4f_1L_1v_1^2}{2gd_1} + \frac{4f_2L_2v_2^2}{2gd_2} + \frac{4f_3L_3v_3^2}{2gd_3} + ...$$

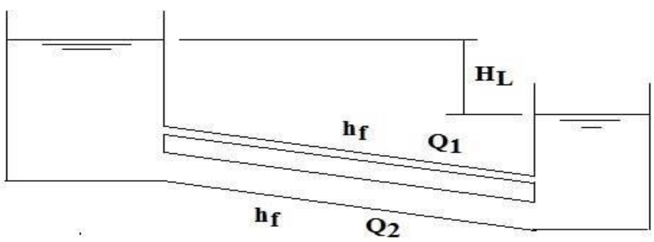
OR:

$$HL = \frac{32f_1L_1Q^2}{\pi^2 g d_1^5} + \frac{32f_2L_2Q^2}{\pi^2 g d_2^5} + \frac{32f_3L_3Q^2}{\pi^2 g d_3^5} + ...$$

Pipes in Parallel:

Pipes in parallel are pipes with different diameters and same lengths, where each pipe is connected separately to increase the discharge. Consider pipes in parallel discharging water from a tank with higher water level to another with lower water level, as shown in the figure.

Neglecting minor losses, it is obvious that the total head loss HL between the two tanks is the same as the friction losses through each pipe.



The friction losses through all pipes are the same, and all pipes discharge water independently.

$$HL = h_{f1} = h_{f2} = ...$$

$L_1 = L_2 = L$

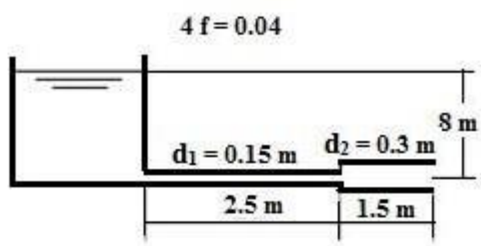
$$HL = \frac{4 f_1 L v_1^2}{2 g d_1} = \frac{4 f_2 L v_2^2}{2 g d_2} = ...$$

$$HL = \frac{32 f_1 L Q_1^2}{\pi^2 g d_1^5} = \frac{32 f_2 L Q_2^2}{\pi^2 g d_2^5} = ...$$

$$Q = Q_1 + Q_2$$

<u>Example 5:</u>
A pipe, 40 m long, is connected to a water tank at one end and flows freely in atmosphere at the other end. The diameter of pipe is 15 cm for first 25 m from the tank, and then the diameter is suddenly enlarged to 30 cm. Height of water in the tank is 8 m above the centre of pipe. Darcy's coefficient is 0.01. Determine the discharge neglecting minor losses?

<u>Solution</u>



Loss due to friction, $h_{Lf} = h_{f1} + h_{f2}$

$$h_f = \frac{32\, f\, L\, Q^2}{\pi^2\, g\, d^5} \qquad f = 0.01$$

Total losses,

$$h_T = Q^2 \left(\frac{32\, f\, L_1}{\pi^2\, g\, d_1^5} + \frac{32\, f\, L_2}{\pi^2\, g\, d_2^5} \right)$$

$$8 = Q^2 \left(\frac{(32 \times 0.01) \times (25)}{\pi^2\, g\, (0.15)^5} + \frac{(32 \times 0.01)\,(15)}{\pi^2\, g\, (0.3)^5} \right)$$

$$\therefore Q = 0.087\ m^3/sec$$

Example

6:

Two pipes are connected in parallel between two reservoirs that have difference in levels of 3.5 m. The length, the diameter, and friction factor (4 f) are 2400 m, m, and 0.026 for the first pipe and 2400 m, 1 m, and 0.019 for the secondpipe.

Calculate the total discharge between the two reservoirs?

Solution

$$HL = \frac{32\ f1\ L\ Q1\ 2}{\square 2\ g\ d1\ 5} = \frac{32\ f2\ L\ Q2\ 2}{\square 2\ g\ d2\ 5}$$

$$3.5 = \frac{32\ f1\ L\ Q1\ 2}{\square 2\ g\ d1\ 5} = \frac{8x0.026x2400xQ1\ 2}{\square 2x9.81\ x1.2\ 5}$$

Q1 = 1.29 m3/sec

$$3.5 = \frac{32\ f2\ L\ Q2\ 2}{\square 2\ g\ d2\ 5} = \frac{8x0.019x2400xQ2\ 2}{\square 2x9.81\ x1\ 5}$$

Q2 = 0.96 m3/sec

□ Q = Q1 + Q2 = 1.29 + 0.96 = 2.25 m3/sec

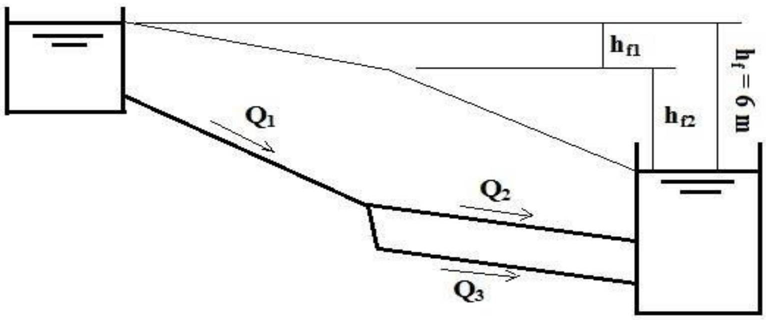
Example

7:

Two reservoirs have 6 m difference in water levels, and are connected by a pipe 60 cm diameter and 3000 m long. Then, the pipe branches into two pipes each 30 cm diameter and 1500 m long. The friction coefficient is 0.01.

Neglecting minor losses, determine the flow rates in the pipe system?

Solution



$h_f = h_{f1} + h_{f2}$

$6 = h_{f1} + h_{f2}$

$6 = k_1 Q_1^2 + k_2 Q_2^2$

$$k_1 = \frac{32\ f1\ L1}{\square 2\ g\ d15} = \frac{32*0.01*3000}{\square 2*9.81*0.65} = 127.64$$

$$k_2 = \frac{32\ f2\ L2}{\square 2\ g\ d2\ 5} = \frac{32*0.01*1500}{\square 2*9.81*0.35} = 4084.48$$

Example

$k_2 = 32\ k_1$

□ $6 = k_1\ Q_1^2 + 32\ k_1\ Q_2^2$

$h_{f2} = h_{f3}$ & $k_2 = k_3$ □ $Q_2 = Q_3$

$Q_1 = Q_2 + Q_3 = 2\ Q_2$

□ $6 = k_1\ Q_1^2 + 8\ k_1\ Q_1^2 = 9\ k_1\ Q_1^2 = (9 * 127.64)\ Q_1^2 = 1148.76\ Q_1^2$

□ $Q_1 = 0.072\ m^3/s$

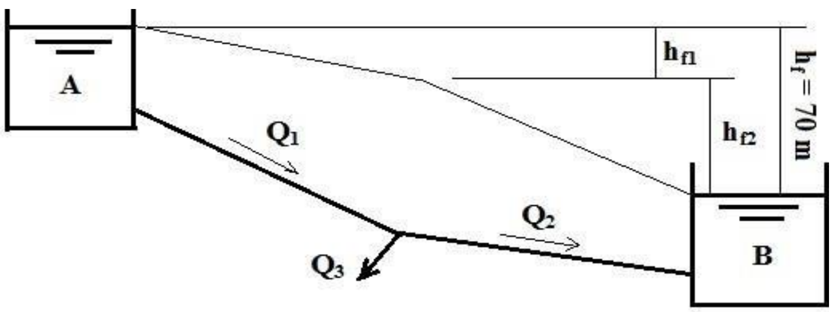
& $Q_2 = 0.036\ m^3/s$

8:

Two tanks A and B have 70 m difference in water levels, and are connected by a pipe 0.25 m diameter and 6 km long with 0.002 friction coefficient. The pipe is tapped at its mid point to leak out 0.04 m^3/s flow rate. Minor losses are ignored.

Determine the discharge leaving tank A?
Find the discharge entering tank B?

Solution



$h_f = h_{f1} + h_{f2}$

$70 = h_{f1} + h_{f2}$

Example

$70 = k_1 Q_1^2 + k_2 Q_2^2$

$$k_1 = k_2 = \frac{32 f L}{\square 2 g d5} = \frac{32*0.002*3000}{\square 2*9.81*0.255} = 2032.7$$

$\square$ $70 = k_1 Q_1^2 + k_1 Q_2^2$

$Q_1 = Q_2 + Q_3 = Q_2 + 0.04$

$\square$ $70 = k_1 (Q_2 + 0.04)^2 + k_1 Q_2^2$

$= k_1 (Q_2^2 + 0.08 Q_2 + 0.0016) + k_1 Q_2^2$

$= k_1 Q_2^2 + 0.08 k_1 Q_2 + 0.0016 k_1 + k_1 Q_2^2$

$= 2 k_1 Q_2^2 + 0.08 k_1 Q_2 + 0.0016 k_1$

$= 4065.4 Q_2^2 + 162.6 Q_2 + 3.25$

$0.0172 = Q_2^2 + 0.04 Q_2 + 0.0008$

$Q_2^2 + 0.04 Q_2 - 0.0164 = 0$

$$Q2 = \frac{-0.04 \pm \sqrt{(-0.04)^2 - 4(1)(-0.0164)}}{2(1)}$$

$\square$ $Q_2 = 0.11 \text{ m}^3/\text{s}$ & $Q_1 = 0.15 \text{ m}^3/\text{s}$

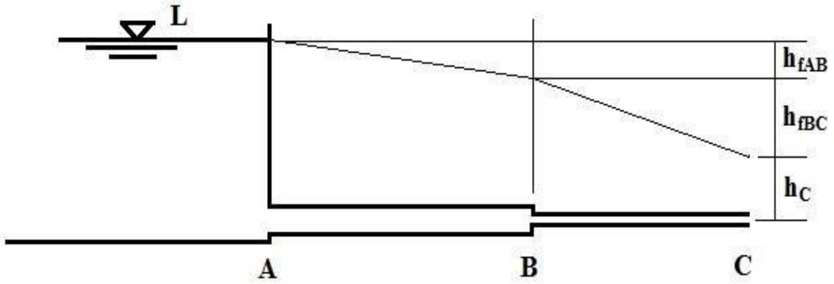
9:

A tank transmits 100 L/s of water to the point C where the pressure is maintained at 1.5 kg/cm^2. The first part AB of the pipe line is 50 cm diameter and 2.5 km long, and the second part BC is 25 cm diameter and 1.5 km long. The friction coefficient is 0.005 and minor losses are ignored.

Example

Assuming level at C is (0.0); find the water level (L) in the tank?

Solution



$h_C = P_C / _\gamma = 1500 / 1 = 1500$ cm = 15 m

$h_C = 15 = L - h_{fAB} - h_{fBC}$

h_{fAB} = 32 f1 L1 = 32*0.005*2500 = 1.32

□2 g d15 □2*9.81*0.55

h_{fBC} = 32 f2 L2 = 32*0.005*1500 = 25.38

□2 g d2 5 □2*9.81*0.255

15 = L – 1.32 – 25.38

Example

 L = 41.7 m

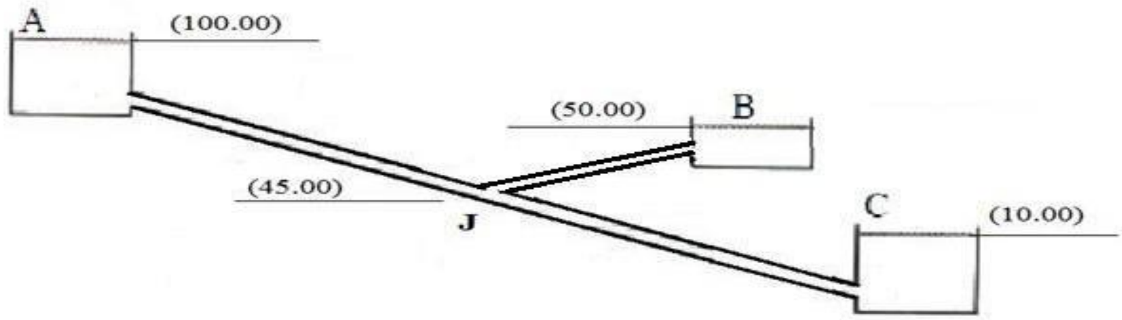
Example

10:

Three water tanks A, B and C with water surface levels (100.00), (50.00) and (10.00) m are connected by pipes AJ, BJ and CJ to a common joint J of a level (45.00) m. The three pipes have the same length, diameter and friction coefficient.

a) Calculate the head at the joint J?
b) Determine the discharge in each pipe?

Solution



Assume, QAJ = QJB + QJC

Applying Bernoulli's equation between A and J:

HA = HJ + hf AJ

100 + 0 + 0 = HJ + hf AJ

100 - HJ = hf AJ = K Q2AJ

where, K = 32 f l / □2 g d5

Q AJ = (100 - H J) 1/2 / (K) 1/2 .... (1)

Similarly, applying Bernoulli’s equation between J and B:

HJ = HB + hf JB

HJ - 50 = hf JB = K Q2JB

QJB = (HJ - 50) 1/2 / (K) 1/2 .... (2)

Also, applying Bernoulli’s equation between J and C:

HJ = HC + hf JC

HJ - 10 = hf JC = K Q2JC

QJC = (HJ - 10) 1/2 / (K) 1/2 .... (3)

Solving equations 1, 2 and 3 by trial and error, we get:

Assumed HJ	**QAJ x** (K)1/2	**JB x** (K)1/2	**QJC x** (K)1/2	**(QJB+QJC)x**(K)1/2
70	5.48	4.47	7.745	12.216
60	6.325	3.162	7.07	10.233
53	6.855	1.732	6.557	8.289
51	7	1	6.4	7.4

50.5	7.036	0.707	6.364	7.07
50.45	**7.039**	0.671	6.36	**7.031**
50.4	7.043	0.632	6.356	6.988
50	7.071	**0**	6.324	6.324

From the table:

HJ = 50.45 m

QAJ = 7.039 / (K)1/2

QJB = 0.671 / (K)1/2

QJC = 6.36 / (K)1/2

It has to be noted that if HJ < 50, then the flow will be from B to J.

Exercise:

Three water tanks A, B and C are connected to a joint J by three pipes AJ, BJ and CJ such that the water level in tank A is 40 m higher than tank B and 55 m higher than tank C. Each pipe is 1500 m long, 0.3 m diameter and f = 0.01.

Calculate the discharges and directions of flow?

Solution

Taking the water level in the tank C as a datum, *the results are:*

HJ = 18 m

QAJ = 0.134 m3/sec

QJB = 0.038 m3/sec

QJC = 0.094 m3/sec

Chapter-I

HYDROSTATICS

Hydrostatic is that branch of science which relating to fluids at rest or to the pressures they exert or transmit **Hydrostatic Pressure**.

Fluid:-

Fluid is a substance that continuously deforms (flows) under an applied shear stress. Fluids are a subset of the **phase of matter and include liquids, gases, plasmas and, to some extent, plastic solids. Fluids can be defined as substances which have zero shear modulus or in simpler terms a fluid is a substance which cannot resist any shear force applied to it.**

- **Fluid is a substance which is capable of flowing**
- **Conform the shape of the containing vessel**
- **Deform continuously under application of small shear force**

PROPERTIES OF FLUID:-

Density:-

The density of a fluid, is generally designated by the Greek symbol $\rho(rho)$ is defined as the mass of the fluid over a unit volume of the fluid at standard temperature and pressure. It is expressed in the SI system as kg/m^3.

$$\rho = \lim \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

If the fluid is assumed to be uniformly dense the formula may be simplified as:

$$\rho = \frac{m}{V}$$

Example: - setting of fine particles at the bottom of the container.

Specific Weight:-

The specific weight of a fluid is designated by the Greek symbol γ (gamma), and is generally defined as the weight per unit volume of the fluid at standard temperature and pressure. In SI systems the units is N/m^3.

$$\lambda = \rho * g$$

g = local acceleration of gravity and ρ = density

Note: It is customary to use:

g = 32.174 ft/s^2 = 9.81 m/s^2

ρ = 1000 kg/m^3

Relative Density (Specific Gravity):-

The relative density of any fluid is defined as the ratio of the density of that fluid to the density of the standard fluid. For liquids we take water as a standard fluid with density ρ=1000 kg/m^3. For gases we take air or O_2 as a standard fluid with density, ρ=1.293 kg/m^3.

Specific volume:-

Specific volume is defined as the volume per unit mass. It is just reciprocal of mass density. It is expressed in m^3/kg.

Viscosity:-

Viscosity (represented by μ, Greek letter mu) is a material property, unique to fluids, that measures the fluid's resistance to flow. Though a property of the fluid, its effect is understood only when the fluid is in motion. When different elements move with different velocities, each element tries to drag its neighboring elements along with it. Thus, shear stress occurs between fluid elements of different velocities.

Viscosity is the property of liquid which destroyed the relative motion between the layers of fluid.

- ❖ It is the internal friction which causes resistance to flow.
- ❖ Viscosity is the property which control the rate of flow of liquid

Viscosity is due to two factors-

a) Cohesion between the liquid molecules.

b) Transfer of momentum between the molecules.

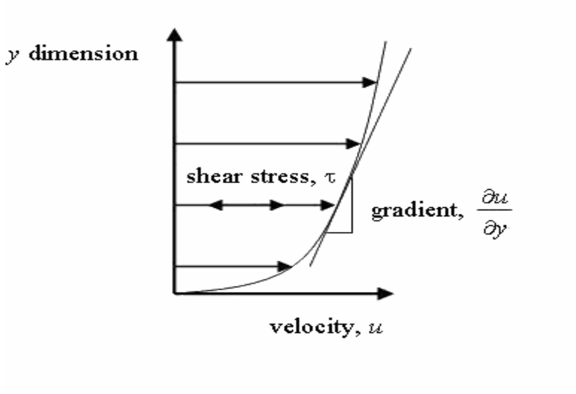


Fig. 1.1

The relationship between the shear stress and the velocity field was that the shear stresses are directly proportional to the velocity gradient. The constant of proportionality is called the coefficient of dynamic viscosity.

$$\tau = \mu \frac{\partial u}{\partial y}$$

UNIT OF VISCOSITY

- In mks system unit of viscosity is kgf-sec/m^2
- In cgs system unit of viscosity is dyne-sec/cm^2
- In S.I system unit of viscosity is Newton-sec/m^2

Kinematic viscosity:-

Another coefficient, known as the kinematic viscosity (ν, Greek nu) is defined as the ratio of dynamic viscosity and density.

I.et, $\upsilon = \mu / \rho$ = viscosity/density

In mks & S.I system unit of kinematic viscosity is meter^2/sec

In cgs system unit of kinematic viscosity is stoke.

SURFACE TENSION:-

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter sigma(σ). In MKS units, it is expressed as kgf/m while in SI unit is N/m.

It is also defined as force per unit length, or of energy per unit area. The two are equivalent□but when referring to energy per unit of area, people use the term surface energy□which is a more general term in the sense that it applies also to solids and not just liquids.

Capillarity:-

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Pressure and its measurement:-

INTENSITY OF PRESSURE:-

Intensity of pressure is defined as normal force exerted by fluid at any point per unit area. It is also called specific pressure or hydrostatic pressure

P=df/da

- ❖ If intensity of pressure is uniform over an area A then pressure force exerted by fluid equal to

  Mathematically F=PA

- ❖ If intensity of pressure is not uniform or vary point to point then pressure force exerted by fluid equal to integration of P*A

  Mathematically F=∫ PA

- ❖ Unit of pressure
  - $1N/m^2$ =1 Pascal
  - $1KN/m^2$= 1 kilo Pascal
  - Kilo Pascal= 1kpa = 10^3 Pascal
  - 1 bar = 10^5 Pascal = 10^5 N/m^2

Pascal’s law:-

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all direction.

Atmospheric Pressure:-

The atmospheric air exerts a normal pressure upon all surface with which it is in contact and it is called atmospheric pressure. It is also called parametric pressure.

Atmospheric pressure at the sea level is called standard atmospheric pressure.

S.A.P = 101.3 KN/m^2 = 101.3 kpa = 10.3m of H_2O

= 760 mm of Hg

=10.3 (milli bar)

Gauge pressure:-

It is the pressure which measure with help of pressure measuring device in which atmospheric pressure taken as datum.

The atmospheric pressure on scale is marked as zero.

Absolute pressure:-

Any pressure measure above absolute zero pressure is called absolute pressure.

Vacuum pressure:-

Vacuum pressure is defined as the pressure below the atmospheric pressure.

RELATIONSHIP BETWEEN ABSOLUTE PRESSURE, GAUGE PRESSURE,VACUUM PRESSURE:-

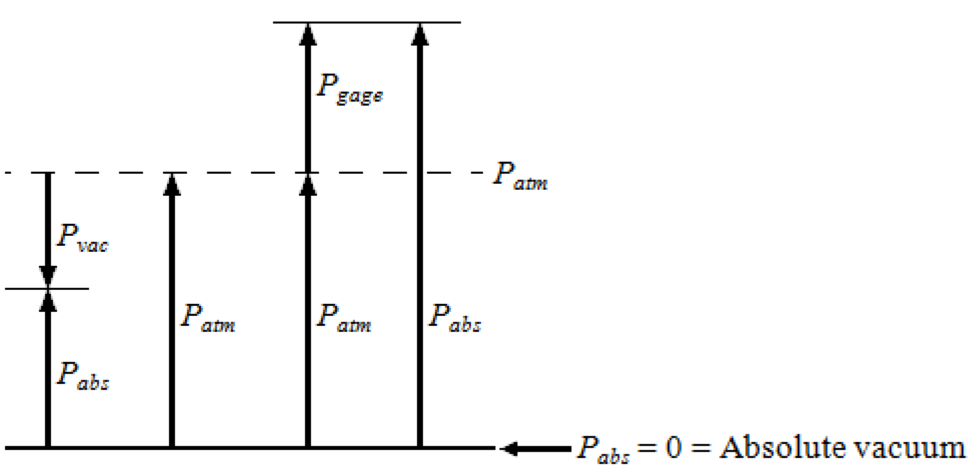


Fig. 1.2

- Equations

$P_{gage} = P_{abs} - P_{atm}$	gauge pressure
$P_{vac} = P_{atm} - P_{abs}$	vacuum pressure
$P_{abs} = P_{atm} + P_{gage}$	absolute pressure

- Nomenclature

P_{abs}	absolute pressure
P_{gage}	gage pressure
P_{vac}	vacuum pressure
P_{atm}	atmospheric pressure

Pressure Head:-

pressure head is the internal energy of a fluid due to the pressure exerted on its container. It may also be called **static pressure head** or simply **static head** (but not **static head pressure**). It is mathematically expressed as:

$$\psi = \frac{p}{\gamma} = \frac{p}{\rho g}$$

where

ψ is pressure head (Length, typically in units of m);
p is fluid pressure (force per unit area, often as Pa units); and
γ is the specific weight (force per unit volume, typically N/m^3 units)
ρ is the density of the fluid (mass per unit volume, typically kg/m^3)
g is acceleration due to gravity (rate of change of velocity, given in m/s^2)

If intensity of pressure express in terms of height of liquid column, which causes pressure is also called pressure head.

Mathematically, $h = P/w$

Pressure Gauges :-

The pressure of a fluid is measured by the following devices:-

1. manometers
2. mechanical gauges

Manometers:-Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:

a) Simple manometers
b) Differential manometer

Mechanical gauges:-mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical gauges are:-

a) Diaphragm pressure gauge
b) Bourdon tube pressure gauge
c) Dead weight pressure gauge
d) Bellows pressure gauge

PRESSURE EXERTED ON IMMERSED

SURFACE:-Hydrostatic forces on surfaces:-

Hydrostatic means the study of pressure exerted by a liquid at rest. The direction of such pressure is always perpendicular to the surface to which it acts.

Forces on Submerged Surfaces in Static Fluids

These are the following features of statics fluids:-

- Hydrostatic vertical pressure distribution
- Pressures at any equal depths in a continuous fluid are equal
- Pressure at a point acts equally in all directions (Pascal's law).
- Forces from a fluid on a boundary acts at right angles to that boundary.

Fluid pressure on a surface:-

Pressure is defined as force per unit area. If a pressure p acts on a small area δA then the force exerted on that area will be

$F = p\delta A$

TOTAL PRESSURE:-

Total pressure is defined as the force exerted by a static fluid on a surface when the fluid comes in contact with the surface.

Mathematically **total pressure**,

$P = p_1 a_1 + p_2 a_2 + p_3 a_3$....

Where,

- p_1, p_2, p_3 = Intensities of pressure on different strips of the surface, and
- a_1, a_2, a_3 = Areas of corresponding strips.

The position of an immersed surface may be,

- Horizontal
- Vertical
- Inclined

Total Pressure On A Horizontal Immersed Surface

Consider a plane horizontal surface immersed in a liquid as shown in figure 1.

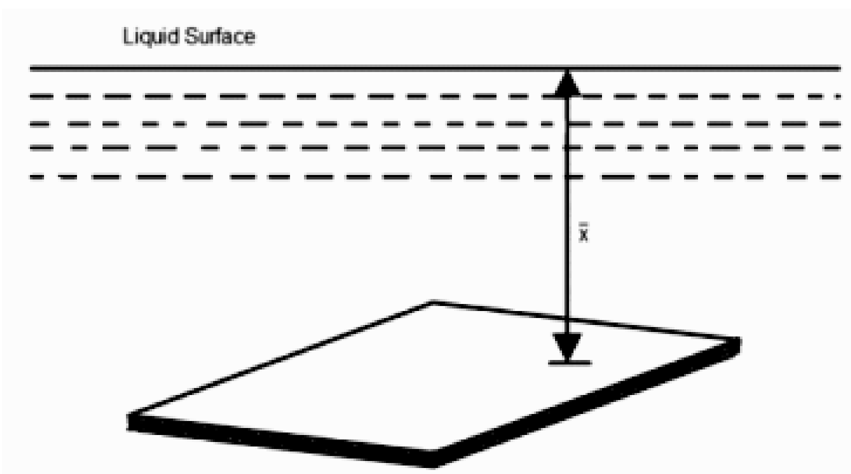


Fig. 1.3

- ω = Specific weight of the liquid
- A = Area of the immersed surface in in^2
- χ = Depth of the horizontal surface from the liquid level in meters

We know that the **Total pressure** on the surface,

P = Weight of the liquid above the immersed surface

= Specific weight of liquid * Volume of liquid

= Specific weight of liquid * Area of surface * Depth of liquid

= $\omega A \chi$ kN

Total Pressure On A Vertically Immersed Surface

Consider a plane vertical surface immersed in a liquid shown in figure 2.

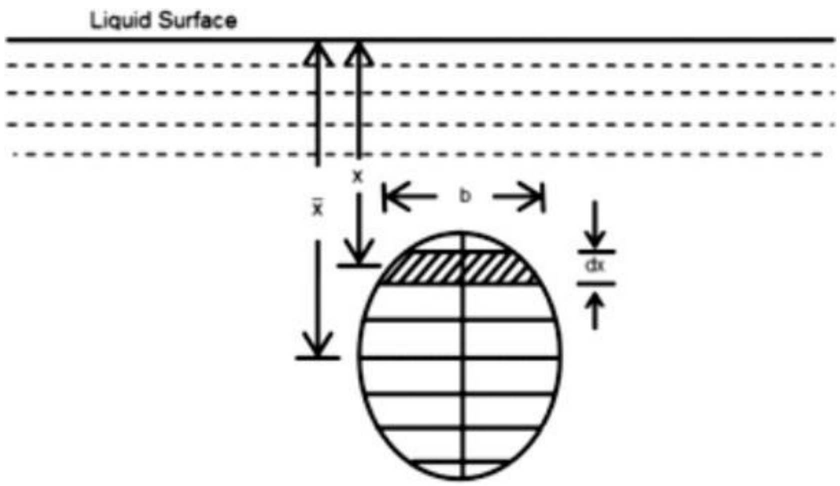


Fig. 1.4

Let the whole immersed surface is divided into a number of small parallel stripes as shown in figure.

Here,

- $\omega =$ Specific weight of the liquid
- A = Total area of the immersed surface
- χ = Depth of the center of gravity of the immersed surface from the liquid surface

Now, consider a strip of thickness dx, width b and at a depth x from the free surface of the liquid.

The intensity of pressure on the strip = $\omega\chi$

and the area of strip =b.dx

$\therefore$Pressure on the strip = Intensity of pressure * Area = $\omega\chi$.bd$x$

Now, Total pressure on the surface,

$P = \int wx.bdx$

$= w\int x.bdx$

But, $w\int x.bdx$ = Moment of the surface area about the liquid level = $A\bar{x}$

$P = wA\bar{x}$

FLOTATION AND BUOYANCY:-

Archimedes Principle:-

Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces. Archimedes' principle is a law of physics fundamental to fluid mechanics. Archimedes of Syracuse formulated this principle, which bears his name.

Buoyancy:-

When a body is immersed in a fluid an upward force is exerted by the fluid on the body. This is upward force is equal to weight of the fluid displaced by the body and is called the force of buoyancy or simple buoyancy.

Centre of pressure:-

The center of pressure is the point where the total sum of a pressure field acts on a body, causing a force to act through that point. The total force vector acting at the center of pressure is the value of the integrated pressure field. The resultant force and center of pressure location produce equivalent force and moment on the body as the original pressure field. Pressure fields occur in both static and dynamic fluid mechanics. Specification of the center of pressure, the reference point from which the center of pressure is referenced, and the associated force vector allows the moment generated about any point to be computed by a translation from the reference point to the desired new point. It is common for the center of pressure to be located on the body, but in fluid flows it is possible for the pressure field to exert a moment on the body of such magnitude that the center of pressure is located outside the body.

Center of buoyancy:-

It is define as the point through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the center of buoyancy will be the center of gravity of the fluid displaced.

METACENTER:-

The metacentric height (GM) is a measurement of the initial static stability of a floating body. It is calculated as the distance between the centre of gravity of a ship and its metacentre. A larger metacentric height implies greater initial stability against overturning. Metacentric height also has implication on the natural period of rolling of a hull, with very large metacentric heights being associated with shorter periods of roll which are uncomfortable for passengers. Hence, a sufficiently high but not excessively high metacentric height is considered ideal for passenger ships.

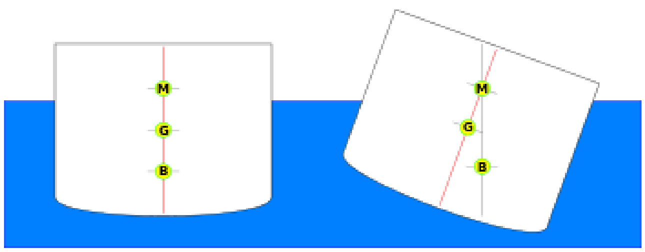


Fig. 1.5

The metacentre can be calculated using the formulae:

$$KM = KB + BM$$

$$BM = \frac{I}{V}$$

Metacentric height:-

The distance between the meta-center of a floating body and a center of gravity of the body is called metacentric height.

MG =BM-BG

MG=I/V-BG

Stability of a submerged body:-

Stable condition:-

- For stable condition w = f_b and the point B above the CG of the body.

Unstable equilibrium;-

- For unstable equilibrium w = f_b and the point B is below the CG of the body.

Neutral equilibrium:-

- If the force of buoyancy is act as CG of the body.

Stability of a floating body:-

- For stable condition w = f_b and the meta centre m is about the CG of the body.
- For unstable equilibrium w = f_b and the metacentre m is below CG of the body.

- In neutral equilibrium w = f_b and metacentre m is acting at CG of the body.

Chapter-II

KINEMATICS OF FLUID FLOW

Basic equation of fluid flow and their application:-

Energy of a Liquid in Motion:-

The energy, in general, may be defined as the capacity to do work. Though the energy exits in many forms, yet the following are important from the subject point of view:

1. Potential energy,
2. Kinetic energy, and
3. Pressure energy.

Potential Energy of a Liquid Particle in Motion:-

It is energy possessed by a liquid particle by virtue of its position. If a liquid particle is $Z\,m$
above the horizontal datum (arbitrarily chosen), the potential energy of the particle will be Z metre-kilogram (briefly written as mkg) per kg of the liquid. The potential head of the liquid, at
point, will be Z metres of the liquid.

Kinetic Energy of a Liquid Particle in Motion:-

It is the ehergy, possessed by a liquid particle, by virtue of its motion or velocity. If a liquid particle is flowing with a mean velocity of v metres per second; then the kinetic energy of the particle will be $V^2/2g$ mkg per kg of the liquid. Velocity head of the liquid, at that velocity, will be $V^2/2g$ metres of the liquid.

Pressure Energy of a Liquid Particle in Motion:-

It is the energy, possessed by a liquid particle, by virtue of its existing pressure. If a liquid particle is under a pressure of p kN/m2 (i.e., kPa), then the pressure energy of the particle-will be $\frac{p}{w}$. mkg per kg of the liquid, where w is the specific weight of the liquid. Pressure head of the liquid under that pressure will be $\frac{p}{w}$. metres of the liquid.

Total Energy of a Liquid Particle in Motion:-

The total energy of a liquid, in motion, is the sum of its potential energy, kinetic energy and pressure energy, Mathematically total energy,

$$E = Z + V^2/2g + \frac{p}{w}\text{. m of Liquid.}$$

Total Head of a Liquid Particle in Motion:-

The total head of a liquid particle, in motion, is the sum of its potential head, kinetic head and pressure head. Mathematically, total head,

$H = Z + V^2/2g + \frac{p}{w}$ m of liquid.

Example

Water is flowing through a tapered pipe having end diameters of 150 mm and 50 mm respectively. Find the discharge at the larger end and velocity head at the smaller end, if the velocity of water at the larger end is 2 m/s. Solution. Given: d_1 = 150mm = 0□15 m ; d_2= 50 mm= 0□05 m and V_1 = 2□5 m/s. Discharge at the larger end We know that the cross-sectional area of the pipe at the larger end,

$a_1 = \frac{\pi}{4} \times (0.15)2 = 17.67 \times 10^{-3} m^2$

and discharge at the larger end,

$Q_1 = a_1.v_1 = (17.67xl0^{-3})x2.5 = 44.2 \times 10^{-3} \ m^3/s$

= 44.2Jitres/s Ans.

Velocity head at the smaller end

We also know that the cross-sectional area of the pipe at the smaller end,

$A_2 = \frac{\pi}{4} \times (0.15)2 = 1.964 \times 10^{-3} m^2$

Since the discharge through the pipe is continuous, therefore

$a_1.v_1 = a_2.v_2$

or $v2 = \frac{a1.v1}{a2} = [(17.67 \times 10^{-3}) \times 2.5]/1.964 \times 10^{-3} = 22.5\ m/s$

:. Velocity head at the smaller end

$V_2^2/2g=(22.5)^2/2 \times 9.81=25.8\ m$ *Ans*

Bernoulli s Equation:-

It states, For a perfect incompressible liquid, flowing in a continuous stream, the total nergy; of a particle remains the same, while the particle moves from one point to another. This statement is based on the assumption that there are no losses due to friction in the pipe. Mathematically,

$Z + V2/2g + \frac{p}{w}$ = Constant

where

Z = Potential energy,

$V^2/2g$ =Kinetic energy, and

$\frac{p}{w}$ = Pressure energy.

Proof

Consider a perfect incompressible liquid, flowing through a non-uniform pipe as shown in Fig-

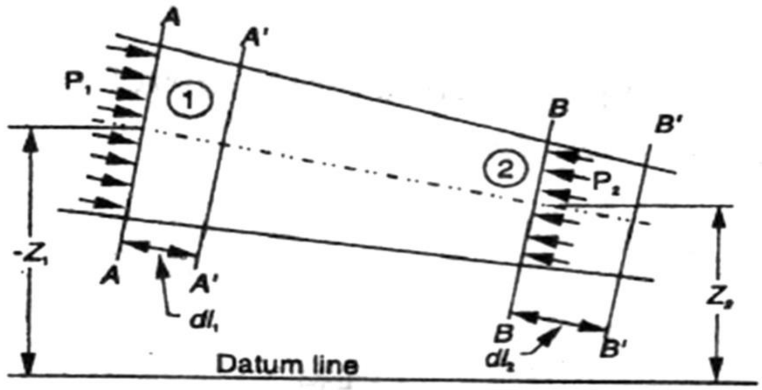


Fig. 2.1

Let us consider two sections AA and BB of the pipe. Now let us assume that the pipe is running full and there is a continuity of flow between the two sections.

Let
Z_1 = Height of AA above the datum,
P_1= Pressure at AA,
V_1 = Velocity of liquid at AA,
A_1 = Cross-sectional area of the pipe at AA, and
Z_2, P_2, V_2, A_2= Corresponding values at BB.
Let the liquid between the two sections AA and BB move to A' A' and B' B' through very small lengths dl_1 and dl_2 as shown in Fig. This movement of the liquid between AA and BB is equivalent to the movement 'of the liquid between AA and A' A' to BB and B' B', the remaining liquid between A' A' and BB being uneffected.
Let W be the weight of the liquid between AA and A' A'. Since the flow is continuous, therefore
$W = wa_1 dI_1 = wa_2 dL_2$

or $a_1 \times dl_1 = \frac{W}{w}$ ...(i)

Similarly $a_2 dl_2 = \frac{W}{w}$

$\therefore a_1 . dL_1 = a_2 dL_2$ ...(ii)

We know that work done by pressure at AA, in moving the liquid to A' A'
= Force x Distance = $P_1 . a_1 . dL_1$
Similarly, work done by pressure at BB, in moving the liquid to B' B'
$= -P_2 a_2 dl_2$
...(Minus sign is taken as the direction of P_2 is opposite to that of P_1)
:. Total work done by the pressure
$= P_1 a1 dl_1 - P_2 a_2 dl_2$
$= P_1 a1 dl_1 - p_{2a} 1 dl_1$

□$(a_1dl_1=a_2dl_2)$

$= a_1.dl_1 (P_1 - P_2) = \frac{W}{w} (P_1 - P_2)$ □$(a_1.dl_1 = \frac{W}{w})$

Loss of potential energy $= W (Z_1-Z_2)$

and again in kinetic energy $=W[(V_2^2/2g)-(V_1^2/2g)]=\frac{W}{2g}(v_2^2-v_1^2)$

We know that loss of potential energy + Work done by pressure
= Gain in kinetic energy

∴ $W (Z_1-Z_2)+ \frac{W}{w} (P_1 - P_2) = \frac{W}{2g}(v_2^2-v_1^2)$

$(Z_1-Z_2)+(p_1/w)-(p_2/w)=v_2^2/2g-v_1^2/2g$

Or $Z_1 + v_1^2/2g+ (p_1/w)=Z_2 + v_2^2/2g+(p_2/w)$

which proves the Bernoulli's equation.

Euler's Equation For Motion

The "Euler's equation for steady flow of an ideal fluid along a streamline is based on the
Newton's Second Law of Motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit weight of the flowing fluid. It is based on the 'following assumptions:

1. The fluid is non-viscous (i.e., the frictional losses are zero).
2. The fluid is homogeneous and incompressible (i.e., mass density of the fluid is constant).
3. The flow is continuous, steady and along the streamline.
4. The velocity of flow is uniform over the section.
5. No energy or force (except gravity and pressure forces) is involved in the flow.

Consider a steady' flow of an ideal fluid along a streamline. Now consider a small element
AB of the flowing fluid as shown in Fig.

Let
dA = Cross-sectional area of the fluid element,
ds = Length of the fluid element,
dW = Weight of the fluid 5!1ement,
p = Pressure on the element at A,
p + dp = Pressure on the element at B, and
v = Velocity of the fluid element.

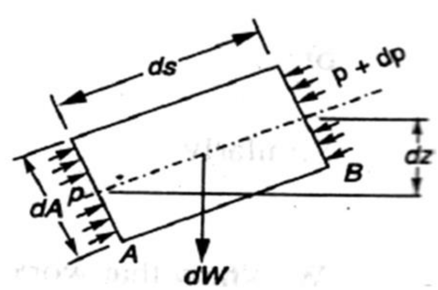


Fig. 2.2

We know that the external forces tending to accelerate element in the direction of the streamline
$= p. dA - (p + dp) dA$
$= -dp.dA$
We also know that the weight of the fluid element,

$dW = \rho$ g. dA . ds

From the geometry of the figure, we find that the component of the weight of the fluid element ,in the direction of flow

$= - \rho$ g . dA . ds $\cos\theta$

$= - \rho$ g . dA . ds$\left(\frac{dz}{ds}\right)$ $\quad\quad$ □$\cos\theta = \frac{dz}{ds}$

$= - \rho$ g. dA. dz

$\therefore$ *mass of the fluid element* $= \rho$ .dA.ds

,We see that the acceleration of the fluid element

$$\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v.\frac{dv}{ds}$$

Now, as per Newton's Second Law of Motion, we know that

Force = Mass x Acceleration

$$(-dp\,.\,dA) - (\rho\, g\,.\,dA\,.\,dz) = \rho\,.\,dA\,.\,ds \times \frac{dv}{ds}$$

$$\frac{dp}{\rho} + g.dz = v.dv$$

□(dividing both side by $-\rho dA$)

Or $\frac{dp}{\rho} + g.dz + v.dv = 0$

This is the required Euler's equation for motion and is in the form of a differential equation. Integrating the above equation, '

$$\frac{1}{\rho}\int dp + \int g.dz + \int v.dv = constant$$

$$\frac{p}{\rho} + gz + v^2/2 = constant$$

$$P + wZ + Wv^2/2g = constant$$

$\frac{p}{w}$+Z+ $v^2/2g$=constant (Dividing by w)

or in other words, $\frac{p_1}{w} + Z_1 + (v_1^2/2g) = \frac{p_2}{w} + Z_2 + (v_2^2/2g)$

which proves the Bernoulli's equation.

Limitations of Bernoulli's Equation:-

The Bernoulli's theorem or Bernoulli's equation has been derived on certain assumptions, which are rarely possible. Thus the Bernoulli's theorem has the following limitations:

1. The Bernoulli's equation has been derived under the assumption that the velocity of every liquid particle, across any cross-section of a pipe, is uniform. But, in actual practice, it is not so. The velocity of liquid particle in the centre of a pipe is maximum and gradually decreases towards the walls of the pipe due to the pipe friction. Thus, while using the Bernoulli's equation, only the mean velocity of the liquid should be

taken into account.

2. The Bernoulli's equation has been derived under the assumption that no external force, except the gravity force, is acting on the liquid. But, in actual practice, it is not so. There are always some external forces (such as pipe friction etc.) acting on the liquid, which effect the flow of the liquid. Thus, while using the Bernoulli's equation, all such external forces should be neglected. But, ifsorne energy is supplied to, or, extracted from the flow, the same should also be taken into account.
3. The Bernoulli's equation has been derived, under the assumption that there is. no loss of energy of the liquid particle while flowing. But, in actual practice, -it is rarely so. In a turbulent flow, some kinetic energy is converted into heat energy. And in a viscous flow, some energy is lost due to shear forces. Thus, while using Bernoulli's equation, all such losses should be neglected.
4. If the liquid is flowing in a curved path, the energy due to centrifugal force should also be taken into account.

Example

The diameter of a pipe changes from 200 mm at a section 5 metres-above datum = to 50 mm at a section 3 metres above datum. The pressure of water at first section is 500 kPa. If the velocity of flow at the first section is 1 m/s, determine the intensity of pressure at the second section.

Solution.'Gi~en: d_1= 200 mm = 0.2 m; Z_1= 5 m; d2 = 50 mm = 0.05 m z_2 = 3 m; p = 500/
kPa = 500 kN/m2 and V_1 = 1 mls.

Let

V_2 = Velocity of flow at section 2, and

P_2 = Pressure at section 2. We know that area of the pipe at section 1 $a_1=\frac{\pi}{4}\times 0.2^2=31.42\times 10^{-3}m^2$

and area of pipe at section 2 $a_1=\frac{\pi}{4}\times 0.05^2=1.964\times 10^{-3}m^2$

Since the discharge through the pipe is continuous,therefore $a_1 . V_1 = a_2 . V_2$

$V_2=\frac{a1.v1}{a2}$ =[(31.42× 10 -3)× 1]/ 1.964× 10 -3=16m/s

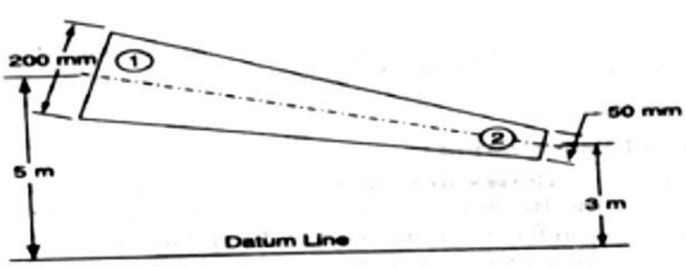


Fig. 2.3

Applying Bernoulli's equation for both the ends of the pipe,

$Z_1 + v_1^2/2g + (p_1/w) = Z_2 + v_2^2/2g + (p_2/w)$

$5+(1)^2/(2\times 9.81)+500/9.81=3+(16)^2/2X9.81+\frac{p2}{9.81}$

P2 = 40 x 9.81 = 392.4 kN/m^2 = 392.4 kPa Ans

practical Applications of Bernoulli's Equation

The Bernoulli's theorem or Bernoulli's equation is the basic equation which has the widest applications in Hydraulics and Applied Hydraulics. Since this equation is applied for the derivation
.of many formulae, therefore its clear understanding is very essential. Though the Bernoulli's equation has a number of practical applications. yet in this chapter we shall discuss its applications on the following hydraulic devices :

1. Venturi meter.
2. Orifice meter.
3. Pitot tube.

Venturimeter

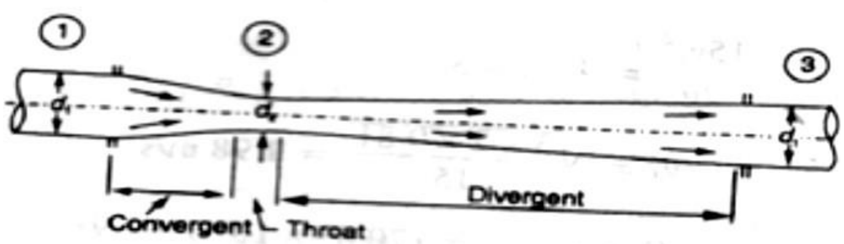


Fig. 2.4

A venturi meter is an apparatus for finding out the discharge of a liquid flowing in a pipe. A- venture meter, in its simplest form, consists of the following three parts:

(a) Convergent cone.
(b) Throat.
(c) Divergent cone.

(a) Convergent cone

It is a short pipe which converges from a diameter d_1 (diameter of the pipe. in which the venture meter is fitted) to a smaller diameter d_2: The convergent cone is also known as inlet of the venturi meter. The slope of the converging sides is between 1 in 4 or 1 in 5 as shown in Fig.

(b) Throat

It is a small portion of circular pipe in which the diameter d_2 is kept constant as shown in Fig.

(c) Divergent cone

It is a pipe, which diverges from a diameter d_2 to a large diameter d_1. The divergent cone is also known as outlet of the venture meter. The length of the divergent cone is about 3 to 4 times than that of the convergent cone as shown in Fig.

A little consideration will show that the liquid, while flowing through the venture meter, is accelerated between the sections 1 and 2 (i.e., while flowing through the convergent cone). As a result of the acceleration, the velocity of liquid at section 2 (i.e., at the throat) becomes higher than that at section 1. This increase in velocity results in considerably decreasing the pressure at section 2.1fthe pressure head at the throat falls below the separation head (which is 2.5 metres of water), then there will be a tendency of separation of the liquid flow, In order to avoid the tendency of separation at throat, there is always a fixed ratio of the diameter of throat and the pipe (i.e., dz/dt). This ratio varies from 1/4 to 3/4, but the most suitable value is 1/3 to 1/2.

The liquid, while flowing through the venture meter, is decelerated (i.e., retarded) between the sections 2 and 3 (i.e., while flowing through the divergent cone). As a result of this retardation, the velocity of liquid decreases which, consequently, increases the pressure. If the pressure is rapidly recovered, then there is every possibility for the stream of liquid to break away from the walls of the metre due to boundary layer effects. In order to avoid the tendency of breaking away the stream of liquid, the divergent cone is made sufficiently longer. Another reason for making the divergent cone longer is to minimise the frictional losses. Due to these reasons, the divergent cone is 3 to 4 times longer than convergent cone as shown in Fig.

Discharge through a Venturi meter

Consider a venture meter through which some liquid is flowing as shown in Fig.

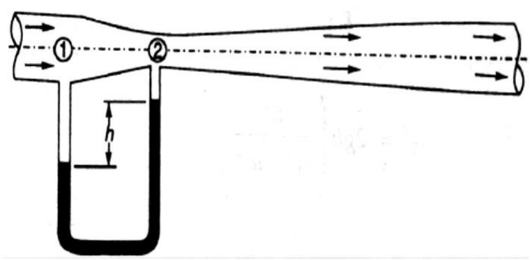


Fig. 2.5

Let

P_1 = Pressure at section 1,

V_1 = Velocity of water at section 1,

Z_1 = Datum head at section 1,

a_1 = Area of the venturi meter at section 1, and

p_2, v_2, z_2, a_2 = Corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2. i.e

$Z_1 + v_1^2/2g + (p_1/w) = Z_2 + v_2^2/2g + (p_2/w)$ □□..(1)

Let us pass our datum line through the axis of the venture meter as shown in Fig.

Now $Z_1=0$ and $Z_2=0$

$\therefore\ v_1^2/2g + (p_1/w) = v_2^2/2g + (p_2/w)$

Or $(p_1/w) - (p_2/w) = v_2^2/2g - v_1^2/2g$ □□.(2)

Since the discharge at sections 1 and 2 is continuous, therefore

$V_1 = a_2v_2/a_1$ $(a_1v_1 = a_2v_2)$

$V_1^2 = a_2^2v_2^2/a_1^2$ □□.(3)

Substituting the above value of v_1^2 in equation (2),

$\frac{p1}{w} - \frac{p2}{w} = v_2^2/2g - (a_2^2/a_1^2 X\ v_2^2/2g)$

$= v_2^2/2g(1 - a_2^2/a_1^2) = v_2^2/2g[(a_1^2 - a_2^2)/a_1^2]$

We know that $\frac{p1}{w} - \frac{p2}{w}$ is the difference between the pressure heads at sections 1 and 2 when the pipe is horizontal, this difference represents the venturi head and is denoted by h.

Or $h = v_2^2/2g[(a_1^2 - a_2^2)/a_1^2]$

Or $v_2^2 = 2gh[a_1^2/(a_1^2 - a_2^2)]$

$\therefore\ v_2 = \sqrt{2gh}[a_1/\sqrt{}(a_1^2 - a_2^2)]$

We know that the discharge through a venture meter,

Q = Coefficient of venturi meter x $a_2 v_2$

$= C.a_2v_2 = [Ca_1a_2/\sqrt{}(a_1^2 - a_2^2)] \times \sqrt{2gh}$

Example

A venture meter with a 150 mm diameter at inlet and 100 mm at throat is,laid with its axis horizontal and is used for measuring the flow of oil specific gravity O.9. The oil-mercury differential manometer shows a gauge difference of200 mm. Assume coefficient of the metre as O.9 Calculate the discharge in litres per minute.

Solution. Given: d_1 = 150 mm = 0.15 m; d_2 = 100 mm = 0.1 rn; Specific gravity of oil = 0.9

h = 200 mm = O.2 m of mercury and C = 0.98.

We know that the area at inlet,

$a_1 = \frac{\pi}{4} \times 0.15^2 = 17.67 \times 10^{-3} m^2$

and the area at throat,

$a_2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3} m^2$

We also know that the difference of pressure head,

H=0.2(13.6-0.9/0.9)=2.82 m of oil

and the discharge through the venturi meter,

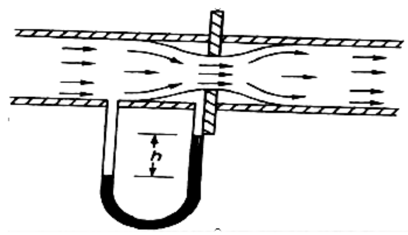
$Q = [Ca_1a_2/\sqrt{}(a_1^2-a_2^2)] \times \sqrt{2gh}$

$=63.9 \times 10^{-3} m^3/s = 63.9 lit/s$ Ans.

0rifice Metre

An orifice metre is used to measure the discharge in a pipe. An orifice metre, in its simplest form, consists of a plate having a sharp edged circular hole known as an orifice. This plate is fixed inside a pipe as shown in Fig. c A mercury manometer is inserted to know the difference

of pressures between the pipe an? the throat (i.e., or i



Let

h = Reading of the mercury manometer,

P_1 = Pressure at inlet,

V_1 = Velocity of liquid at inlet,

a_1 = Area of pipe at inlet, and

P_2, v_2, a_2 = Corresponding values at the throat.

Fig. 2.6

Now applying Bernoulli's equation for inlet of the pipe and the throat,

$Z_1 + v_1^2/2g + (p_1/w) = Z_2 + v_2^2/2g + (p_2/w)$ □□□(i)

$(p_1/w) - (p_2/w) = v_2^2/2g - v_1^2/2g$

Or $h = v_2^2/2g - v_1^2/2g = 1/2g(v_2^2 - v_1^2)$ □□□(ii)

Since the discharge is continuous, therefore $a_1.v_1 = a_2v_2$

$V_1 = a_2/a_1 \times v_2$ or $v_1^2 = a_2^2/a_1^2 \times v_2^2$

Substituting the above value of v_1^2 in equation (ii)

$h = 1/2g(v_2^2 - a_2^2/a_1^2 \times v_2^2) = v_2^2/2g \times (1 - a_2^2/a_1^2) = v_2^2/2g[(a_1^2 - a_2^2)/a_1^2]$

$\therefore$ $v_2^2 = 2gh[a_1^2/(a_1^2 - a_2^2)]$ or $v_2 = \sqrt{}2gh[a_1/\sqrt{}(a_1^2 - a_2^2)]$

We know that the discharge,

Q = Coefficient of orifice metre x $a_2 . v_2$

$=[Ca_1a_2/\sqrt{}(a_1^2-a_2^2)]\times\sqrt{2gh}$

Example. An orifice metre consisting of 100 mm diameter orifice in a 250 mm diameter pipe has coefficient equal to 0□65. The pipe delivers oil (sp. gr. 0□8). The pressure difference on the two sides of the orifice plate is measured by a mercury oil differential inano meter.lfthe differential gauge reads 80 mm of mercury, calculate the rate of flow in litresls.
Solution. Given: d_2 = 100 mm = 0.1 m; d_1 = 250 mm = o.25 m; C = 0.65 ; Specific gravity
of oil = 0.8 and h = 0.8 m of mercury.

We know that the area of pipe,

$a_1=\frac{\pi}{4}\times 0.25^2=49.09\times 10^{-3}m^2$

and area of throat

$a_2=\frac{\pi}{4}\times 0.1^2=7.854\times 10^{-3}m^2$

We also know that the pressure difference,

h = 0.8[(13.6-0.8)/0.8]=12.8 m of oil

and rate of flow,

$Q=[Ca_1a_2/\sqrt{}(a_1^2-a_2^2)]\times\sqrt{2gh}$

$=82\times 10^{-3}\ m^3/s$=82 lit/s Ans

Pitot Tube.

A Pitot tube is an instrument to determine the velocity of flow at the required point in a pipe or a stream. In its simplest form, a pitot tube consists of a glass tube bent a through 90° as shown in Fig.
The lower end of the tube faces the direction of the flow as shown in Fig. The liquid rises up in the tube due to the pressure exerted by the flowing liquid. By measuring the rise of liquid in the tube, we can find out the velocity of the liquid flow.
Let h = Height of the liquid in the pitot tube above the surface,

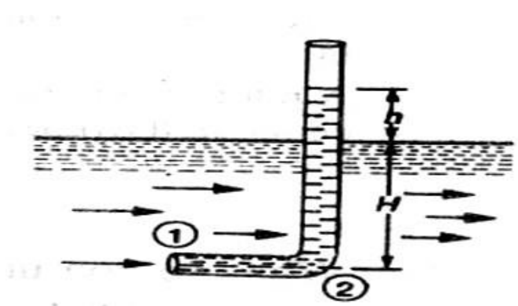


Fig. 2.7

H = Depth of tube in the liquid, and

v = Velocity of the liquid.

Applying Bernoulli's equation for the sections 1 and 2,

$H+v^2/2g=H+h$

□.$(z_1=z_2)$

$h = v^2/2g$

∴ $v = \sqrt{2gh}$

Example .

A pltot tube was inserted in a pipe to measu!e the velocity of water in it. If (

water rises the tube is 200 mm, find the velocity of water.

Solution. Given: $h = 200$ mm $= 0.2$ m.

We know that the velocity of water in the pipe,

$v = \sqrt{2gh} = \sqrt{(2 \times 9.81 \times 0.2)} = 1.98$ m/s Ans.

Rate of Discharge

The quantity of a liquid, flowing per second through a section of a pipe or a channel, is known as the rate of discharge or simply discharge. It is generally denoted by Q. Now consider a liquid flowing through a pipe.

Let, a = Cross-sectional area of the pipe, and

v = Average velocity of the liquid,

:. Discharge, $Q = \text{Area} \times \text{Average velocity} = a.v$

Notes: 1. If the area is in m^2 and velocity in m/s, then the discharge,

$Q = m^2 \times m/s = m^3/s$ = cumecs

2. Remember that $1m^3 = 1000$ litres.

Equation of Continuity of a Liquid Flow

If an incompressible liquid is continuously flowing through a pipe or a channel (whose cross- sectional area may or may not be constant) the quantity of liquid passing per second is the same at all sections. This is known as the equation of continuity of a liquid flow. It is the first and fundamental equation of flow.

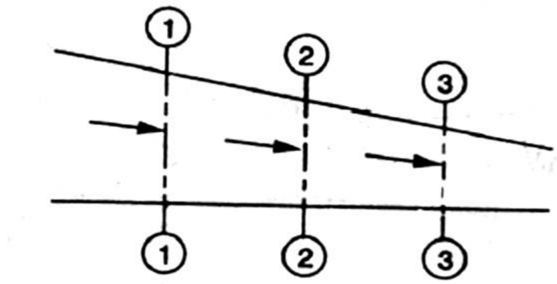


Fig. 2.8

CONTINUITY OF A LIQUID FLOW

Consider a tapering pipe through which some liquid is flowing as shown in Fig

Let , a_1 = Cross-sectional area of the pipe at section 1-1, and
v_1= Velocity of the liquid at section 1-1,
Similarly , a_2 , v_2= Corresponding values at section 2-2,
and a_3 , v_3 = Corresponding values at section 3-3.
We know that the total quantity of liquid passing through section 1-1,

$Q_1 = a_1.v_1$ □□□□□□□□□□□□(i)

Similarly, total quantity of liquid passing through section 2-2,

$Q_2= a_1.v_1$ □□□□□□□(ii)

and total quantity of the liquid passing through section 3-3,

$Q_3 = a_3.v_3$ □□□□□□□.(iii)

From the law of conservation of matter, we know that the total quantity of liquid passing through the sections 1-1, 2-2 and 3-3 is the same. Therefore
$Q_1 = Q_2 = Q_3 =$ .... or $a_1. v_1 = a_2.v_2 = a_3.v_3$ .... and so on.

Example : Water is flowing through a pipe of 100 mm diameter with an average velocity
10 m/s. Determine the rate of discharge of the water in litres/s. Also determine the velocity of water
At the other end of the pipe, if the diameter of the pipe is gradually changed to 200 mm.

Solution. Given: d_1= 100 mm = 0.1 m; V_1 = 10 m/s and d_2= 200 mm = 0.2 m.
Rate of discharge
We know that the cross-sectional area of the pipe at point 1,

$a_1= \left(\frac{\pi}{4}\right) x (0.1)^2 = 7.854x10^{-3}$ m^2

and rate of discharge, Q = $a_1.v_1$ = $(7.854 \text{ x } 10^{-3})$x 10 = 78.54 X 10^{-3} m^3/s
= 78.54 litres/s **Ans.**

Velocity of water at the other end of the pipe

We also know that cross-sectional area of the pipe at point 2,

$a_2 = \left(\frac{\pi}{4}\right) x (0.2)^2 = 31.42 x 10^{-3}\ m^2$

and velocity of water at point 2, $v_2 = \frac{Q}{a2}$ = ((78.54 X 10^{-3})/(31.42x10^{-3}))=2.5m/s **Ans.**

Flow over Notches:-

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonry structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonry structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

Classification Of Notches And Weirs:-

The notches are classified as :

I. According to the shape of the opening:

(a) Rectangular notch,

(b) Triangular notch,

(c) Trapezoidal notch, and

(d) Stepped notch.

2. According to the effect of the sides on the nappe:

(a) Notch with end contraction.

lb) Notch without end contraction or suppressed notch e,

Weirs are classified according to the shape of the opening the' shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

Discharge Over A Rectangular Notch Or Weir

The expression for discharge over a rectangular notch or weir is the same.

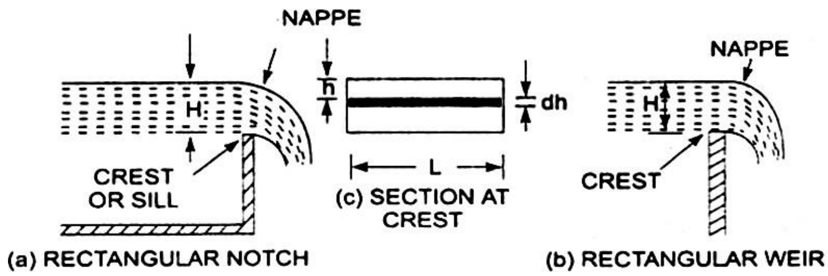


Fig. 2.9

Rectangular notch and 'weir:-

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig Let H = Head of water over the crest L = Length of the notch or weir

The total discharge, $Q=\frac{2}{3}\times c_d \times L \times \sqrt{2g[H]}_{3/2}$

Problem - 1

Find the discharge of water flowing over a rectangular notch 0/2 In length when the constant head over the notch is 300 mm. Take *cd* = 0.60.

Solution. Given:

Length of the notch, L=2.0m

Head over notch, H = 300 m = 0.30 m

C_d=0.06

Discharge $Q=\frac{2}{3}\times c_d \times L \times \sqrt{2g[H]}_{3/2}$

$=\frac{2}{3}\times 0.6 \times 2.0 \times \sqrt{2} \times 9.81 \times [0.30]_{1.5}$ m3/s

= 3.5435 x 0.1643 = 0.582 m3/s. Ans,

Problem 2

Determine the height of a rectangular weir of length 6 m to be built across a Rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8m and discharge is 2000 litres/s. Take Cd = 0.6 and neglect end contractions.

Solution. Given:

Length of weir, L=6m

Depth of water, H1=1.8m

Discharge, Q = 2000 litls = 2 m3/s

Cd/=0.6

Let H is the height of water above the crest of weir and H2 =height of weir

The discharge over the weir is given by the equation .

$Q=\frac{2}{3}\times c_d \times L \times \sqrt{2g}[H]_{3/2}$

$2=\frac{2}{3}\times 0.6 \times 6 \times \sqrt{2}\times 9.81 \times [H]_{3/2}$

$=10.623\ H^{3/2}$

$= H^{3/2}=\frac{2.0}{10.623}$

$H=\left(\frac{2.0}{10.623}\right)^{2/3}=0.328$ m

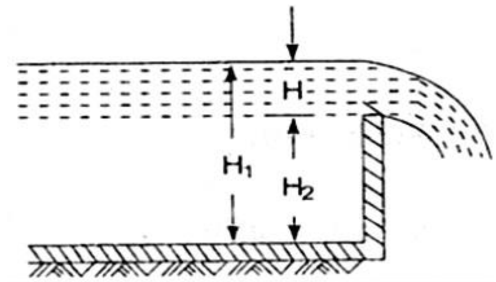


Fig. 2.10

Height of weir, H2 = H1- H

= Depth of water on upstream side - H

= 1.8 - .328 = 1.472 m. Ans.

Discharge Over A Triangular Notch Or Weir:-

The expression for the discharge over a triangular notch or weir is the same. It is derived as : Let H = head of water above the V- notch

θ = angle of notch

Total discharge, $Q=\frac{8}{15}\times C_d \times \frac{\tan\theta}{2}\times\sqrt{2g}\times H^{5/2}$

For a right angle V Notch ,if C_d=0.6

$\theta = 90^0$, $\tan\frac{\theta}{2} = 1$

Discharge, $Q = \frac{8}{15}\times 0.6 \times 1 \times \sqrt{2\times 9.81}\times H^{5/2}$

$=1.417\times H^{5/2}$

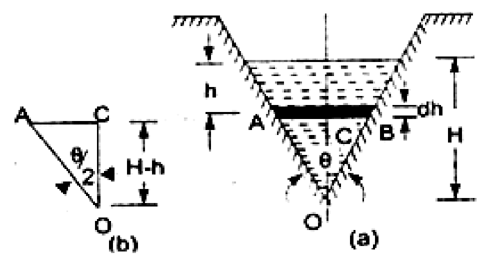


Fig. 2.11

Problem -1

Find the discharge over a triangular notch of angle 60° when the head over the

V-notch is 0.3 m. Assume C_d = 0.6.

Solution. Given an Angle of V-notch, e = 60°

Head over notch, H=0.3 m

C_d = 0.6

Discharge, Q over a V-notch is given by equation

$$Q=\frac{8}{15}\times C_d \times \frac{\tan\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\frac{8}{15}\times C_d \times \frac{0.6 \tan 60}{2} \times \sqrt{2\times 9.81} \times (0.3)^{5/2}$$

= 0.8182 x 0.0493 = 0.040 m3/s. Ans,

Problem -2

Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.

Solution. Given:

For rectangular weir. Length= L = 1 m

Depth of water, H = 150mm=0.15m

C_d = 0.62

For triangular weir.

θ = 90°

C_d = 0.59

Let depth over triangular weir = H_1

The discharge over the rectangular weir IS given by equation

$$Q=\frac{2}{3}\times c_d \times L \times \sqrt{2g}[H]^{3/2}$$

$$=\frac{2}{3}\times 0.62 \times 1.0 \times \sqrt{2\times 9.81} \times (0.15)^{3/2}$$

=0.10635 m^3/s

The same discharge passes through the triangular right-angled weir. But discharge. Q. is given by the equation

$$Q=\frac{8}{15}\times C_d\times\frac{\tan\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

$$0.10635=\frac{8}{15}\times 0.59\times\frac{\tan 90}{2}\times\sqrt{2g}\times H_1^{5/2}$$ { $\theta = 90^0$ and $H=H_1$}

$$=\frac{8}{15}\times 0.59\times 1\times 4.429\times H_1^{5/2}$$

$$=1.3936\ H_1^{5/2}$$

$$H_1^{5/2}=\frac{0.10635}{1.3936}=0.07631$$

$H_1=(0.07631)^{0.4}=0.3572$ m , Ans

Discharge Over A Trapezoidal Notch Or Weir:-

A trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = Height of water over the notch

L = Length of the crest of the notch

C_{d1} = Co-efficient or discharge. for rectangular portioo ABCD of Fig.

C_{d2} = Co-efficient of discharge for triangular portion [FAD and BCE]

The-discharge through rectangular portion ABCD is given by

or $$Q_1=\frac{2}{3}\times C_{d1}\times L\times\sqrt{2g}\times H^{3/2}$$

The discharge through two triangular notches FDA and BCE is equal to the discharge through a single triangular notch of angle e and it is given by equation

$$Q_2=\frac{2}{3}\times C_{d2}\times\frac{\tan\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

Discharge through trapezoldal notch or weir FDCEF = $Q_1 + Q_2$

$$=\frac{2}{3}\times C_{d1}L\sqrt{2g}\times H^{3/2}+\frac{8}{15}C_{d2}\times\frac{\tan\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

Problem -1 Find the discharge through a trapezoidal notch which is 1 m wide at the tap and 0.40 m at the bottom and is 30 cm in height. The head of water On the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.

Solution. Given:

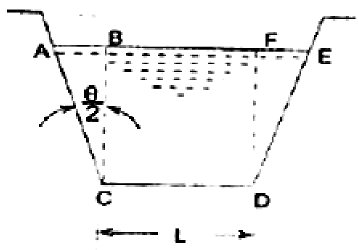
Top width AE=1 m

Base width, CD=L=0.4 m

Head of water, H=0.20 m

For rectangular portion, C_{d1}=0.62

From ΔABC ,we have



$$\frac{\tan\theta}{2}=\frac{AB}{BC}=\frac{\frac{AE-CD}{2}}{H}$$

$$=\frac{\frac{1.0-0.4}{2}}{0.3}=\frac{\frac{0.6}{2}}{0.3}=\frac{0.3}{0.3}=1$$

Fig. 2.12

Discharge through trapezoidal notch is given by equation

$$Q=\frac{2}{3}C_{d1}\times L\times\sqrt{2g}\times H^{3/2}+\frac{8}{15}C_{d2}\times\frac{\tan\theta}{2\sqrt{2g}}\times H^{5/2}$$

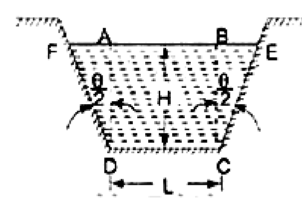
$$=\frac{2}{3}\times 0.62\times 0.4\times\sqrt{2\times 9.81}\times(0.2)^{3/2}+\frac{8}{15}\times 60\times 1\times\sqrt{2\times 9.81}\times(0.2)^{5/2}$$

= 0.06549 + 0.02535 = 0.09084 m^3/s = 90.84 litres/s. Ans

Discharge Over A Stepped Notch:-

A stepped notch is a combination of rectangular notches. The discharge through 'stepped notch is equal to the sum of the discharges' through the different rectangular notches.

Consider a stepped notch as shown in Fig.



Let H_1 = Height of water above the crest of notch (1).

L_1 = Length of notch 1,

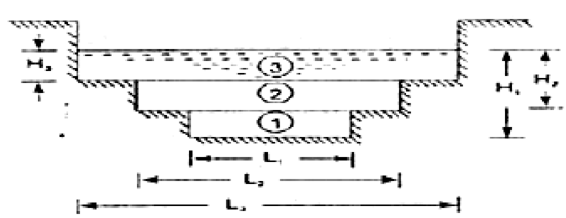
H_2,L_2 and H_3,L_3 are corresponding values for notches 2

C_d=Co-efficient of discharge for all notches

Total discharge $Q=Q_1+Q_2+Q_3$

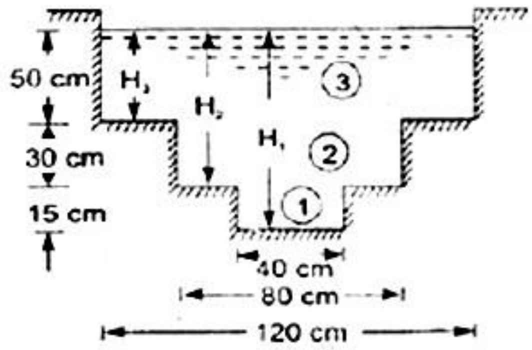
Fig. 2.12

$$Q=\frac{2}{3}\times C_d\times L_1\times\sqrt{2g}[H_1^{3/2}-H_2^{3/2}]+\frac{2}{3}\times C_d\times L_2\times\sqrt{2g}[H_2^{3/2}-H_3^{3/2}]+\frac{2}{3}C_d\times L_3\times\sqrt{2g}\times H_3^{3/2}$$



Problem Fig. 2.13

Fig. 1 shows a stepped notch. Find the discharge through the notch if Cd for all
section = 0.62.



Solution. Given:

L_1 = 40 cm, L_2 = 80 cm,

L_3 = 120cm

H_1 = 50 + 30 + 15 = 95 crn,

H_2=80 cm,H_3=50 cm,

C_d=0.62

Total Discharge ,$Q=Q_1+Q_2+Q_3$

where

$Q_1=\frac{2}{3}\times C_d\times L_1\times\sqrt{2g}[H_1^{3/2}-H_2^{3/2}]$

$=\frac{2}{3}\times 0.62\times 40\times\sqrt{2\times 981}\times[95^{3/2}-80^{3/2}]$

=154067cm^3/s =154.067 lit/s

$Q_2=\frac{2}{3}\times C_d\times L_2\times\sqrt{2g}[H_2^{3/2}-H_3^{3/2}]$

$=\frac{2}{3}\times 0.62\times 80\times\sqrt{2\times 981}\times[80^{3/2}-50^{3/2}]$

=530141 cm^3/s

=530.144 lit/s

$Q_3=\frac{2}{3}C_d\times L_3\times\sqrt{2g}\times H_3^{3/2}$

$=\frac{2}{3}\times 0.62\times 120\times\sqrt{2\times 981}\times 50^{3/2}$

=776771 cm^3/s

=776.771 lit/s

∴ $Q=Q_1+Q_2+Q_3$

=154.067+530.144+776.771

=1460.98 lit/s Ans.

Velocity Of Approach

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it. Thus if V_a is the velocity of approach, then an additional head h_a equal

to V_a^2 /2g due to velocity of approach, is acting on the water. flowing over the notch. Then initial height of water over the notch becomes ($H+ h_a$) and final height becomes equal to h_a,' Then all the formulae are

changed taking into consideration of velocity of approach.

The velocity of approach, V_a is determined by finding the discharge over the notch or weir neglecting velocity of approach. Then dividing the -discharge-by the cross-sectional area of the channel .on the' upstream side of the weir or notch, the velocity of approach is obtained . Mathematically,

$$V_a=\frac{Q}{Area\ of\ Channel}$$

This velocity of approach is used to find an additional head ($h_a= V_a^2$ /2g).Again the discharge is calculated and above process is repeated for more accurate discharge.

Discharge over a rectangular weir, with velocity of approach

$$=\frac{2}{3}\times C_d \times L \times \sqrt{2g}\,[(H_1+h_a)^{3/2}-h_a^{3/2}]$$

Problem:-

Water is flowing in a rectangular channel of 1 m wide and 0.75 m deep. Find the discharge over a rectangular weir of crest length 60 cm if the head of water over the crest of weir is

20 cm and water from channel flows over the weir. Take Cd = 0.62. Neglect end contractions. Take

velocity of approach into consideration.

Solution. Given:

Area of channel, A = Width x depth = 1.0 x 0.75 = 0.75 m^2

Length of weir, L = 60 cm = 0.6 m

Head of water, H_1 = 20 cm = 0.2 m

C_d = 0.62

Discharge over a rectangular weir without velocity of approach is given by

$Q=\frac{2}{3} C_d \times L \times \sqrt{2g} \times H_1^{3/2}$

$=\frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2}$

$=0.0982\ m^3/s$

velocity of approach $V_a=\frac{Q}{A}=\frac{0.0982}{0.75}=0.1309\frac{m}{s}$

Additional head $h_a=V_a^2/2g$

$=(0.1309)^2/2 \times 9.81=0.0008733\ m$

Then discharge with velocity of approach is given by equation

$Q=\frac{2}{3} \times C_d \times L \times \sqrt{2g}\,[(H_1+h_a)^{3/2}-h_a^{3/2}]$

$=2/3 \times 0.62 \times 0.6 \times \sqrt{(2 \times 9.81}[(0.2+0.00087)^{3/2}-(0.00087)^{3/2}]$

$= 1.098\ [0.09002- .00002566]$

$= 1.098 \times 0.09017$

$=0.09881\ m^3/s$. Ans

Types of Weirs :-

Though there are numerous types of weirs, yet the following are important from the subject point of view :

1. Narrow-crested weirs,
2. Broad-crested weirs,
3. Sharp-crested weirs,
4: Ogee weirs, and
5. Submerged or drowned weirs.

Discharge over a Narrow-crested Weir :-

The weirs are generally classified according to the width of their crests into two types. i.e.
narrow-crested weirs and broad crested weirs.

Let b = Width of the crest of the weir, and
H = Height of water above the weir crest.

If 2b is less than H,the weir is called a narrow-crested weir. But if 2b is more than H . it is called a broad-crested weir.
A narrow-crested weir is hydraulically similar to an ordinary weir or to a rectangular weir .Thus, the same formula for discharge over a narrow-crested weir holds good, which we derived from an ordinary weir .

$$Q = \frac{2}{3} \text{ X } C_d .L\sqrt{2g} \text{ x } H^{3/2}$$

Where, Q = Discharge over the weir,

Cd = Coefficient of discharge,

L = Length of the weir, and

H = Height of water level above the crest of the weir.

Example A narrow-crested weir of 10metres long is discharging water under a constant head of 400 mm. Find discharge over the weir in litresls. Assume coefficient of discharge as 0.623.

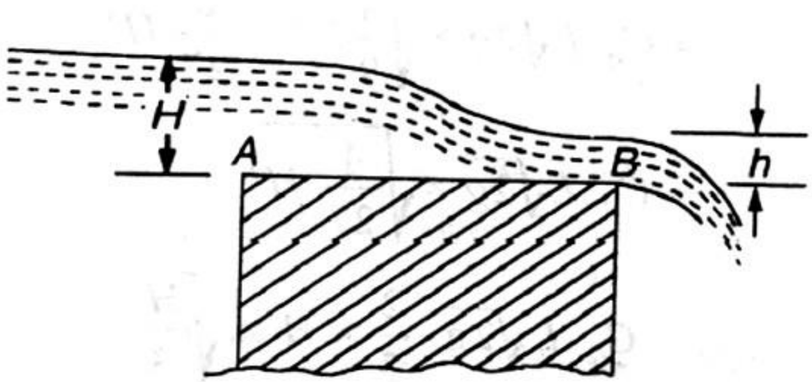
Solution. Given: L = 10 m; H= 400 mm = 0.4 m and C_d = 0.623.We know that the discharge over the weir,

$$Q = \frac{2}{3} \text{ X } C_d .L\sqrt{2g} \text{ x } H^{3/2}$$

$$= \frac{2}{3} \text{ x}0.623\text{x}10\sqrt{(2 \times 9.81)} \times (0.4)^{3/2}$$

$=46.55 \text{ m}^2/\text{s} = 4655 \text{ lit/s}$

Discharge over a Broad-crested Weir :-



Broad-crested weir

Consider a broad-crested weir as shown in Fig. Let A and B be the upstream and downstream ends of the weir.

Let H = Head of water on the upstream side of the weir (i.e., at A),

h = Head of water on the downstream side of the weir (i.e., at B),

v = Velocity of the water on the downstream side of the weir (i.e., at B),

C_d = Coefficient of discharge, and

L = Length of the weir.

$$Q = 1.71 C_d . L \times H^{3/2}$$

Example A broad-crested weir 20 m long is discharging water from a reservoir in to channel. What will be the discharge over the weir, if the head of water on the upstream and downstream sides is 1m and 0.5 m respectively? Take coefficient of discharge for the flow as 0.6 .

Solution. Given: L = 20 m; H = 1 m; h = 0.5 m and C_d = 0.6.

We know that the discharge over the weir,

$Q = C_d \times L . h \sqrt{2g(H-h)}$

$= 0.6 \times 2.0 \times 0.5 \times \sqrt{2 \times 9.81(1-0.5)}$ m^3/s

$= 6 \times 3.13 = 18.8$ m^3/s **Ans.**

Discharge over a Sharp-crested Weir :-

It is a special type of weir, having a sharp-crest as shown in Fig. The water flowing over the crest comes in contact with the crest -line and then springs up from the crest and falls as a trajectory as shown in Fig.

In a sharp-crested weir, the thickness of the weir is kept less than half of the height of water on the weir. i.e.,

$$b<(H/2)$$

where , b = Thickness of the weir,

and H = Height of water, above the crest of the weir.

The discharge equation, for a sharp crested weir, remains the same as that of a rectangular weir. i.e.,

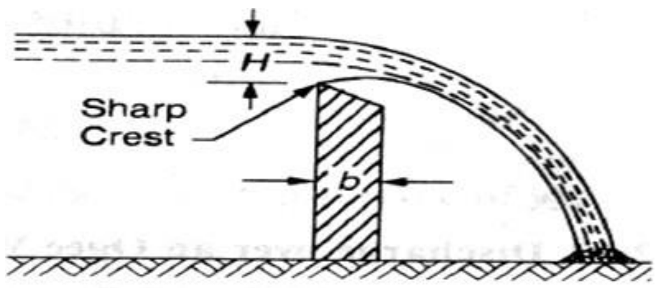


Fig. 2.16

Sharp-crested weir :-

$$Q = \frac{2}{3} \text{ X } C_d .L\sqrt{2g} \text{ x } H^{3/2}$$

Where, C_d = Coefficient of discharge, and
L = Length of sharp-crested weir

Example In a laboratory experiment, water flows over a sharp-crested weir 200 mm long under a constant head of 75mm. Find the discharge over the weir in litres/s, if C_d = 0.6.

Solution. Given: L = 200 mm = 0.2 m; H = 75 mm = 0.075 m and C_d = 0.6.

We know that the discharge over the weir,

$$Q = \frac{2}{3} \text{ X } C_d .L\sqrt{2g} \text{ x } H^{3/2}$$

$$= \frac{2}{3} \times 0.6 \times 0.2 \times \sqrt{2 \times 9.81} \times (0.075)^{3/2}$$

$= 0.0073\ m^3/s = 7.3$ litres/s. **Ans.**

Discharge over an Ogee Weir :-

It is a special type of weir, generally, used as a spillway of a dam as shown in Fig.

, The crest of an agee weir slightly rises up from the

point A ,(i.e., crest of the sharp-crested weir) and after reaching the maximum rise of 0.115 H (where H is the height of a water above the point A) falls in a parabolic form as shown in Fig.

The discharge equation for an agee weir remains the same as that of a rectangular weir. i.e.,

$Q = \frac{2}{3}$ X C_d .L$\sqrt{2g}$ x $H^{3/2}$

Where C_d = Co-efficient of discharge and

L= Length of an ogee weir

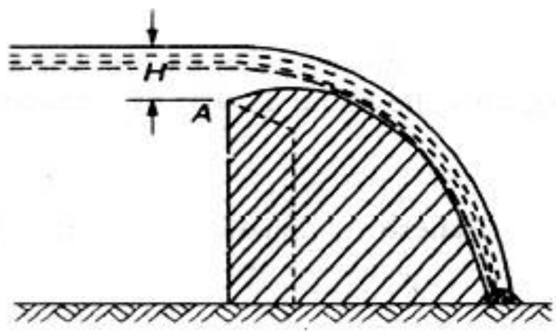


Fig. 2.17

Example

An ogee weir 4 metres long has 500 mm head of water. Find the discharge over the weir, if $C_d = 0.62$.

Solution. Given: L = 4 m; H = 500 mm = 0.5 m and C_d= 0.62.

We know that the discharge over the weir,

$Q = \frac{2}{3}$ X C_d .L$\sqrt{2g}$ x $H^{3/2}$

$= \frac{2}{3}$ X 0.62 X 4$\sqrt{2 X 9.81}$ X $(0.5)^{3/2}$ m^3/s

= 7.323 x 0.354 = 2.59 m^3/s= 2590 litres/s Ans

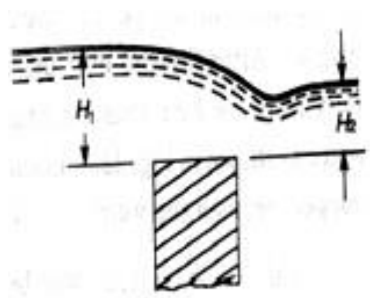
Discharge over a Submerged or Drowned Weir :-

When the water level on the downstream side of a weir is above the top surface of weir, it is known a submerged or drowned weir as shown in Fig

The total discharge, over such a weir, is found out by splitting up the height of water, above the sill of the weir, into two portions as discussed below:

Let H_1 = Height of water on the upstream side of the weir, and

H_2 =height of water on the downstream side of the weir.



The discharge over the upper portion may be considered as a free discharge under a head of water equal to (H_1 □ H_2). And the discharge over the lower portion may be considered as a submerged discharge under a head of H_2 . Thus discharge over the free portion (i.e., upper portion),

$$Q_1 = \frac{2}{3} \times C_d . L\sqrt{2g} \times (H_1 - H_2)^{3/2}$$

Submerged weir :-

and the discharge over the submerged (i.e., lower portion),

$Q_2 = C_d . L. H_2 . \sqrt{2g(H_1 - H_2)}$

:. Total discharge, $Q = Q_1 + Q_2$

Example A submerged sharp crested weir 0.8 metre high stands clear across a channel having vertical sides and a width of 3 meters. The depth of water in the channel of approach is 1.2 meter. And 10 meters downstream from the weir, the depth of water is 1 meter. Determine the discharge over the weir in liters per second. Take C_d as 0.6.

Solution. Given: L = 3 m and Cd = 0.6.

From the geometry of the weir, we find that the depth

of water on the upstream side,

H_1 = 1.25 - 0.8 = 0.45 m and depth of water on the downstream side,

H_2 = 1 - 0.8 = 0.2 m

We know that the discharge over the free portion of the weir

$Q_1 = \frac{2}{3} X\, Cd\, .L\sqrt{2g}\ x\ (H_1 - H_2)^{3/2}$

$= \frac{2}{3} \times 0.6 \times 3 \times \left(\sqrt{2 \times 9.81}\right)(0.45 - 0.20)_{3/2}$

= 5.315 x 0.125 = 0.664 m^3/s = 664 liters/s ... (i)

and discharge over the submerged portion of the weir,

$Q_2 = C_d\ .\ L.\ H_2. \sqrt{2g(H_1 - H_2)}$

= 0.6 x 3 x 0.2 $\sqrt{2 \times 9.81(0.45 - 0.2)}$ m^3/s

= 0.36 x 2.215 = 0.797 m^3/s = 797 liters/s ... (ii)

:. Total discharge: Q = $Q_1 + Q_2$ = 664 + 797 = 1461 liters/ s **Ans.**

Flow over Weirs:-

An open channel is a passage through which the water flows under the force of gravity - atmospheric pressure. Or in other words, when the free surface of the flowing water is in contact, with the atmosphere as in the case of a canal, a sewer or an aquaduct, the flow is said to be through an open channel. A channel may be covered or open at the top. As a matter of fact, the flow of water in an open channel, is not due to any pressure as in the case of pipe flow. But it is due to the slope the bed of the channel. Thus during the construction of a channel, a uniform slope in its bed is provided to maintain the flow of water.

Chezy's Formula for Discharge through an Open Channel :-

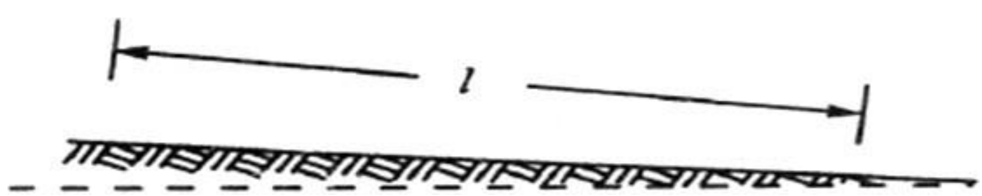


Fig. 2.19

Sloping bed of a channel :-

Consider an open channel of uniform cross-section and bed slope as shown in Fig.

Let

I = Length of the channel,

A = Area of flow,

v = Velocity of water,

p = Wetted perimeter of the cross-section, m=

f = Frictional resistance per unit area at unit velocity, and

i = Uniform slope in the bed.

$m = \frac{A}{P}$ □□.(known as hydraulic mean depth or hydraulic radious)

∴ Discharge $Q = A \times v = AC\sqrt{mi}$

Example.

A rectangular channel is 1. 5 metres deep and 6 metres wide. Find the discharge through channel, when it runs full. Take slope of the bed as 1 in 900 and Chezy's constant as 50.

Solution. Given: d = 1.5 m; b = 6 m; i = 1/900 and C = 50.

We know that the area of the channel,

$A = b.d = 6 \times 1.5 = 9\ m^2$

and wetted perimeter, $D = b + 2d = 6 + (2 \times 1.5) = 9$ m

:. Hydraulic mean depth, $m = \frac{A}{P} = 1$ m

and the discharge through the channel,

$Q = AC\sqrt{mi} = 9 \times 50\sqrt{(1 \times 1/900)} = 15 m^3/s$ Ans.

Manning Formula for Discharge :-

Manning, after carrying out a series of experiments, deduced the following relation for the value of C in Chezy's formula for discharge:

$C = \frac{1}{N} \times m^{1/6}$

where N is the Kutter's constant

Now we see that the velocity,

$v = C \sqrt{mi} = M \times m^{2/3} \times i^{1/2}$

where

M =1/N and is known as Manning's constant.

Now the discharge,

$Q = \text{Area} \times \text{Velocity} = A \times 1/N \times m^2 \times i^{1/2}$

$= A \times M \times m^{2/3} \times i^{1/2}$

Example

An earthen channel with a 3 m wide base and side slopes 1 : 1 carries water with a depth of 1 m. The bed slope is 1in 1600. Estimate the discharge. Take value of N in Manning's formula as 0.04.

Solution.

Given: b = 3 m; Side slopes = 1 : 1; d = 1 m; i = 1/1600 and N = 0.04.

We know that the area of flow,

$A = \frac{1}{2} \times (3 + 5) \times 1 = 4 \ m^2$

and wetted perimeter,

$P = 3 + 2 \times \sqrt{(1)^2+(1)^2} = 5.83 \ m$

∴ hydraulic mean depth m = A/P=4/5.83=0.686 m

We know that the discharge through the channel

$Q = \text{Area} \times \text{Velocity} = A \times 1/N \times m^{2/3} \times i^{1/2}$

$= 4 \times 1/0.04 \times 0.686^{2/3} \times (1/1600)^{1/2}$

$=1.945 \ m^3/s$ Ans

Channels of Most Economical Cross-sections :-

A channel, which gives maximum discharge for a given cross-sectional area and bed slope is called a channel of most economical cross-section. Or in other words, it is a channel which involves least excavation for a designed amount of discharge. A channel of most economical cross-section is, sometimes: also defined as a channel which has a minimum wetted perimeter; so that there is a minimum resistance to flow and thus resulting in a maximum discharge. From the above definitions,

it is obvious that while deriving the condition for a channel of most economical cross-section, the cross-sectional area is assumed tobe constant. The relation between depth and breadth of the section is found out to give the maximum discharge.

The conditions for maximum discharge for the following sections will be dealt with in the succeeding pages :

1. Rectangular section,
2. Trapezoidal section, and

3. Circular section.

Condition for Maximum Discharge through a Channel of Rectangular Section :-

A rectangular section is, usually, not provided in channels except in rocky soils where the faces of rocks can stand vertically. Though a rectangular section is not of
much practical importance, yet we shall discuss it for its theoretical importance only.

Consider a channel of rectangular section as shown in Fig.

Let

b = Breadth of the channel, and
d = Depth of the channel.
A= b X d
Discharge Q = A x v = AC $\sqrt{mi}$
m=A/P
=d/2

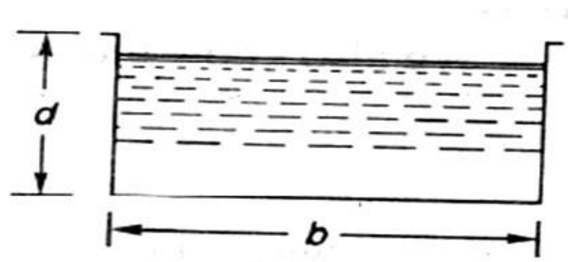


Fig. 2.20

Hence, for maximum discharge or maximum velocity, these two conditions (i.e., b = 2d and
m = d/2) should be used for solving the problems of channels of rectangularcross-sections.

Example

A rectangular channel has a cross-section of 8 square metres. Find its size and discharge through the most economical section, if bed slope is 1 in 1000. Take C = 55.

Solution. Given: A = 8 m2

i= 1/1000 = 0.001 and C = 55.
Size of the channel
Let
b = Breadth of the channel, and
d = Depth of the channel.
We know that for the most economical rectangular section,
b = 2d
:. Area (A) 8=b X d= 2d X d =$2d^2$
= b=2 m

And b=2d=2 X 2= 4 m

Discharge through the channel

We also know that for the most economical rectangular section, hydraulic mean depth,

m=d/2=2/2=1 m

and the discharge through the channel,

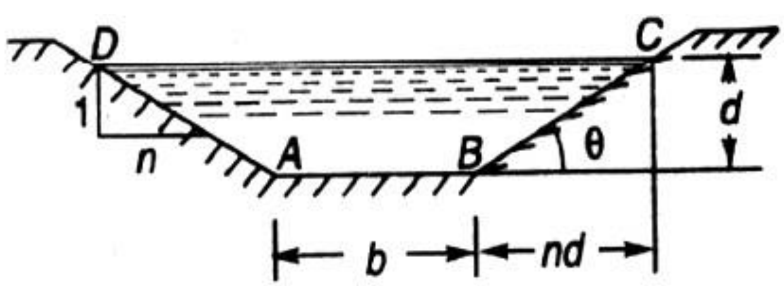
$Q = AC \sqrt{mi} = 8 \times 55\sqrt{1 \times 0.001}$ m^3/s

$= 440 \times 0.0316 = 13.9$ m^3/s , Ans.

Condition for Maximum Discharge through a Channel of Trapezoidal Section :-

A trapezoidal section is always provided in the earthen channels. The side slopes, in a channel of trapezoidal cross-section are provided, so that the soil can stand safely. Generally, the side slope in a particular soil is decided after conducting experiments on that soil. In a soft soil, flatter side slopes should be provided whereas in a harder one, steeper side slopes may be provided.

consider a channel of trapezoidal cross- section ABCD as shown in FIg.



Let

b = Breadth of the channel at the bottom,

d = Depth of the channel and

$\frac{1}{n}$ =side slope (i.e., 1 vertical to n horizontal)

Hence, for maximum discharge or maximum velocity these two (i.e., $\frac{b+2nd}{2} = d\sqrt{n^2+1}$ and m = d/2) should be used for solving

problems on channels of
trapezoidal cross-sections.

Example .

A most economical trapezoidal channel has an area of flow 3.5 m^2 discharge in the channel, when running 1 metre deep. Take C = 60 and bed slope 1 in 800.

Solution. Given: A = 3.5 m2 ; d = 1 m; C = 60 and i = 1/800.

We know that for the most economical trapezoidal channel the hydraulic mean depth

m = d/2 = 0.5 m

and discharge in the channel,

Q= A.C.$\sqrt{mi}$ = 5.25 m^3/s Ans.

Example .

A trapezoidal channel having side slopes of 1 : 1 and bed slope of 1 in 1200 is required to carry a discharge of 1800 m^3/min. Find the dimensions of the channel for cross-section. Take Chezy's constant as 50.

Solution.

Given side slope (n)=1

(i.e. 1 vertical to n horizontal),

i= 1/1200 ,Q= 180m^3/min = 3m^3/sec

and C = 50

Let b=breadth of the channel of its bottom and d= depth of the water flow.

We know that for minimum cross section the channel should be most economical and for economical trapezoidal section half of the top width is equal to the slopping side. i.e.

b +2nd/2 = d $\sqrt{n2 + 1}$

or b = 0.828d

∴ Area A = d X (b + nd) = 1.828d^2

We know that in the case of a most economical trapizodial section the hydraulic mean depth m=d/2

And discharge through the channel (Q)= A.C. $\sqrt{mi}$ =1.866$d^{5/2}$

∴ $d^{5/2}$ =3/1.866 = 1.608

Or d = 1.21 m

∴ b = 0.828 d = 0.828 X 1.21 = 1 m ANS

Condition for Maximum Velocity through a Channel of Circular Section :-

Consider a channel 'of circular section, discharging water under the atmospheric pressure shown in Fig.

Let r = Radius of the channel,

h = Depth of water in the channel, and

2θ = Total angle (in radians) subtended at the centre by the water

From the geometry of the figure, we find that the wetted perimeter of the channels,

$$P = 2r\theta \quad \text{...(i)}$$

and area of the section, through which the water is flowing,

$$A = r^2\theta - \frac{r^2 \sin 2\theta}{2} = r^2\left(\theta - \frac{\sin 2\theta}{2}\right) \quad \square\text{(ii)}$$

We know that the velocity of flow in an open channel,

Q= A.C.$\sqrt{mi}$

We know that the velocity of flow in an open channel , v= C$\sqrt{mi}$

Problem: Find the maximum velocity of water in a circular channel of 500 mm radius, if the bed slope is 1 in 400. Take manning s constant as 50.

Solution:-

Given d= 500mm = 0.5m or r = 0.5/2 = 0.25m, i=1/400 and M= 50

Let **2** θ = total angle (in radian) subtended by the water surface at the centre of the channel.

Now we know that for maximum velocity , the angle subtended by the water surface at the centre of the channel.

2 $\theta = 257^0 30$ or $\theta = 128.75^0 = 128.75 X \frac{\pi}{180} = 2.247$rad

∴ Area of flow, **A** $= r^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 171m^2$

And perimeter $P = 2r\theta = 1.124m$

∴ hydraulic mean depth $m = A/P = 0.171/1.124 = 0.152m$

And velocity of water $v = M X m^{2/3} X i^{1/2} = 0.71m/s$ ANS

PUMPS

Centrifugal Pumps:-

The hydraulic machines which convert the mechanical energy to hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted, into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point (i.e. , rise in pressure head = $\frac{v^2}{2g}$ or $\frac{\omega^2 r^2}{2g}$) . Thus at the outlet of the impeller, where radius is more , the rise in pressure head will be more & the liquid will be more & the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a high level.

Main Parts Of A Centrifugal Pump:-

The followings are the main parts of a centrifugal pump:

1. Impeller
2. Casing
3. Suction pipe with a foot valve & a strainer
4. Delivery Pipe

All the main parts of the centrifugal pump are shown in Fig 19.1

1. **Impeller**: The rotating part of a centrifugal pump is called impeller . It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

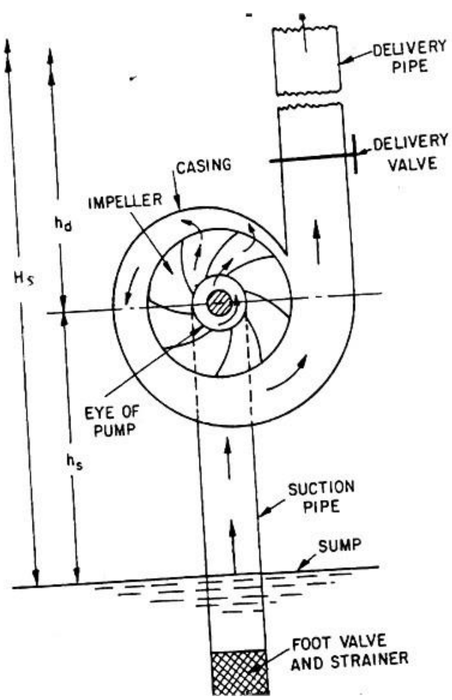
2. **Casing:** The casing of a centrifugal pump is similar to the casing of a reaction turbine. It is an air-tight passage surrounding the impeller & is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing & enters the delivery pipe. The following three types of the casings are commonly adopted:

a. Volute **casing** as shown in Fig.19.1

b. Vortex casing as shown in Fig.19.2(a)

c. Casing with guide blades as shown in Fig.19.2(b)

a) Volute casing as shown in Fig.3.1the Volute casing, which is surrounding the impeller. It is of spiral type in which area of flow increases gradually. The increase in area of flow decrease velocity of flow. Decrease in velocity increases the pressure of water flowing through casing. it has been observed that in case of volute casing, the efficiency of pump increases.



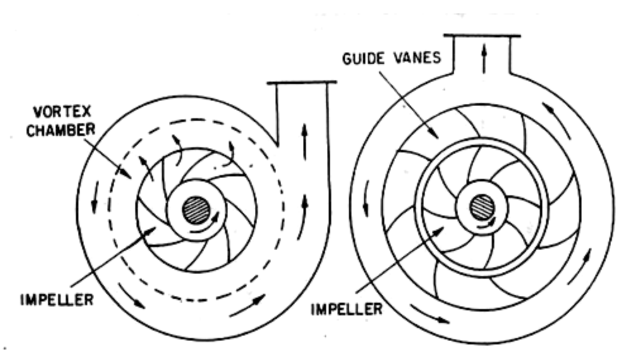
Main parts of a centrifugal pump

Fig. 3.1

b) Vortex casing. if a circular chamber is introduced between the casing and impeller as shown in fig.3.1,the casing is known as vortex casing .by introducing the circular chamber, loss of energy due to formation of eddies is reduced to a considerable extent. thus efficiency of pump is more than the efficiency when only volute casing is provided.

c) Casing with guide blades. This casing is shown in fig.3.1 in which the impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser. the guide vanes are designed in which a way that the water from the impeller enters the guide vanes without stock. Also the area of guide vanes increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water. the water from guide vanes then passes through the surrounding casing which is in most of cases concentric with the impeller as shown in fig.3.1.

3. **suction pipe with foot-valve and a strainer:** A pipe whose one end is connected to the inlet of pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one □way type valve is fitted at lower end of suction pipe. Foot valve opens only in upward direction. A strainer is also fitted at lower end of suction pipe.



(a) VORTEX CASING (b) CASING WITH GUIDE BLADES

Different type of casing

Fig: 3.2

4. **Delivery pipe:** a pipe whose one end is connected to outlet of pump and other end delivers water at a required height is known as delivery pipe.

Efficiencies of a centrifugal pump: Efficiencies of a centrifugal pump: In case of a centrifugal pump , the power is transmitted from the shaft of the electric motor to the shaft of the pump & then to the impeller. From the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller & then to the water. The following are the important efficiencies of a centrifugal pump:

a. Manometric efficiencies η_{man}

b. Mechanical efficiencies η_m

c. Overall efficiencies η_o

a) **Manometric Efficiencies** η_{man}**:** The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency. It is written as

η_{max} = Manometric head/Head imparted by impeller to water

$$= \frac{H_m}{\frac{V_{w2}u_2}{g}} = \frac{gH_m}{V_{w2}u_2} \text{.................}$$

The impeller at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

The power given to water at outlet of the pump= $\frac{WH_m}{1000}$ kW

The power at the impeller = $\frac{Work\ done\ by\ impeller\ per\ second}{1000} kW$

$$\frac{W}{g} \times \frac{V_{w2}u_2}{1000} \text{ kW}$$

$$\eta_{max} = \frac{\frac{WH_m}{1000}}{\frac{W}{g} \times \frac{V_{w2}u_2}{1000}} = \frac{gH_m}{V_{w2} \times u_2}$$

b) Mechanical efficiencies:-

The power at the shaft of the centrifugal pump is more than the power available at the impeller of the pump . The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency. It is written as

$\eta_m =$ Power at the impeller/Power at the shaft

The power at the impeller in kW=Work done by impeller per second/10000

$$= \frac{W}{g} \times \frac{V_{w2}u_2}{1000}$$

$$\eta_m = \frac{\frac{W}{g}\frac{V_{w2}u_2}{1000}}{S.P.} \text{..............}$$

Where S.P.= Shaft Power

c) **Overall efficiencies** η_o

It is defined as the ratio of power output of the pump to the power input to the pump . The power output of the pump in kW

$$= \frac{\text{Weight of water lifted} * H_m}{1000} = \frac{WH_m}{1000}$$

Power input to the pump =Power supplied by the electric motor

= S.P. of the pump

$$\eta_o = \frac{\frac{WH_m}{1000}}{S.P.} \text{................}$$

$$= \eta_{man} \times \eta_m \text{..................}$$

Problem 3.1: The internal & external diameters of the impeller of a centrifugal pump are 200mm & 400mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet & outlet are 20^0 & 30^0 respectively. The water enters the impeller radially & velocity of flow is constant. Determine the velocity of flow per metre sec.

Solution: **I**nternal Dia. Of impeller,=D_1=200mm=0.20m

External Dia. Of impeller ,=D_2=400mm=0.40m

Speed N=1200r.p.m

Vane angle at inlet , $\theta = 20^0$

Vane angle at outlet, $\varphi = 30^0$

Water enter s radially means, $\alpha = 90^0$ and $V_{w1} = 0$

Velocity of flow , =$V_{f1} = V_{f2}$

Tangential velocity of impeller at inlet & outlet are,

$$u_1 = \frac{\Pi D_1 N}{60} = \frac{\Pi \times .20 \times 1200}{60} = 12.56 m/s$$

$$u_2 = \frac{\Pi D_2 N}{60} = \frac{\Pi \times .40 \times 1200}{60} = 25.13 m/s$$

From inlet velocity triangle,

$$\tan\varphi = \frac{V_{f1}}{u_1} = \frac{V_{f2}}{12.56}$$

$$V_{f1} = 12.56 \tan\theta = 12.56 \times \tan 20 = 4.57 m/s$$

$$V_{f2} = V_{f1} = 4.57 m/s$$

Problem 3.2: A centrifugal pump delivers water against a net head of 14.5 metres & a design speed of 1000r.p.m .The values are back to an angle of 30^0 with the periphery. The impeller diameter is 300mm & outlet width 50mm. Determine the discharge of the pump if manometric efficiency is 95%.

Solution: Net head, $H_m = 14.5$m

Speed, N =1000r.p.m

Vane angle at outlet, $\varphi = 30^0$

Impeller diameter means the diameter of the impeller at outlet

Diameter, $D_2 = 300\text{mm} = 0.30\text{m}$

Outlet width, $B_2 = 50\text{mm} = 0.05\text{m}$

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

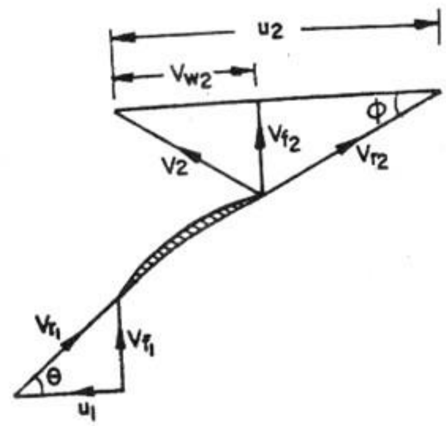
Tangential velocity of impeller at outlet, $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .30 \times 1000}{60} = 15.70\text{m/s}$

Now using equation

$$\eta_{max} = \frac{gH_m}{V_{w2}u_2}$$

$$0.95 = \frac{9.81 \times 14.5}{V_{w2} \times 15.70}$$

$$V_{w2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 m/s$$



Refer to fig(3.3). From outlet velocity triangle, we have

$$\tan\varphi = \frac{V_{f2}}{(u_2 - V_{w2})}$$

$$\tan 30^0 = \frac{V_{f2}}{(15.70 - 9.54)} = \frac{V_{f2}}{6.16}$$

$$V_{f2} = 6.16 \times \tan 30^0 = 3.556 m/s$$

$$\text{Discharge} = Q = \pi \times D_2 \times B_2 \times V_{f2}$$

$$= \pi \times 0.30 \times 0.05 \times 3.556 m^3/s = 0.1675 m^3/s$$

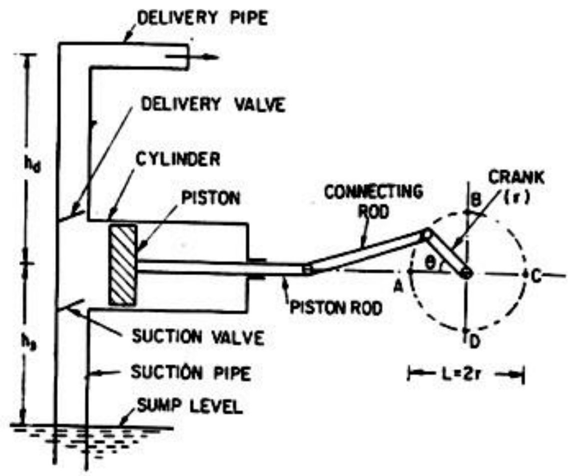
Reciprocating Pump:-

Introduction:-

We have defined the pumps as the hydraulic machines which convert the mechanical energy to hydraulic energy which is mainly in the form of pressure energy. If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on the liquid & increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump.

Main parts of a reciprocating pump:-

The following are the main parts of a reciprocating pump as shown in fig (3.4)



Main parts of a reciprocating pump.

1. A cylinder with a piston, piston rod, connecting rod and a crank,
2. Suction pipe,
3. Delivery pipe,
4. Suction valve, and
5. Delivery valve.

Fig. 3.4

Discharge through a Reciprocating Pump: Consider a single acting reciprocating pump as shown in fig ().

Let D= dia. Of the cylinder

A= C/s area of the piston or cylinder

$= \frac{\pi}{4} D^2$

r= Radius of crank

N=r.p.m of the crank

L=Length of the stroke=2*r

h_s = height of the axis of the cylinder from water surface in sump

h_d = Height of the delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$$= \text{Area} * \text{Length of stroke} = A*L$$

Number of revolution per second, $= \frac{N}{60}$

Discharge of the pump per second , Q= Discharge in one direction × No. of revolution per second

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60}$$ □□□□□□□□□□□.

Wt. of water delivered per second, W = $\rho g Q = \frac{\rho g ALN}{60}$ □□□□□□□□□□□□

Work done by Reciprocating Pump : Work done by the reciprocating pump per sec. is given by the reaction as

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d)$$

Where $(h_s + h_d)$ = Total height through which water is lifted

From equation () Weight, W is given by $W = \frac{\rho g ALN}{60}$

Substituting the value of W in equation () we get

Work done per second =

$$\frac{\rho g ALN}{60}(h_s + h_d)$$ □□□□□□□□□□□□□□□□□

Power required to drive the pump, in kW $P = \frac{\text{Work done per second}}{1000} = \frac{\rho \times g \times ALN(h_s + h_d)}{60 \times 1000}$

$$= \frac{\rho g ALN(h_s + h_d)}{60{,}000} \text{kW}$$ □□□□□□□□□□

Classification of reciprocating pumps:

The reciprocating pumps may be classified as:

1. According to the water being in contact with one side or both sides of the piston, and
2. According to the number of cylinders provided

If the water is in contact with one side of the piston, the pump is known as single-acting. On the other hand,

If the water is in contact with both sides of the piston, the pump is called double □acting. Hence, classification according to the contact of water is:

I. Single-acting pump

II. Double □acting pump

According to the number of cylinder provided, the pumps are classified as:

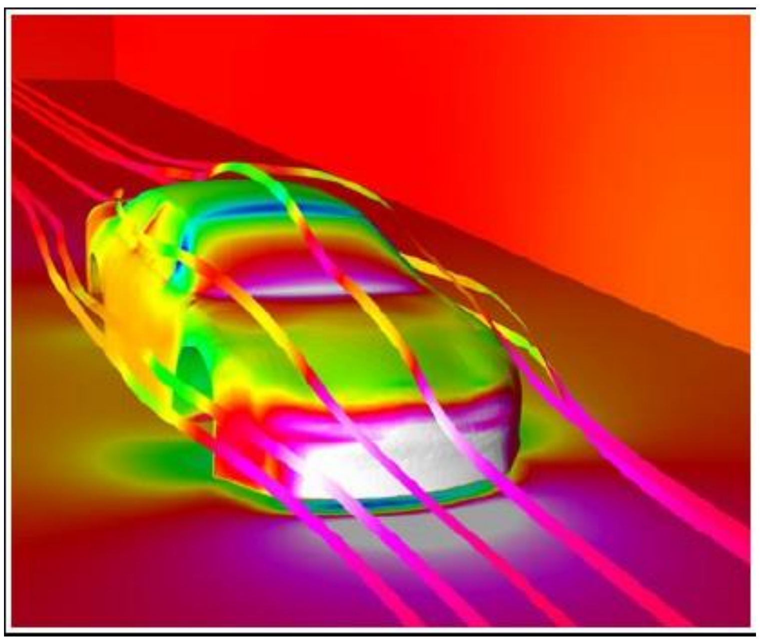
I. Single cylinder pump

II. Double cylinder pump

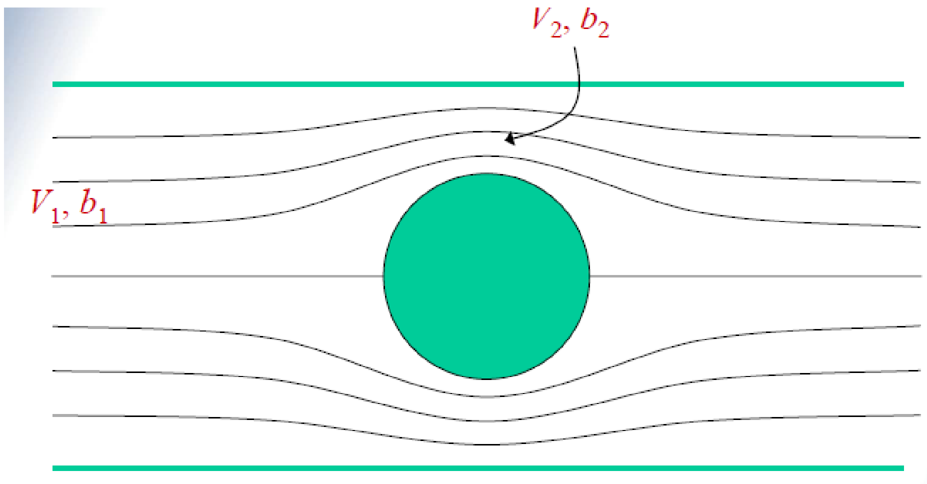
III. Triple cylinder pump

Fluid Kinematics

- Branch of fluid mechanics which deals with response of fluids in motion without considering forces and energies in them.
- The study of *kinematics* is often referred to as the *geometry of motion*.



CAR surface pressure contours and streamlines



Flow around cylindrical object

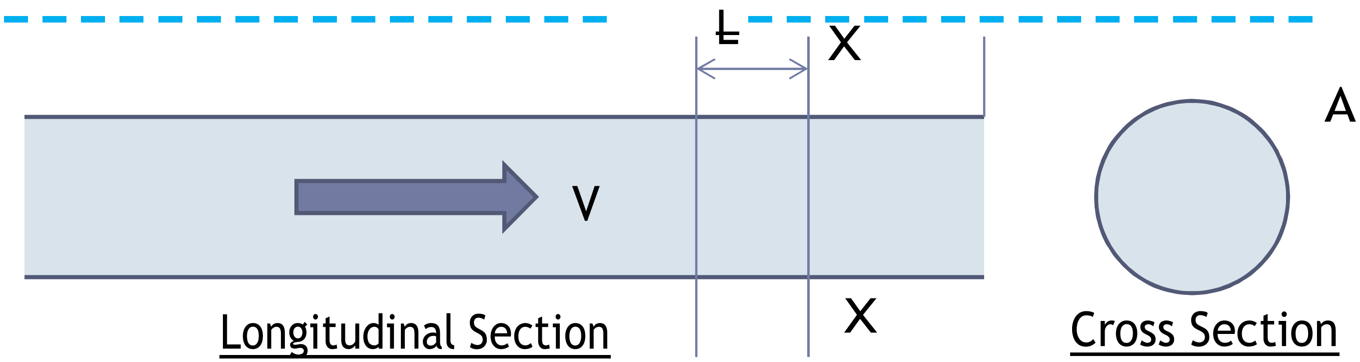
Fluid Flow

- **Rate of flow:** Quantity of fluid passing through any section in a unit time.

$$\text{Rate of flow} = \frac{\text{Quantity of fluid}}{\text{time}}$$

- **Type:**
  - 1.Volume flow rate: $= \frac{\text{volume of fluid}}{\text{time}}$
  - 2. Mass flow rate $= \frac{\text{mass of fluid}}{\text{time}}$
  - 3.Weigh flow rate $= \frac{\text{weight of fluid}}{\text{time}}$

Fluid Flow



- Let's consider a pipe in which a fluid is flowing with mean velocity, V.
- Let, in unit time, t, volume of fluid (AL) passes through section X-X,

Units

- 1.Volume flow rate: $$Q = \frac{\text{volume of fluid}}{\text{time}} = \frac{AL}{t}$$
- 2. Mass flow rate $$M = \frac{\text{mass of fluid}}{\text{time}} = \frac{\rho(AL)}{t}$$
- 3.Weigh flow rate $$G = \frac{\text{weight of fluid}}{\text{time}} = \frac{\rho g(AL)}{t} = \frac{\gamma(AL)}{t}$$

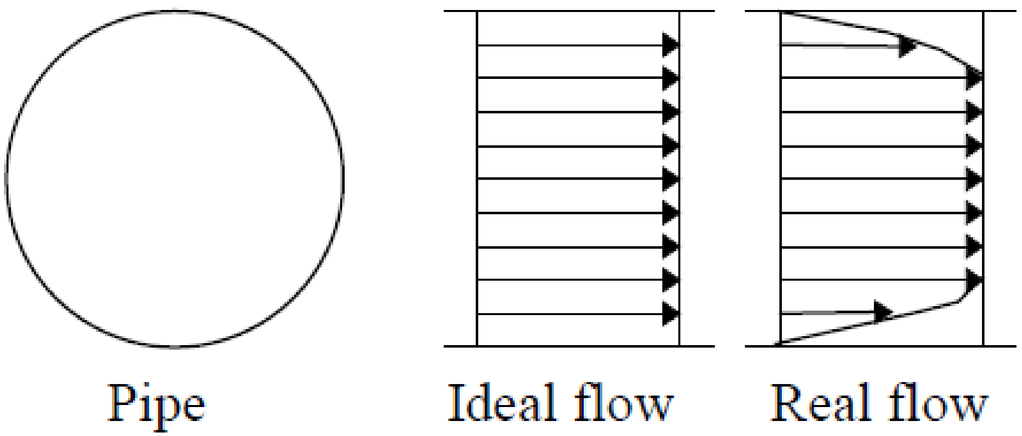
Types of Flow

- **<u>Depending upon fluid properties</u>**
- Ideal and Real flow
- Incompressible and compressible

- **<u>Depending upon properties of flow</u>**
- Laminar and turbulent flows
- Steady and unsteady flow
- Uniform and Non-uniform flow

Ideal and Real flow

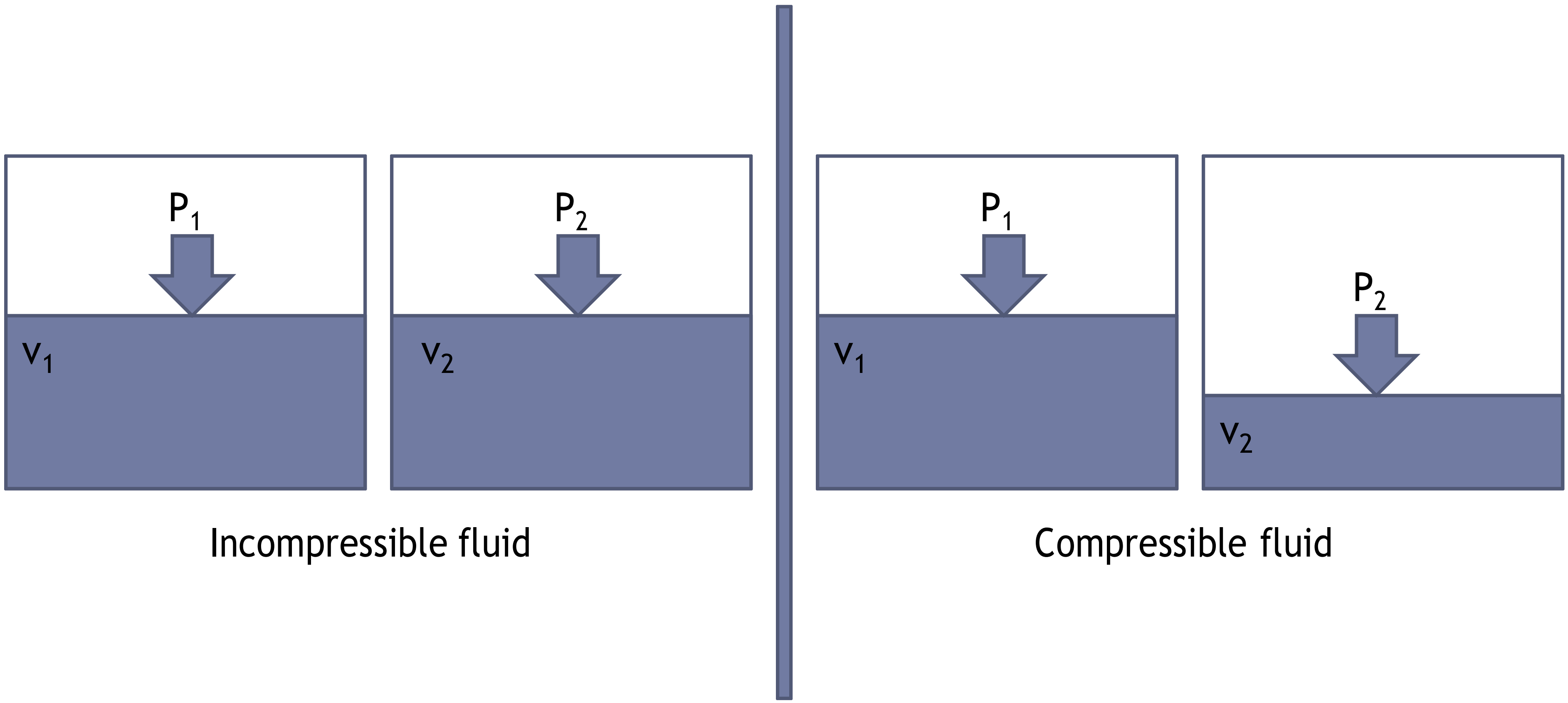
- Real fluid flows implies friction effects. Ideal fluid flow is hypothetical; it assumes no friction.



Velocity distribution of pipe flow

Compressible and incompressible flows

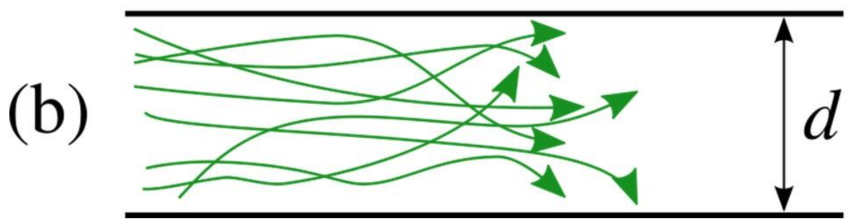
- Incompressible fluid flows assumes the fluid have constant density while in compressible fluid flows density is variable and becomes function of temperature and pressure.



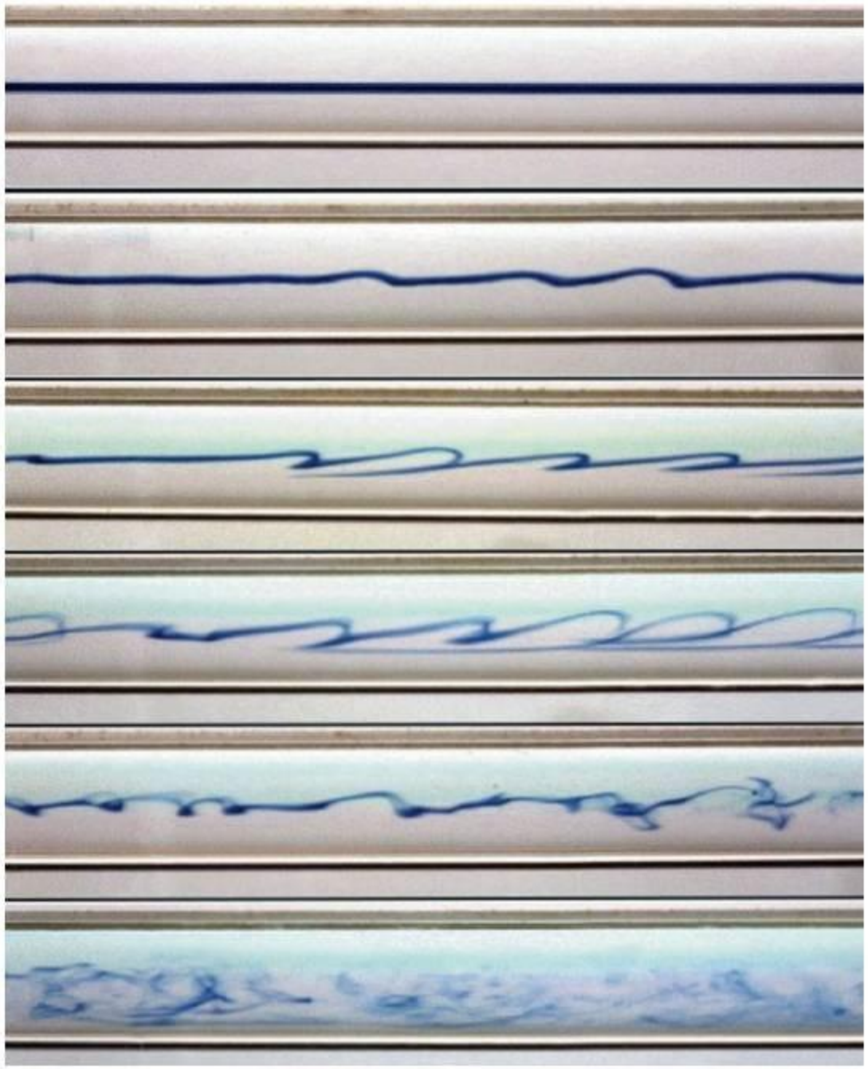
Laminar and turbulent flow

- The flow in laminations (layers) is termed as laminar flow while the case when fluid flow layers intermix with each other is termed as turbulent flow.

Laminar flow



Turbulent flow

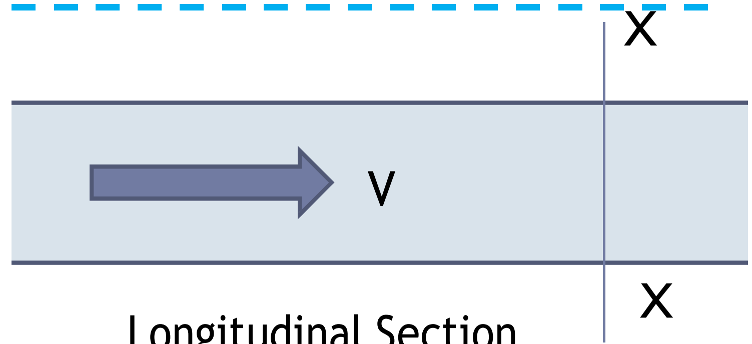


Transition of flow from Laminar to turbulent

- Reynold's number is used to differentiate between laminar and turbulent flows.

Steady and Unsteady flows

- **Steady flow:** It is the flow in which conditions of flow remains constant w.r.t. time at a particular section but the condition may be different at different sections.
- Flow conditions: velocity, pressure, density or cross-sectional area etc.
- e.g., A constant discharge through a pipe.



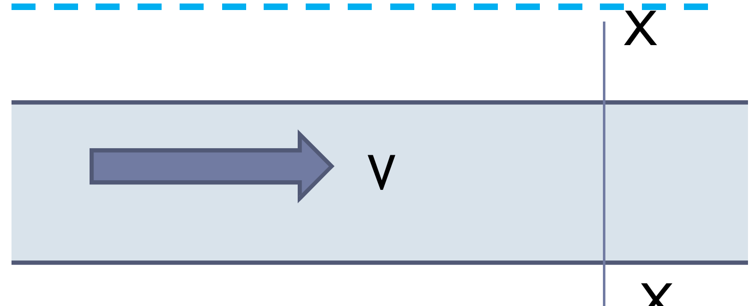
$$\frac{\partial V}{\partial t} = 0; \Rightarrow V = contt$$

- **Unsteady flow:** It is the flow in which conditions of flow changes w.r.t. time at a particular section.
- e.g.,A variable discharge through a pipe

$$\frac{\partial V}{\partial t} \neq 0; \Rightarrow V = \text{variable}$$

Uniform and Non-uniform flow

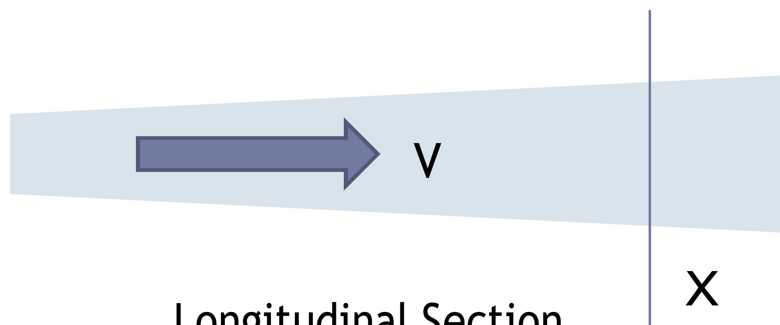
- **Uniform flow:** It is the flow in which conditions of flow remains constant from section to section.
- e.g., Constant discharge though a constant diameter pipe



Longitudinal Section

$$\frac{\partial V}{\partial x} = 0; \Rightarrow V = contt$$

- **Non-uniform flow:** It is the flow in which conditions of flow does not remain constant from section to section.
- e.g., Constant discharge through variable diameter pipe

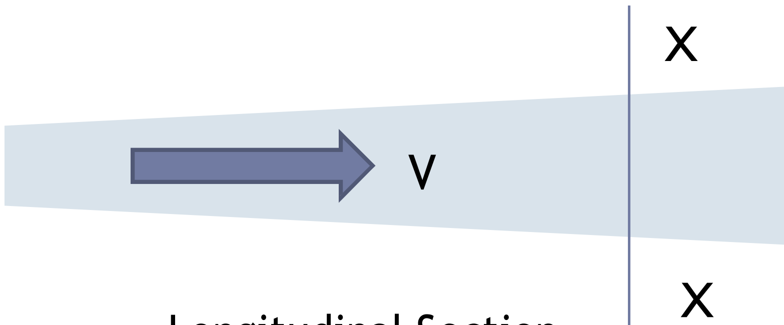


Longitudinal Section

$$\frac{\partial V}{\partial x} \neq 0; \Rightarrow V = \text{variable}$$

Describe flow condition

- Constant discharge though non variable diameter pipe



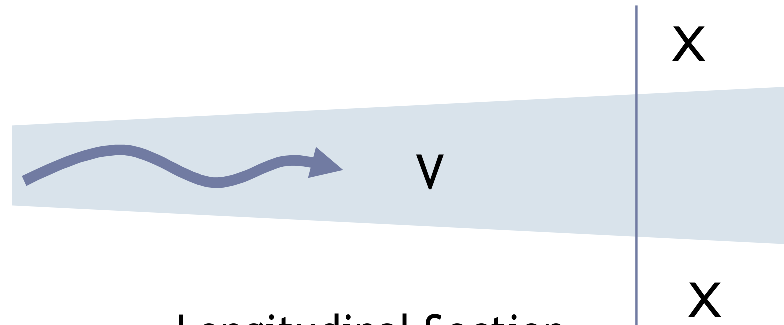
Longitudinal Section

Steady flow !!

Non-uniform flow !!

Steady-non-uniform flow

- Variable discharge though non variable diameter pipe



Longitudinal Section

Unsteady flow !!

Non-uniform flow !!

unsteady-non-uniform flow

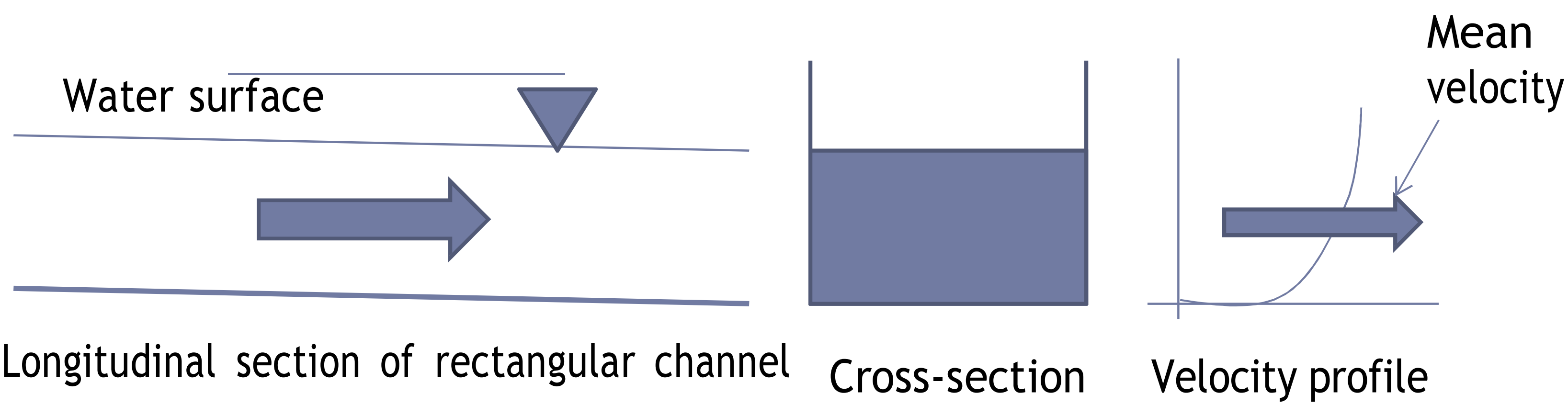
11

Flow Combinations

Type	Example
1. Steady Uniform flow	Flow at constant rate through a duct of uniform cross-section
2. Steady non-uniform flow	Flow at constant rate through a duct of non-uniform cross-section (tapering pipe)
3. Unsteady Uniform flow	Flow at varying rates through a long straight pipe of uniform cross-section.
4. Unsteady non-uniform flow	Flow at varying rates through a duct of non-uniform cross-section.

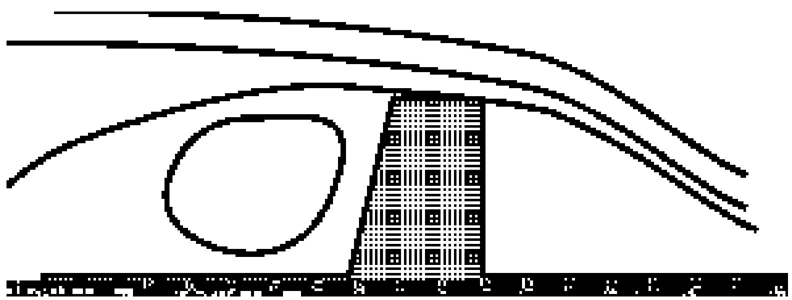
One, Two and Three Dimensional Flows

- Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much more simple.
- Flow is *one dimensional if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in* time only vary in the direction of flow and not across the cross-section



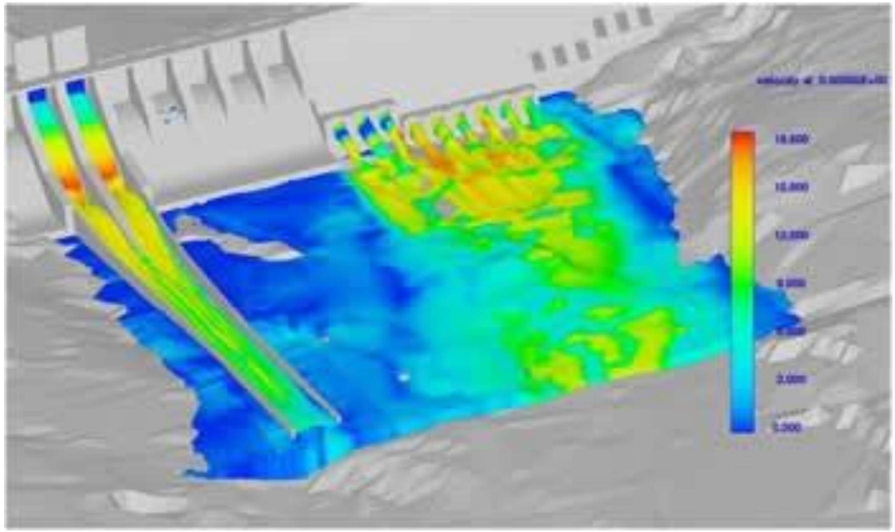
One, Two and Three Dimensional Flows

- Flow is *two-dimensional if it can be assumed that the flow parameters vary in the direction of flow and in* one direction at right angles to this direction



Two-dimensional flow over a weir

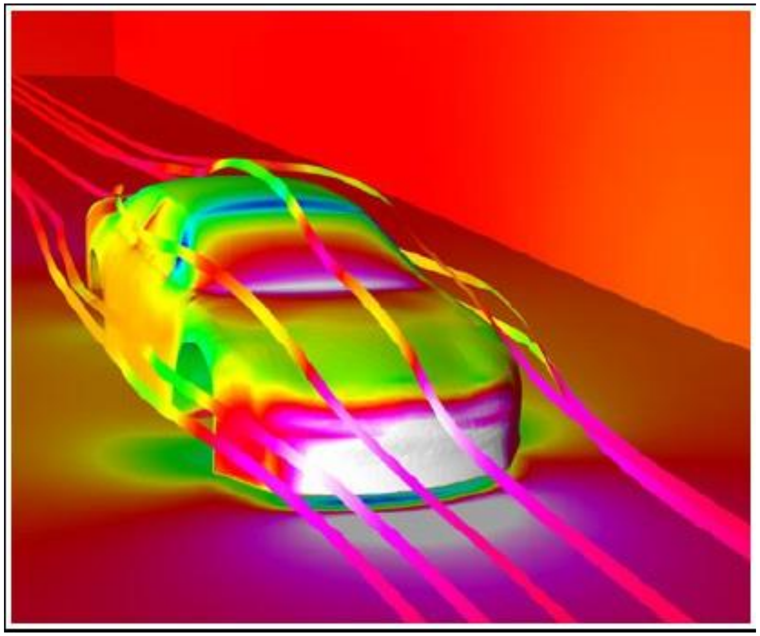
- Flow is *three-dimensional if the flow parameters vary in all three directions of flow*



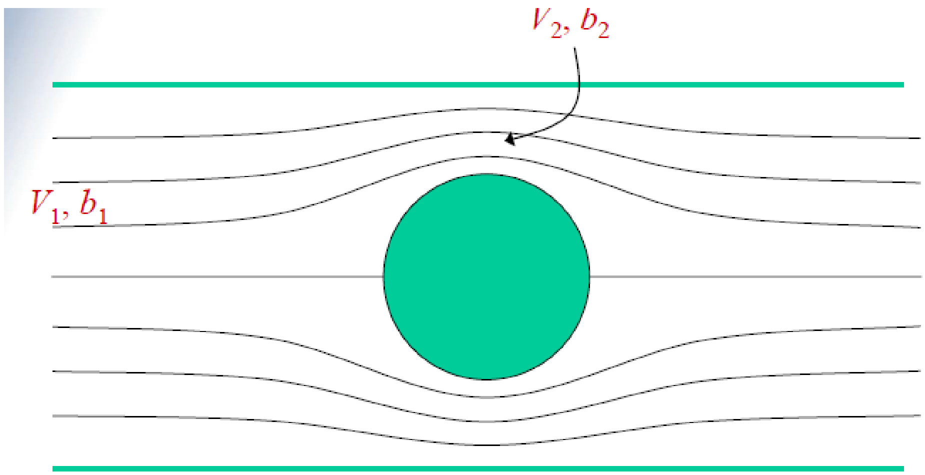
Three-dimensional flow in stilling basin

Visualization of flow Pattern

- The flow velocity is the basic description of how a fluid moves in time and space, but in order to **visualize the flow pattern** it is useful to define some other properties of the flow. These definitions correspond to various experimental methods of visualizing fluid flow.

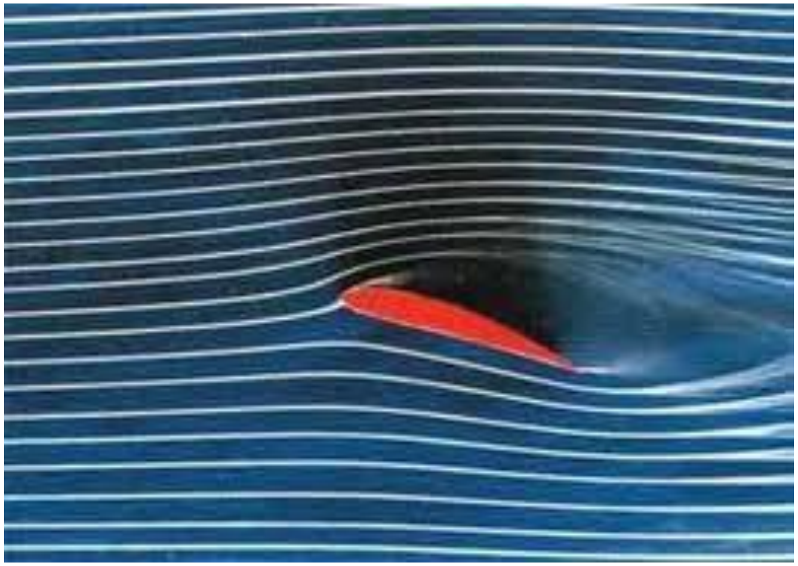


CAR surface pressure contours and streamlines



Flow around cylindrical object

Visualization of flow Pattern

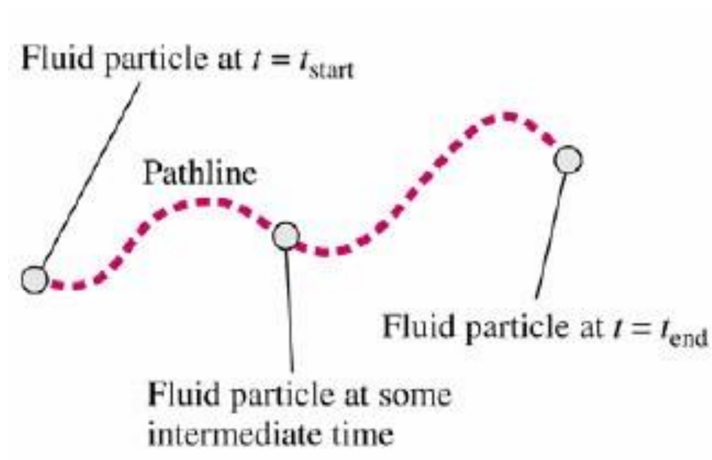


Streamlines around a wing shaped body

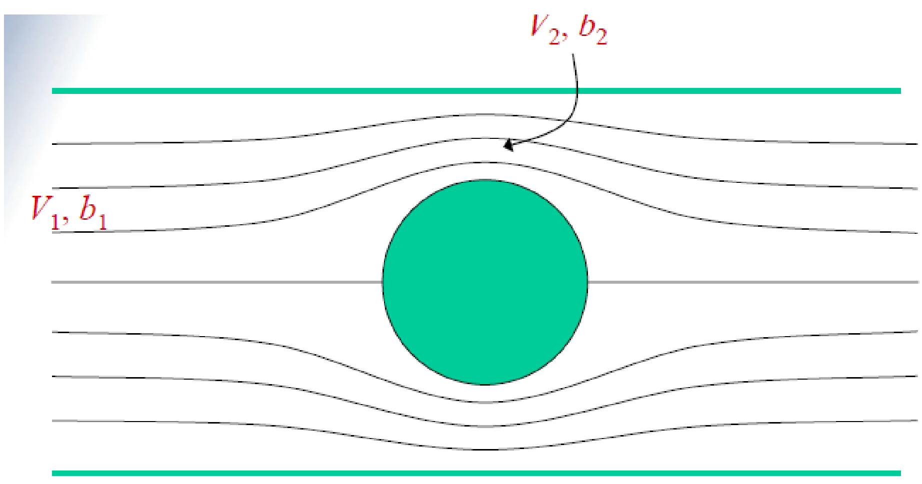
Flow around a skiing athlete

Path line and stream line

- **Pathline:** It is trace made by single particle over a period of time.
- **Streamline** show the mean direction of a number of particles at the same instance of time.



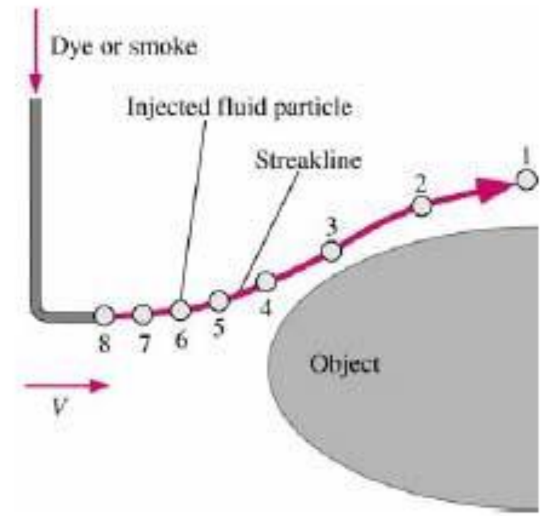
- **Character of Streamline**
- 1. Streamlines can not cross each other. (otherwise, the cross point will have two tangential lines.)
- 2. Streamline can't be a folding line, but a smooth curve.
- 3. Streamline cluster density reflects the magnitude of velocity. (Dense streamlines mean large velocity; while sparse streamlines mean small velocity.)



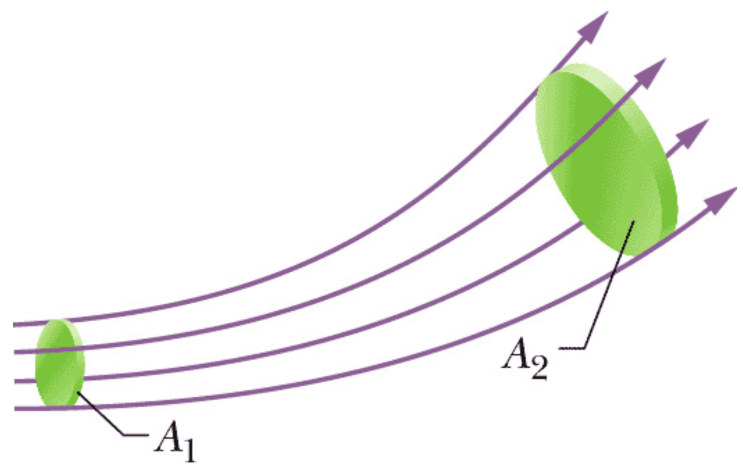
Flow around cylindrical object

Streakline and streamtubes

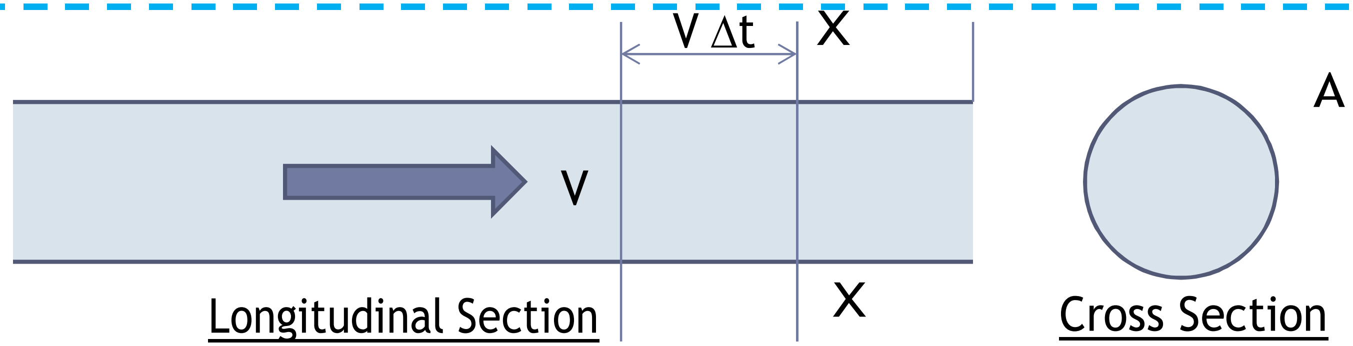
- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- It is an instantaneous picture of the position of all particles in flow that have passed through a given point.



- **Streamtube** is an imaginary tube whose boundary consists of streamlines.
- The volume flow rate must be the same for all cross sections of the stream tube.



Mean Velocity and Discharge



- Let's consider a fluid flowing with mean velocity, V, in a pipe of uniform cross-section. Thus volume of fluid that passes through section XX in unit time, Δt, becomes;

$$\text{Volume of fluid} = (\Delta t V)A$$

- Volume flow rate: $Q = \dfrac{\text{volume of fluid}}{\text{time}} = \dfrac{(\Delta t V)A}{\Delta t}$

$$Q = AV$$

Similarly

$$M = \rho AV$$

$$G = \gamma AV$$

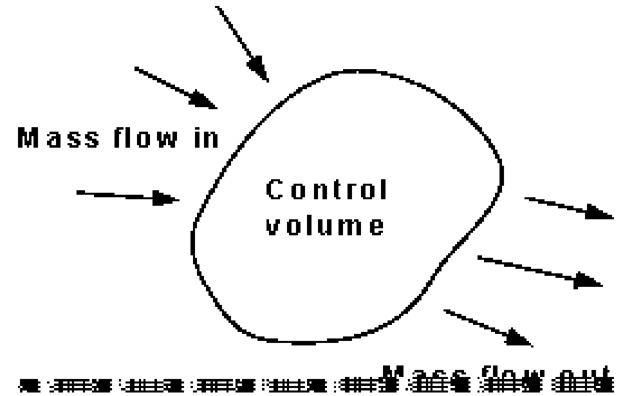
Fluid System and Control Volume

- **Fluid system** refers to a specific mass of fluid within the boundaries defined by close surface. The shape of system and so the boundaries may change with time, as when fluid moves and deforms, so the system containing it also moves and deforms.

- **Control volume** refers to a fixed region in space, which does not move or change shape. It is region in which fluid flow into and out.

Continuity

- Matter cannot be created or destroyed - (it is simply changed in to a different form of matter).
- This principle is know as the *conservation of mass* and we use it in the analysis of flowing fluids.
- The principle is applied to fixed volumes, known as control volumes shown in figure:



An arbitrarily shaped control volume.

For any control volume the principle of conservation of mass says

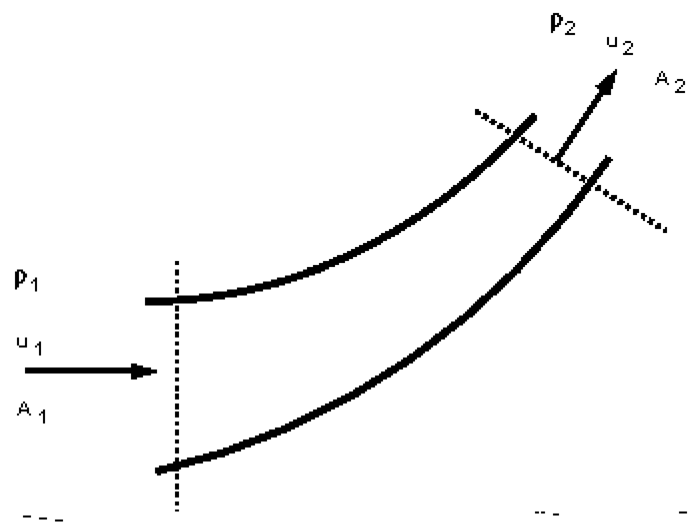
Mass entering per unit time -Mass leaving per unit time
= Increase of mass in the control volume per unit time

Continuity Equation

- For steady flow there is no increase in the mass within the control volume, so

Mass entering per unit time = Mass leaving per unit time

- **Derivation:**
- Lets consider a stream tube.
- ρ_1, v_1 and A_1 are mass density, velocity and cross-sectional area at section 1. Similarly, ρ_2, v_2 and A_2 are mass density, velocity and cross-sectional area at section 2.
- According to mass conservation



A stream tube

$$M_1 - M_2 = \frac{d(M_{CV})}{dt}$$

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = \frac{d(M_{CV})}{dt}$$

$$M_1 = \rho_1 A_1 V_1$$

$$M_2 = \rho_2 A_2 V_2$$

Continuity Equation

- For steady flow condition $d\left(M_{CV}\right)/dt = 0$

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = 0 \Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$M = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- Hence, for stead flow condition, mass flow rate at section 1= mass flow rate at section 2. i.e., mass flow rate is constant.
- Similarly $G = \rho_1 g A_1 V_1 = \rho_2 g A_2 V_2$

- Assuming incompressible fluid, $\rho_1 = \rho_2 = \rho$

$$A_1 V_1 = A_2 V_2 \quad \Rightarrow \quad Q_1 = Q_2 \quad \Rightarrow \quad Q_1 = Q_2 = Q_3 = Q_4$$

- Therefore, according to mass conservation for steady flow of incompressible fluids volume flow rate remains same from section to section.

Hydrostatic Forces on Surfaces .

3.1/ Vertical Plane Surface Submerged in Liquid :

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig.(3.1) :

Let , A – Total area of the Surface. h_c - Distance of center of gravity(C.G)of the area from free surface of liquid. G – Center of gravity of plane surface. P – Center Of pressure. h_p – Distance of center of pressure from free surface of liquid.

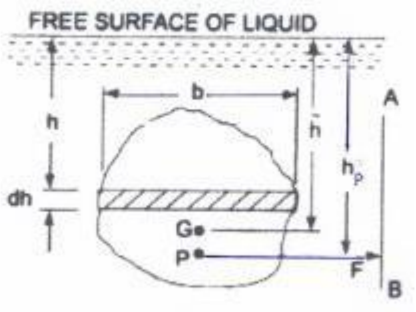


Fig.(3.1)

Consider a strip of thickness (dh) and width(b) at a depth of (h) from free surface of liquid as shown in Fig.(3.1) :

Pressure on the strip $P = \rho\, g\, h$

Area of the strip $A = b \times dh$

Total Force on strip $dF = p \times Area = \rho\, g\, h \times b \times dh$

Total Force on the whole surface $F = \int dF = \int \rho\, gh \times b \times dh$

$$= \rho\, g \int b \times h \times dh$$

$$\int b \times h \times dh = \int h \times dA$$

= Moment of surface area about the free surface of liquid.

= Area of surface × Distance of (C.G) from free surface .

$$= A \times h_c$$

$$F = \rho\, g\, A\, h_c \qquad (3.1)$$

<u>Center of pressure (p)</u> : Center of pressure is calculated by using the (principle of moments) , which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis . (The distance of center of pressure (p) from the free surface is h_p).

Moment of Force = dF × h

$$= \rho\ g\ h \times b \times dh \times h \qquad (3.2)$$

Sum of momentum of all such forces = $\int \rho\ g\ h \times b \times dh \times h$

$$= \rho\ g \int bh^2\ dh = \rho\ g \int h^2\ dA$$

$$= \rho\ g \int bh^2\ dh = \rho\ g\ I_o \qquad (3.3)$$

(In which I_o is moment of inertia of the surface about free surface of liquid)

But , the moment of the force F about free surface of the liquid = $F \times h_p$

Therefore , $F \times h_p = \rho\ g\ I_o$

But , $F = \rho\ g\ A\ h_c$

Therefore , $\rho\ g\ A\ h_c \times h_p = \rho\ g\ I_o$

$$h_p = \frac{\rho\ g\ I_o}{\rho\ g\ A\ h_c} = \frac{I_o}{A\ h_c} \qquad (3.4)$$

By the theorem of parallel axis , we have

$$I_o = I_G + A \times h_c^2$$

Where I_G = Moment of Inertia of area about an axis passing through the C.G of the area and parallel to the free surface of liquid.

Substituting I_o in equation (3.4) , we get ,

$$h_p = \frac{I_G + A\ h_c^2}{A\ h_c} = \frac{I_G}{A\ h_c} + h_c \qquad (3.5)$$

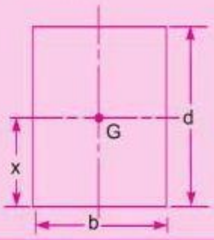
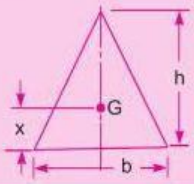
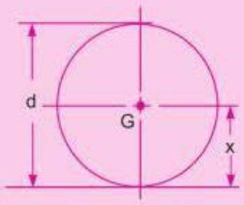
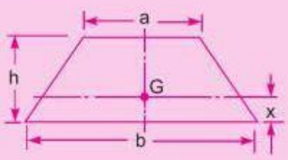
The center of pressure h_p lies below the center of gravity of the vertical surface h_c .

/ <u>Horizontal plane surface submerged in liquid</u> :

Consider a plane horizontal surface immersed in a static fluid. As every point of he surface is at the same depth from the free surface of the liquid, the pressure will be equal on the entire surface and equal to :

$P = \rho g h$ **(where h is depth of the surface)**

Table (3.1) The moments of inertia and other geometric properties of some important plane surfaces .

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Contd...

/ <u>Inclined Plane surface submerged in liquid</u> :

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle ϴ with the free surface of the liquid as shown in Fig.(3.2).

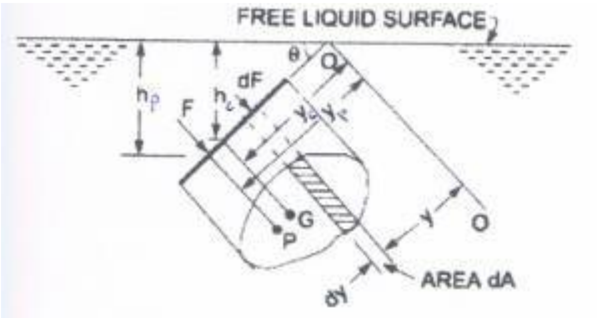


Fig.(3.2) Inclined immersed surface

Let , A total area of inclined surface , h_c depth of C.G of inclined area from free surface , h_p distance of center of pressure from free surface of liquid , ϴ angle made by the plane of the surface with free surface , y_c distance of the C.G of the inclined surface from O – O , y_p distance of the center of pressure from the O – O .

Consider a small strip of area dA at a depth (h) from free surface and at a distance y from the axis O – O as shown in Fig.(3.2).

Force dF on the strip = p × Area of strip = ρ g h ×dA

Total Force on the whole area , F = ∫ dF = ∫ ρ g h dA

But from Fig.(3.2) , $\sin\Theta = \frac{h}{y} = \frac{h_c}{y_c} = \frac{h_p}{y_p}$

Therefore , h = y sin ϴ

$$F = \int \rho\, g \times y \sin\Theta \times dA = \rho\, g \sin\Theta \int y\, dA$$

But , $\int y\, dA = A\, y_c$

Therefore , $F = \rho\, g \sin\Theta \times A \times y_c$

$$F = \rho\, g\, A\, h_c \qquad (3.6)$$

Force on the strip , dF = ρ g h dA

$\sin\Theta = \frac{h}{y}$, **h = y sin ϴ**

$dF = \rho\, g\, y \sin\Theta\, dA$

Moment of force (dF) about axis O – O ,

$dF \times y = \rho\, g\, y \sin\Theta\, dA \times y = \rho\, g \sin\Theta\, y^2\, dA$

Sum of moments of all such forces about O - O ,

$M = \int \rho\, g \sin\Theta\, y^2\, dA = \rho\, g \sin\Theta \int y^2\, dA$

But $\int y^2\, dA = I_o$

Therefore , $M = \rho\, g \sin\Theta\, I_o$ (3.7)

Moment of the total force F , about O – O is given by : $F \times y_p$ (3.8)

Equating the two values given by equations (3.7) & (3.8)

$F \times y_p = \rho\, g \sin\Theta\, I_o$

$$y_p = \frac{\rho\, g \sin\theta\, I_o}{F} \qquad (3.9)$$

But , $\sin\Theta = \frac{h_p}{y_p}$, $y_p = \frac{h_p}{\sin\theta}$, and $F = \rho\, g\, A\, h_c$

And $I_o = I_G + A\, y_o^2$ Substituting these values in eq.(3.9) , we get :

$$\frac{h_p}{\sin\theta} = \frac{\rho\, g \sin\theta}{\rho\, g\, A h_c}\left(I_G + A\, y_c^2 \right) \qquad (\times \sin\Theta)$$

But , $\sin\Theta = \frac{hc}{y_c}$, $y_c = \frac{h_c}{\sin\theta}$

$$h_p = \frac{\sin^2\theta}{A h_c}\left(I_G + A \frac{h_c^2}{\sin^2\theta} \right)$$

$$h_p = \frac{I_G \sin^2\theta}{A\, h_c} + h_c \qquad (3.10)$$

If the $\Theta = 90^0$, equation (3.10) becomes same as equation (3.5) (vertical plane submerged) .

/ Curved Surface Submerged in Liquid :

Consider a curved surface (AB) , submerged in a static liquid as shown in Fig.(3.3) . Let dA is the area of a small strip at a depth of (h) from water surface.

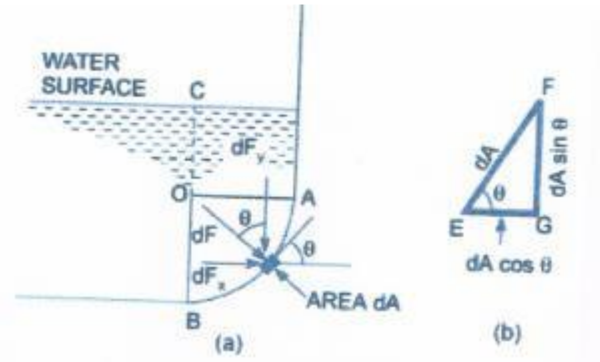


Fig.(3.3)

Then pressure (p) = ρ g h

Force (dF) = p × area = ρ g h × dA (3.11) This force dF acts normal to the surface , hence , total force on the curved surface should be:

$$F = \int \rho g h \, dA \quad (3.12)$$

By resolving the force dF in two components dF , and dF_x and dF_y in the x and y directions respectively . The total force in the x and y directions , i .e , F_x and F_y are obtained by integrating dF_x and dF_y , Then total force on the curved surface is :

$$F = \sqrt{F_x^2 + F_y^2} \quad (3.13)$$

And inclination of resultant with horizontal is ,

$$\tan \Theta = \frac{F_y}{F_x} \quad (3.14)$$

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin\Theta = \rho g h \, dA \sin\Theta$$

$$dF_y = dF \cos \Theta = \rho g h \, dA \cos \Theta$$

Total forces in the x and y directions are :

$$F_x = \int dF_x = \rho g \int h \, dA \sin\Theta \quad (3.15)$$

$$F_y = \int dF_y = \rho g \int h \, dA \cos \Theta \quad (3.16)$$

Fig.(3.3) b , shows the enlarged area dA , from this figure , i.e. , Δ EFG :

$$EF = dA \quad , \quad FG = dA \sin\Theta \quad , \quad EG = dA \cos\Theta$$

Thus , in Eq.(3.15) , $dA \sin\Theta$ = FG = Vertical projection of the area dA .

Therefore , F_x force on the projected area on the vertical plane .

Thus , in Eq.(3.16) , $dA \cos\Theta$ = EG = Horizontal projection of the area dA.

Therefore , $\int h \, dA \cos\Theta$ is the total volume contained between the curved surface , extended up to free surface .

Hence , $\rho g \int h \, dA \cos\Theta$ is the total weight supported by the curved surface , thus , $F_y = \rho g \int h dA \cos \Theta$ = Weight of liquid supported by the curved surface up to free surface of liquid.

Pressure and Its Measurement

Consider a small area dA in large mass of fluid. If the fluid is static , then the force exerted by fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction.

Then the ratio of $\frac{dF}{dA}$ is known as the pressure (P). Hence mathematically the pressure at a point in a fluid at rest (static) is :

$$P = \frac{dF}{dA}$$

If the force (F) is uniformly distributed over the area (A) , the pressure at any point is given by :

$$P = \frac{F}{A} \qquad (2.1)$$

The unit of pressure are (1) kgf / cm^2 (in MKS) (meter – kilogram – second)

(2) Newton / m^2 (N/ m^2) (in SI unit) . N/ m^2 is known as Pascal (1 bar = 100 kpa = 10^5 Pascal)

/ Pascal Law :

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions .This is proved as :

The fluid element is of very small directions , i.e , (dx , dy and ds) .

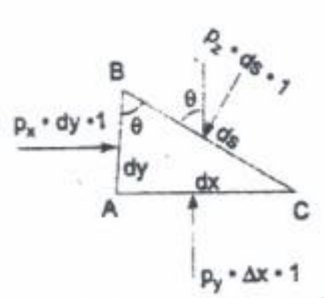


Fig.(2.1) Forces on a fluid element .

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest , as shown in Fig.(2.1) . Let the width of the element perpendicular to the plane of

paper is unity and P_x , P_y and P_z are the pressure acting on the face AB , AC and BC respectively . Let angle ABC is ϴ . Then the forces acting on the element are:

1. **Pressure force normal to the surfaces .**
2. **Weight of the element in the vertical direction.**

Force on the face AB = $P_x \times$ area of face AB

$$= P_x \times dy \times 1$$

Force on the face AC = $P_y \times dx \times 1$

Force on the face BC = $P_z \times ds \times 1$

Weight of element = mass of element × g

$$= (\text{volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$$

$$\sum F_x = 0$$

$$P_x \times dy \times 1 - P_z (ds \times 1) \sin(90 - \Theta) = 0$$

$$P_x \times dy - P_z \times ds \times \cos\Theta = 0$$

But , from Fig.(2.1) , $ds \cos\Theta = AB = dy$

$$P_x \times dy - P_z \times dy = 0$$

$$P_x = P_z$$

Similarly , $\sum F_y = 0$

$$P_y \times dx \times 1 - P_z \times ds \times 1 \times \cos(90 - \Theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$P_y \times dx - P_s \, ds \sin\Theta - \frac{dx\, dy}{2} \times \rho \times g = 0$$

But , $ds \sin\Theta = dx$, and the element has very small , therefore the weight is negligible (third term) , therefore ,

$$P_y = P_s$$

Therefore , $P_x = P_y = P_z$ (2.2)

This equation shows that the pressure at ant point in x , y and z direction is equal .

/ Pressure variation in a fluid at rest (fluid static) :

The pressure at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight (weight density) of the fluid at the point. This is proved as :

Consider a small fluid element as shown in Fig.(2.2) .

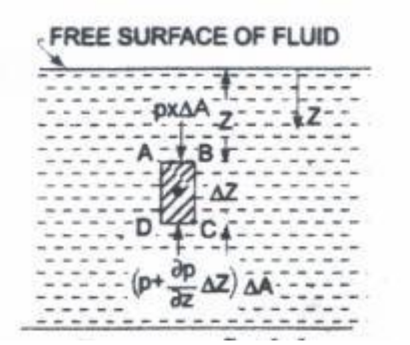


Fig.(2.2) Forces on a fluid element

Let , Δ A - cross – section area of element .

Δ Z - Height of fluid element .

P - pressure on face AB .

Z – distance of fluid element from free surface.

The forces acting on the fluid element are :

1. **Pressure force on AB = p × Δ A (acting perpendicular to face AB in the downward direction).**
2. **Pressure force on CD = (p + $\frac{\partial p}{\partial z}$Δ Z) × Δ A (acting perpendicular to face CD vertically upward direction).**
3. **Weight of fluid element = γ × volume = ρ g (ΔA × ΔZ) .**
4. **Pressure forces on surface BC and AD are equal and opposite. Forequilibrium of fluid element , we have**

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z} \Delta Z \right) \Delta A + \rho\, g\, (\Delta A \times \Delta Z) = 0$$

$$p\Delta A - p\Delta A - \frac{\partial p}{\partial z} \Delta Z\, \Delta A + \rho\, g \times \Delta A \times \Delta Z = 0$$

$$- \frac{\partial p}{\partial z} \Delta Z\, \Delta A + \rho\, g \times \Delta A\, \Delta Z = 0$$

$$\frac{\partial p}{\partial z} \Delta Z \ \Delta A = \rho \ g \times \Delta A \ \Delta Z$$

$$\frac{\partial P}{\partial Z} = \gamma$$

$$\frac{dP}{dZ} = \gamma \quad , \quad d\,p = \gamma \, dz \quad , \quad \int dp = \gamma \int dz$$

$$P = \gamma \, Z \qquad (2.3)$$

Equation (2.3) states that the rate of increase of pressure in vertical direction is equal to weight density (γ) of the fluid at that point. This is Hydrostatic Law . (Z is called pressure head) .

/ Absolute , Gauge , Atmospheric , And Vacuum Pressures

The pressure on the fluid is measured in two difference systems. In one system , it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system , pressure is measured above the atmospheric pressure and is called gauge pressure .

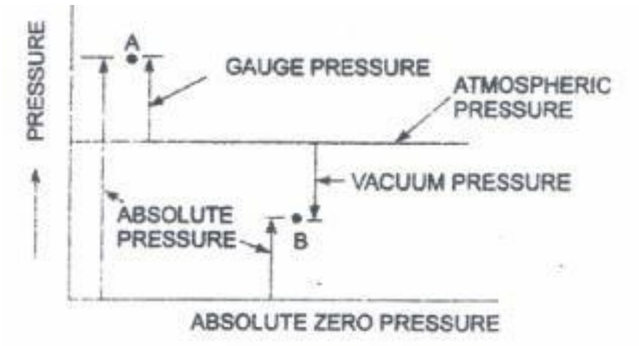


Fig.(2.3) Relationship between pressure.

The relationship between the absolute pressure , gauge pressure and vacuum pressure are shown in Fig.(2.3) .

Mathematically :

$$P_{abs.} = P_{atm.} \pm P_{gauge} \qquad (2.4)$$

$$P_{A(abs)} = P_{atm.} + P_{gauge}$$

$$P_{B(abs)} = P_{atm.} - P_{gauge(vacuum)}$$

The values of atmospheric pressure at sea level at 15°c :

$P_{atm.} = 101.3\ KN/m^2$ (kpa) , $P_{atm.} = 10^5\ N/m^2$ (Pascal)

$P_{atm.}$ = 76 cm Hg. , $P_{atm.}$ = 10 m(water) , $P_{atm.}$ = 14.7 psi

$P_{atm.}$ = 14.7 psi . $P_{atm.}$ = 1bar.

/ Measurement of pressure :

The pressure of a fluid is measured by the following devices :

1. **Manometers .**
2. **Mechanical Gauges .**

2.5.1/ Manometers :

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(1) simple manometers , (2) Differential manometers

2.5.2 / Simple Manometers :

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer. , 2. U – tube Manometer. , 3. Single Column Manometer.

1. Piezometer :

It is simple form of manometer , used for measuring gauge pressures , as shown in Fig.(2.4)

$$P_A = \rho\, g\, h = \gamma\, h \quad N/m^2 \text{ (Pascal)} \qquad (2.5)$$

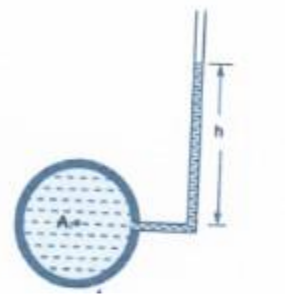


Fig.(2.4) Piezometer.

2. U– tube Manometer :

It consists of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to

the atmosphere as shown in Fig.(2.5) . In this manometer , we can measure positive pressure (gauge pressure) and negative pressure (vacuum).

Let B is the point at which pressure is to be measured (p) . The datum line is A – A .

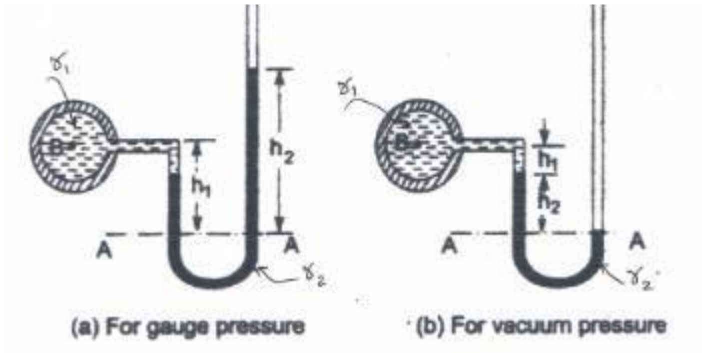


Fig.(2.5) U- tube Manometer.

If we want to measure the pressure (p) at point B .

There are two legs in the manometer , if there is equilibrium between two legs (right and left legs) over the datum (A – A) , i .e the pressure at each leg over the datum are equal .

Mathematically ,

(a) For gauge pressure :

Pressure at left leg = Pressure at right leg

$P + \gamma_1 h_1 = \gamma_2 h_2$

$$P = \gamma_2 h_2 - \gamma_1 h_1 \quad N/m^2 \qquad (2.6)$$

(b) For vacuum (negative) pressure :

Pressure at left leg = pressure at right leg

$P + \gamma_1 h_1 + \gamma_2 h_2 = 0$

$$P = -\gamma_1 h_1 - \gamma_2 h_2 \quad N/m^2 \qquad (2.7)$$

/ Differential Manometers :

Differential manometer are the devices used for measuring the difference of pressures between between two points in a pipe or in two different pipes . A differential manometer consists of a U – tube , containing a heavy liquid (liquid manometer) , frequently is mercury (Hg). Most commonly types of differential manometers are :

1.U-tube differential manometer. , 2 – Inverted U-tube differential manometer . Fig.(2.6) shows the differential manometer of U-tube type.

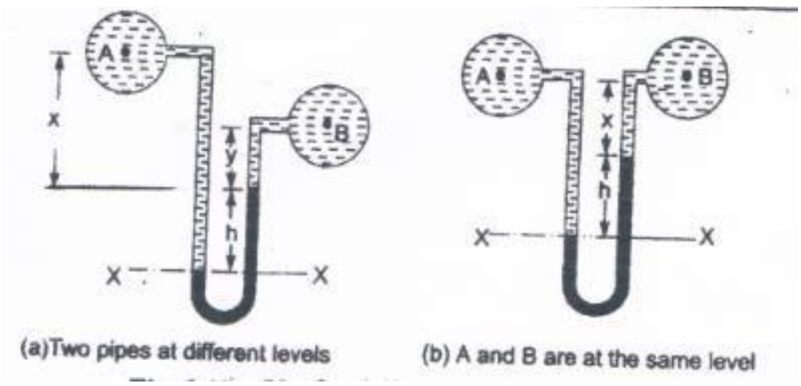


Fig.(2.6) U-tube differential manometer .

In Fig.(2.6) (a) , let the two points A & B are at different level and also contains liquids at different specific gravity (S) (sp. gr.).

Level X - X , level of equilibrium , the pressures in the left leg equal the pressures in right leg :

$$p_A + \gamma_A (x + h) = p_B + \gamma_B y + \gamma_m h$$

$$p_A - p_B = \gamma_B y + \gamma_m h - \gamma_A (x + h) \qquad (2.8)$$

In Fig.(2.6) (b) ,

$$p_A + \gamma_A (x + h) = p_B + \gamma_B x + \gamma_m h$$

$$p_A - p_B = \gamma_B x + \gamma_m h - \gamma_A (x + h) \qquad (2.9)$$

2. Inverted U- tube differential manometer :

It consists of an inverted U-tube . The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measured difference of low pressure . Fig.(2.7) shows an inverted U- tube differential manometer connected to the two points A & B. Let the pressure at A is more than the pressure at B.

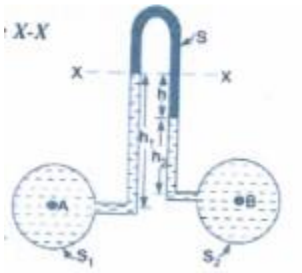


Fig.(2.7)

Taking X –X as datum line , then ,

Pressure at the left leg below the X – X = $p_A - \gamma_1 h_1$

Pressure at the right leg below the X- X = $p_B - \gamma_2 h_2 - \gamma_m h$

Pressure at the left leg = pressure at the right leg

$$P_A - \gamma_1 h_1 = p_B - \gamma_2 h_2 - \gamma_m h$$

$$P_A - p_B = \gamma_1 h_1 - \gamma_2 h_2 - \gamma_m h \qquad (2.10)$$

1.7 / Inclined Single column Manometer :

Fig.(2.8) shows the inclined single column manometer.This manometer is more sensitive . Due to inclination the distance moved by the heavy liquid in the right side will be more.

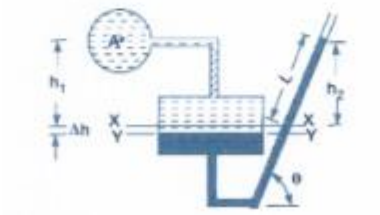


Fig.(2.8) Inclined manometer

Let L – length of heavy liquid moved in right side from X – X

Θ - inclination of right leg with horizontal.

h_2 - vertical rise of heavy liquid in right leg from X – X

(L sin Θ)

$$P_A = \gamma_2 h_2 - \gamma_1 h_1 \qquad (\text{ but } h_2 = L \sin\Theta)$$

$$P_A = \gamma_2 L \sin\Theta - \gamma_1 h_1 \qquad (2.11)$$

2.8/ Micromanometer :

It is used for determine small differences in pressure .With two gage liquids , immiscible in each other and in the fluid to be measured , a large gage difference R , as shown in Fig.(2.9) can be produced for a small pressure difference. The heaver gage liquid fills the lower U-tube up to O - Othen the lighter gage liquid is added to both sides , filling the larger reservoir up to 1 – 1 . The gas or liquid in the system fills the space above 1 – 1 . When the pressure at C is slightly greater than at D, the menisci move as indicated in Fig.(2.9) . The volume of liquid displaced in each reservoir equals the displacement in the U –tube , thus ,

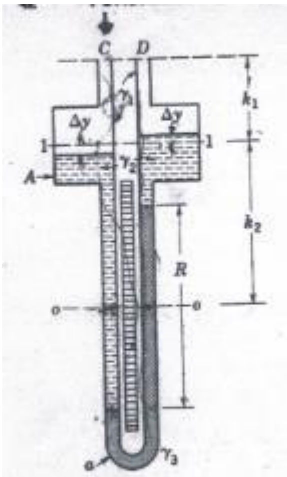


Fig.(2.9) Micromanometer

$$\Delta y . A = \frac{R}{2} . a \quad , \quad \Delta y = \frac{Ra}{2A}$$

In which , A is area of reservoir , a is area of U – tube .

The manometer equation may be written starting at C ,

$$P_c + (k_1 + \Delta y)\gamma_1 + (k_2 - \Delta y + \frac{R}{2})\gamma_2 - R\gamma_3 - (k_2 - \frac{R}{2} + \Delta y)\gamma_2 -$$

$(k_1 - \Delta y)\gamma_1 = p_D$

In which γ_1 , γ_2 , γ_3 are the specific weights . Simplifying and substituting for Δy gives :

$$p_C - p_D = R\left[\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) - \gamma_1 \frac{a}{A}\right] \qquad (2.12)$$

The quantity in bracket is a constant for specified gage and fluids , hence, the pressure difference is directly proportional to R .

1.8 / Bourdon Gage (Mechanical) :

The bourdon pressure gage as shown in Fig.(2.10) is typical of the devices used for measuring gage pressure .

The bourdon gage (shown schematically) in Fig.(2.11) . In the gage , a bent tube (A) of elliptical cross section is held rigidly at (B) and its free end is connected to a pointer (C) by a link (D) . When pressure is

admitted to the tube , its cross section tends to become circular , causing thetube to straighten and move the pointer to the right over the graduated scale .

Fig.(2.10) typical of Bourdon gage

Fig.(2.11) Schematically shown of Bourdon gage

The pointer rests at zero on the scale , when the gauge is disconnected , in this condition the pressure inside and outside of the tube are the same.

Chapter 2
Properties of Fluids

In this chapter we discuss a number of fundamental properties of fluids. An understanding of these properties is essential for us to apply basic principles of fluid mechanics to the solution of practical problems.

2.1 Distinction Between a Solid and a Fluid

The molecules of a ***solid*** are usually closer together than those of a ***fluid.*** The attractive forces between the molecules of a solid are so large that a solid tends to retain its shape. This is not the case for a fluid, where the attractive forces between the molecules are smaller. An ideal elastic solid will deform under load and, once the load is removed, will return to its original state. Some solids are plastic. These deform under the action of a sufficient load and deformation continues as long as a load is applied, providing the material does not rupture. Deformation ceases when the load is removed, but the plastic solid does not return to its original state.

The intermolecular cohesive forces in a fluid are not great enough to hold the various elements of the fluid together. Hence a fluid will flow under the action of the slightest stress and flow will continue as long as the stress is present.

2.2 Distinction Between a Gas and a Liquid

A fluid may be either a ***gas*** or a ***liquid.*** The molecules of a gas are much farther apart than those of a liquid. Hence a gas is very compressible, and when all external pressure is removed, it tends to expand indefinitely. A gas is therefore in equilibrium only when it is completely enclosed. A liquid is relatively incompressible, and if all pressure, except that of its own vapor pressure, is removed, the cohesion between molecules holds them together, so that the liquid does not expand indefinitely. Therefore a liquid may have a free surface, i.e., a surface from which all pressure is removed, except that of its own vapor.

A ***vapor*** is a gas whose temperature and pressure are such that it is very near the liquid phase. Thus steam is considered a vapor because its state is normally not far from that of water. A gas may be defined as a highly superheated vapor; that is, its state is far removed from the liquid phase. Thus air is considered a gas because its state is normally very far from that of liquid air.

The volume of a gas or vapor is greatly affected by changes in pressure or temperature or both. It is usually necessary, therefore, to take account of changes in volume and temperature in dealing with gases or vapors. Whenever significant temperature or phase changes are involved in dealing with vapors and gases, the subject is largely dependent on heat phenomena (***thermodynamics***). Thus fluid mechanics and thermodynamics are interrelated.

2.3 DENSITY, SPECIFIC WEIGHT, SPECIFIC VOLUME, AND SPECIFIC GRAVITY

The ***density*** ρ (rho),[1] or more strictly, ***mass density,*** of a fluid is its *mass* per unit volume, while the ***specific weight*** γ (gamma) is its *weight* per unit volume. In the British Gravitational (BG) system (Sec. 1.5) density ρ will be in slugs per cubic foot (kg/m^3 in SI units), which can also be expressed as units of $lb \cdot sec^2/ft^4$ ($N \cdot s^2/m^4$ in SI units) (Sec. 1.5 and inside covers).

Specific weight γ represents the force exerted by gravity on a unit volume of fluid, and therefore must have the units of force per unit volume, such as pounds per cubic foot (N/m^3 in SI units).

Density and specific weight of a fluid are related as:

$$\rho = \frac{\gamma}{g} \quad \text{or} \quad \gamma = \rho g \tag{2.1}$$

Since the physical equations are dimensionally homogeneous, the dimensions of density are

$$\text{Dimensions of } \rho = \frac{\text{dimensions of } \gamma}{\text{dimensions of } g} = \frac{\text{lb/ft}^3}{\text{ft/sec}^2} = \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}^4} = \frac{\text{mass}}{\text{volume}} = \frac{\text{slugs}}{\text{ft}^3}$$

In SI units

$$\text{Dimensions of } \rho = \frac{\text{dimensions of } \gamma}{\text{dimensions of } g} = \frac{\text{N/m}^3}{\text{m/s}^2} = \frac{\text{N}\cdot\text{s}^2}{\text{m}^4} = \frac{\text{mass}}{\text{volume}} = \frac{\text{kg}}{\text{m}^3}$$

Note that density ρ is absolute, since it depends on mass, which is independent of location. Specific weight γ, on the other hand, is not absolute, since it depends on the value of the gravitational acceleration g, which varies with location, primarily latitude and elevation above mean sea level.

Densities and specific weights of fluids vary with temperature. Appendix A provides commonly needed temperature variations of these quantities for water

[1] The names of Greek letters are given in the List of Symbols on page xix.

and air. It also contains densities and specific weights of common gases at standard atmospheric pressure and temperature. We shall discuss the specific weight of liquids further in Sec. 2.6.

Specific volume v is the volume occupied by a unit mass of fluid.[2] We commonly apply it to gases, and usually express it in cubic feet per slug (m^3/kg in SI units). Specific volume is the reciprocal of density. Thus

$$v = \frac{1}{\rho} \tag{2.2}$$

Specific gravity s of a liquid is the dimensionless ratio

$$s_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water at standard temperature}}}$$

Physicists use 4°C (39.2°F) as the standard, but engineers often use 60°F (15.56°C). In the metric system the density of water at 4°C is 1.00 g/cm^3 (or 1.00 g/mL),[3] equivalent to 1000 kg/m^3, and hence the specific gravity (which is dimensionless) of a liquid has the same numerical value as its density expressed in g/mL or Mg/m^3. Appendix A contains information on specific gravities and densities of various liquids at standard atmospheric pressure.

The specific gravity of a gas is the ratio of its density to that of either hydrogen or air at some specified temperature and pressure, but there is no general agreement on these standards, and so we must explicitly state them in any given case.

Since the density of a fluid varies with temperature, we must determine and specify specific gravities at particular temperatures.

SAMPLE PROBLEM 2.1 The specific weight of water at ordinary pressure and temperature is 62.4 lb/ft^3. The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution

$$\rho_{\text{water}} = \frac{\gamma_{\text{water}}}{g} = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/sec}^2} = 1.938 \text{ slugs/ft}^3 \qquad \textbf{\textit{ANS}}$$

$$\gamma_{\text{mercury}} = s_{\text{mercury}}\gamma_{\text{water}} = 13.56(62.4) = 846 \text{ lb/ft}^3 \qquad \textbf{\textit{ANS}}$$

$$\rho_{\text{mercury}} = s_{\text{mercury}}\rho_{\text{water}} = 13.56(1.938) = 26.3 \text{ slugs/ft}^3 \qquad \textbf{\textit{ANS}}$$

[2] Note that in this book we use a "rounded" lower case v (vee), to help distinguish it from a capital V and from the Greek ν (nu).
[3] One cubic centimeter (cm^3) is equivalent to one milliliter (mL).

SAMPLE PROBLEM 2.2 The specific weight of water at ordinary pressure and temperature is 9.81 kN/m^3. The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution

$$\rho_{\text{water}} = \frac{9.81\ \text{kN/m}^3}{9.81\ \text{m/s}^2} = 1.00\ \text{Mg/m}^3 = 1.00\ \text{g/mL} \qquad \textbf{\textit{ANS}}$$

$$\gamma_{\text{mercury}} = s_{\text{mercury}}\gamma_{\text{water}} = 13.56(9.81) = 133.0\ \text{kN/m}^3 \qquad \textbf{\textit{ANS}}$$

$$\rho_{\text{mercury}} = s_{\text{mercury}}\rho_{\text{water}} = 13.56(1.00) = 13.56\ \text{Mg/m}^3 \qquad \textbf{\textit{ANS}}$$

EXERCISES

2.3.1 If the specific weight of a liquid is 52 lb/ft^3, what is its density?

2.3.2 If the specific weight of a liquid is 8.1 kN/m^3, what is its density?

2.3.3 If the specific volume of a gas is 375 ft^3/slug, what is its specific weight in lb/ft^3?

2.3.4 If the specific volume of a gas is 0.70 m^3/kg, what is its specific weight in N/m^3?

2.3.5 A certain gas weighs 16.0 N/m^3 at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 12.0 N/m^3?

2.3.6 The specific weight of glycerin is 78.6 lb/ft^3. Compute its density and specific gravity. What is its specific weight in kN/m^3?

2.3.7 If a certain gasoline weighs 43 lb/ft^3, what are the values of its density, specific volume, and specific gravity relative to water at 60°F? Use Appendix A.

2.4 COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

Fluid mechanics deals with both incompressible and compressible fluids, that is, with liquids and gases of either constant or variable density. Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. We may also consider gases to be incompressible when the pressure variation is small compared with the absolute pressure.

Ordinarily we consider liquids to be incompressible fluids, yet sound waves, which are really pressure waves, travel through them. This is evidence of the elasticity of liquids. In problems involving water hammer (Sec. 12.6) we must consider the compressibility of the liquid.

The flow of air in a ventilating system is a case where we may treat a gas as incompressible, for the pressure variation is so small that the change in density is of no importance. But for a gas or steam flowing at high velocity through a long pipeline, the drop in pressure may be so great that we cannot ignore the change in density. For an airplane flying at speeds below 250 mph (100 m/s), we may consider the air to be of constant density. But as an object moving through the air approaches the velocity of sound, which is of the order of 760 mph (1200 km/h) depending on temperature, the pressure and density of the air adjacent to the body become materially different from those of the air at some distance away, and we must then treat the air as a compressible fluid (Chap. 13).

2.5 Compressibility of Liquids

The compressibility (change in volume due to change in pressure) of a liquid is inversely proportional to its ***volume modulus of elasticity,*** also known as the ***bulk modulus.*** This modulus is defined as

$$E_v = -v\frac{dp}{dv} = -\left(\frac{v}{dv}\right)dp$$

where v = specific volume and p = pressure. As v/dv is a dimensionless ratio, the units of E_v and p are identical. The bulk modulus is analogous to the modulus of elasticity for solids; however, for fluids it is defined on a volume basis rather than in terms of the familiar one-dimensional stress–strain relation for solid bodies.

In most engineering problems, the bulk modulus at or near atmospheric pressure is the one of interest. The bulk modulus is a property of the fluid and for liquids is a function of temperature and pressure. A few values of the bulk modulus for water are given in Table 2.1. At any temperature we see that the value of E_v increases continuously with pressure, but at any one pressure the value of E_v is a maximum at about 120°F (50°C). Thus water has a minimum compressibility at about 120°F (50°C).

Note that we often specify applied pressures, such as those in Table 2.1, in absolute terms, because atmospheric pressure varies. The units psia or kN/m^2 abs indicate absolute pressure, which is the actual pressure on the fluid, relative

Table 2.1 Bulk modulus of water E_v, psi[a]

	Temperature, °F				
Pressure, psia	**32°**	**68°**	**120°**	**200°**	**300°**
15	293,000	320,000	333,000	308,000	
1,500	300,000	330,000	342,000	319,000	248,000
4,500	317,000	348,000	362,000	338,000	271,000
15,000	380,000	410,000	426,000	405,000	350,000

[a] These values can be transformed to meganewtons per square meter by multiplying them by 0.006895. The values in the first line are for conditions close to normal atmospheric pressure; for a more complete set of values at normal atmospheric pressure, see Table A.1 in Appendix A. The five temperatures are equal to 0, 20, 48.9, 93.3, and 148.9°C, respectively.

to absolute zero. The standard atmospheric pressure at sea level is about 14.7 psia or 101.3 kN/m^2 abs (1013 mb abs) (see Sec. 2.9 and Table A.3). Bars and millibars were previously used in metric systems to express pressure; 1 mb = 100 N/m^2. We measure most pressures relative to the atmosphere, and call them gage pressures. This is explained more fully in Sec. 3.4.

The volume modulus of mild steel is about 26,000,000 psi (170000 MN/m^2). Taking a typical value for the volume modulus of cold water to be 320,000 psi (2200 MN/m^2), we see that water is about 80 times as compressible as steel. The compressibility of liquids covers a wide range. Mercury, for example, is approximately 8% as compressible as water, while the compressibility of nitric acid is nearly six times greater than that of water.

In Table 2.1 we see that at any one temperature the bulk modulus of water does not vary a great deal for a moderate range in pressure. By rearranging the definition of E_v, as an approximation we may use for the case of a fixed mass of liquid at constant temperature

$$\frac{\Delta v}{v} \approx -\frac{\Delta p}{E_v} \tag{2.3a}$$

or

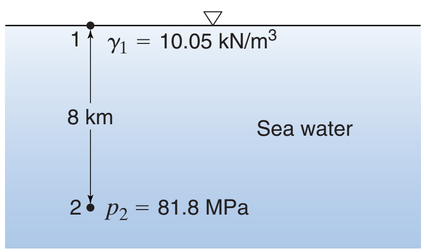
$$\frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v} \tag{2.3b}$$

where E_v is the mean value of the modulus for the pressure range and the subscripts 1 and 2 refer to the before and after conditions.

Assuming E_v to have a value of 320,000 psi, we see that increasing the pressure of water by 1000 psi will compress it only $\frac{1}{320}$, or 0.3%, of its original volume. Therefore we find that the usual assumption regarding water as being incompressible is justified.

SAMPLE PROBLEM 2.3 At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m^3 and that the average volume modulus is 2.34×10^9 N/m^2 for that pressure range. (*a*) What will be the change in specific volume between that at the surface and at that depth? (*b*) What will be the specific volume at that depth? (*c*) What will be the specific weight at that depth?

Solution



(*a*) Eq. (2.2): $v_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10050 = 0.000976\ \text{m}^3/\text{kg}$

Eq. (2.3*a*): $\Delta v = -0.000976(81.8 \times 10^6 - 0)/(2.34 \times 10^9)$

$= -34.1 \times 10^{-6}\ \text{m}^3/\text{kg}$ ***ANS***

(*b*) Eq. (2.3*b*): $v_2 = v_1 + \Delta v = 0.000942\ \text{m}^3/\text{kg}$ ***ANS***

(*c*) $\gamma_2 = g/v_2 = 9.81/0.000942 = 10410\ \text{N/m}^3$ ***ANS***

Exercises

2.5.1 To two significant figures what is the bulk modulus of water in MN/m^2 at 50°C under a pressure of 30 MN/m^2? Use Table 2.1.

2.5.2 At normal atmospheric conditions, approximately what pressure in psi must be applied to water to reduce its volume by 2%? Use Table 2.1.

2.5.3 Water in a hydraulic press is subjected to a pressure of 4500 psia at 68°F. If the initial pressure is 15 psia, approximately what will be the percentage decrease in specific volume? Use Table 2.1.

2.5.4 At normal atmospheric conditions, approximately what pressure in MPa must be applied to water to reduce its volume by 3%?

2.5.5 A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage.

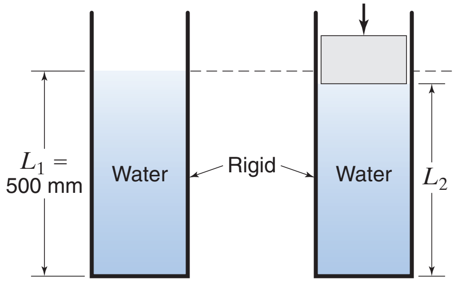


Figure X2.5.5

2.6 Specific Weight of Liquids

The specific weights γ of some common liquids at 68°F (20°C) and standard sea-level atmospheric pressure[4] with $g = 32.2\ \text{ft/sec}^2$ ($9.81\ \text{m/s}^2$) are given in Table 2.2. The specific weight of a *liquid* varies only slightly with pressure, depending on the bulk modulus of the liquid (Sec. 2.5); it also depends on temperature, and the variation may be considerable. Since specific weight γ is equal to ρg, the

[4] See Secs. 2.9 and 3.5.

TABLE 2.2 Specific weights γ of common liquids at 68°F (20°C), 14.7 psia (1013 mb abs) with g = 32.2 ft/sec² (9.81 m/s²)

	lb/ft³	kN/m³
Carbon tetrachloride	99.4	15.6
Ethyl alcohol	49.3	7.76
Gasoline	42	6.6
Glycerin	78.7	12.3
Kerosene	50	7.9
Motor oil	54	8.5
Seawater	63.9	10.03
Water	62.3	9.79

specific weight of a *fluid* depends on the local value of the acceleration of gravity in addition to the variations with temperature and pressure. The variation of the specific weight of water with temperature and pressure, where g = 32.2 ft/sec² (9.81 m/s²), is shown in Fig. 2.1. The presence of dissolved air, salts in solution, and suspended matter will increase these values a very slight amount. Ordinarily

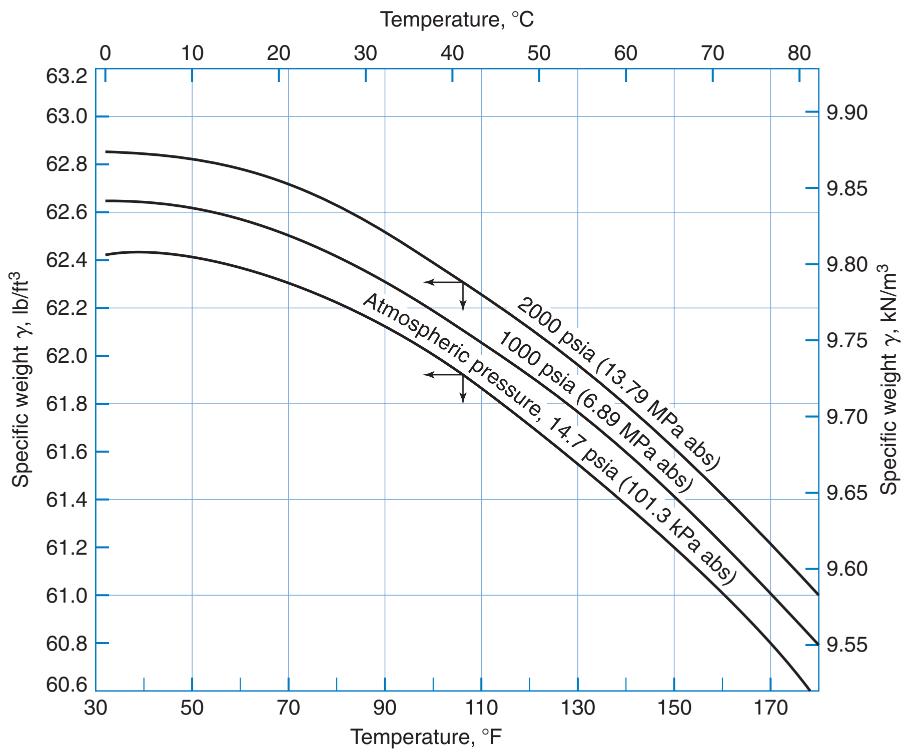
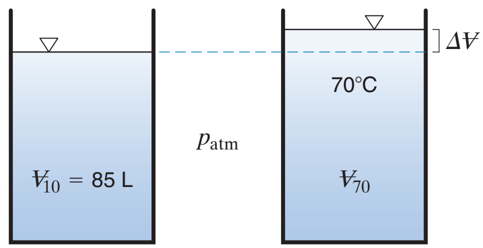


Figure 2.1
Specific weight γ of pure water as a function of temperature and pressure for the condition where g = 32.2 ft/sec² (9.81 m/s²).

we assume ocean water to weigh 64.0 lb/ft^3 (10.1 kN/m^3). Unless otherwise specified or implied by a given temperature, the value to use for water in the problems in this book is γ = 62.4 lb/ft^3 (9.81 kN/m^3). Under extreme conditions the specific weight of water is quite different. For example, at 500°F (260°C) and 6000 psi (42 MN/m^2) the specific weight of water is 51 lb/ft^3 (8.0 kN/m^3).

Sample Problem 2.4 A vessel contains 85 L of water at 10°C and atmospheric pressure. If the water is heated to 70°C, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.

Solution



Volume, $\mathcal{V}_{10} = 85\text{ L} = 0.085\text{ m}^3$

Table A.1: $\gamma_{10} = 9.804\text{ kN/m}^3, \quad \gamma_{70} = 9.589\text{ kN/m}^3$

$$\text{Weight of water, } W = \gamma\mathcal{V} = \gamma_{10}\mathcal{V}_{10} = \gamma_{70}\mathcal{V}_{70}$$

i.e., $9.804(0.085)\text{ kN} = 9.589\,\mathcal{V}_{70}; \quad \mathcal{V}_{70} = 0.08691\text{ m}^3$

$$\Delta\mathcal{V} = \mathcal{V}_{70} - \mathcal{V}_{10} = 0.08691 - 0.08500 = 0.001906\text{ m}^3\text{ at }\gamma_{70}$$

$$\Delta\mathcal{V}/\mathcal{V}_{10} = 0.001906/0.085 = 2.24\%\text{ increase} \qquad \textbf{\textit{ANS}}$$

Must remove (at γ_{70}): $W\left(\frac{\Delta\mathcal{V}}{\mathcal{V}_{70}}\right) = \gamma_{70}\Delta\mathcal{V}$

$$= (9589\text{ N/m}^3)(0.001906\text{ m}^3) = 18.27\text{ N} \qquad \textbf{\textit{ANS}}$$

Exercises

2.6.1 Use Fig. 2.1 to find the approximate specific weight of water in lb/ft^3 under the following conditions: (*a*) at a temperature of 60°C under 101.3 kPa abs pressure; (*b*) at 60°C under a pressure of 13.79 MPa abs.

2.6.2 Use Fig. 2.1 to find the approximate specific weight of water in kN/m^3 under the following conditions: (*a*) at a temperature of 160°F under normal atmospheric pressure; (*b*) at 160°F under a pressure of 2000 psia.

2.6.3 A vessel contains 5.0 ft^3 of water at 40°F and atmospheric pressure. If the water is heated to 80°F, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.

2.6.4 A cylindrical tank (diameter = 8.00 m and depth = 5.00 m) contains water at 15°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Assume the tank does not expand with the change in temperature. Use Appendix A.

2.7 PROPERTY RELATIONS FOR PERFECT GASES

The various properties of a gas, listed below, are related to one another (see, e.g., Appendix A, Tables A.2 and A.5). They differ for each gas. When the conditions of most real gases are far removed from the liquid phase, these relations closely approximate those of hypothetical ***perfect gases.*** Perfect gases, are here (and often) defined to have constant specific heats[5] and to obey the ***perfect-gas law,***

$$\frac{p}{\rho} = pv = RT \tag{2.4}$$

where p = absolute pressure (Sec. 3.4)
ρ = density (mass per unit volume)
v = specific volume (volume per unit mass, $= 1/\rho$)
R = a gas constant, the value of which depends upon the particular gas
T = absolute temperature in degrees Rankine or Kelvin[6]

For air, the value of R is 1715 ft·lb/(slug·°R) or 287 N·m/(kg·K) (Appendix A, Table A.5); making use of the definitions of a slug and a newton (Sec. 1.5), these units are sometimes given as $ft^2/(sec^2\cdot°R)$ and $m^2/(s^2\cdot K)$, respectively. Since $\gamma = \rho g$, Eq. (2.4) can also be written

$$\gamma = \frac{gp}{RT} \tag{2.5}$$

from which the specific weight of any gas at any temperature and pressure can be computed if R and g are known. Because Eqs. (2.4) and (2.5) relate the various gas properties at a particular state, they are known as ***equations of state*** and as ***property relations.***

In this book we shall assume that all gases are perfect. Perfect gases are sometimes also called ideal gases. Do not confuse a perfect (ideal) gas with an ideal fluid (Sec. 2.10).

[5] Specific heat and other thermodynamic properties of gases are discussed in Sec. 13.1.
[6] Absolute temperature is measured above absolute zero. This occurs on the Fahrenheit scale at −459.67°F (0° Rankine) and on the Celsius scale at −273.15°C (0 Kelvin). Except for low-temperature work, these values are usually taken as −460°F and −273°C. Remember that no degree symbol is used with Kelvin.

Avogadro's law states that all gases at the same temperature and pressure under the action of a given value of g have the same number of molecules per unit of volume, from which it follows that the specific weight of a gas[7] is proportional to its molar mass. Thus, if M denotes ***molar mass*** (formerly called ***molecular weight***), $\gamma_2/\gamma_1 = M_2/M_1$ and, from Eq. (2.5), $\gamma_2/\gamma_1 = R_1/R_2$ for the same temperature, pressure, and value of g. Hence for a perfect gas

$$M_1R_1 = M_2R_2 = \text{constant} = R_0$$

R_0 is known as the ***universal gas constant,*** and has a value of 49,709 ft·lb/(slug-mol·°R) or 8312 N·m/(kg-mol·K). Rewriting the preceding equation in the form

$$R = \frac{R_0}{M}$$

enables us to obtain any gas constant R required for Eq. (2.4) or (2.5).

For real (nonperfect) gases, the specific heats may vary over large temperature ranges, and the right-hand side of Eq. (2.4) is replaced by zRT, so that $R_0 = MzR$, where z is a compressibility factor that varies with pressure and temperature. Values of z and R are given in thermodynamics texts and in handbooks. However, for normally encountered monatomic and diatomic gases, z varies from unity by less than 3%, so the perfect-gas idealizations yield good approximations, and Eqs. (2.4) and (2.5) will give good results.

When various gases exist as a mixture, as in air, ***Dalton's law of partial pressures*** states that each gas exerts its own pressure as if the other(s) were not present. Hence it is the partial pressure of each that we must use in Eqs. (2.4) and (2.5) (see Sample Prob. 2.5). Water vapor as it naturally occurs in the atmosphere has a low partial pressure, so we may treat it as a perfect gas with $R = 49{,}709/18 = 2760$ ft·lb/(slug·°R) [462 N·m/(kg·K)]. But for steam at higher pressures this value is not applicable.

As we increase the pressure and simultaneously lower the temperature, a gas becomes a vapor, and as gases depart more and more from the gas phase and approach the liquid phase, the property relations become much more complicated than Eq (2.4), and we must then obtain specific weight and other properties from vapor tables or charts. Such tables and charts exist for steam, ammonia, sulfur dioxide, freon, and other vapors in common engineering use.

Another fundamental equation for a perfect gas is

$$pv^n = p_1v_1^n = \text{constant} \tag{2.6a}$$

or

$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^n = \text{constant} \tag{2.6b}$$

where p is absolute pressure, v $(=1/\rho)$ is specific volume, ρ is density, and n may have any nonnegative value from zero to infinity, depending on the process to

[7] The specific weight of air (molar mass ≈ 29.0) at 68°F (20°C) and 14.7 psia (1013 mb abs) with $g = 32.2$ ft/sec^2 (9.81 m/s^2) is 0.0752 lb/ft^3 (11.82 N/m^3).

which the gas is subjected. Since this equation describes the change of the gas properties from one state to another for a particular process, we call it a ***process equation.*** If the process of change is at a constant temperature (isothermal), $n = 1$. If there is no heat transfer to or from the gas, the process is ***adiabatic.*** A frictionless (and reversible) adiabatic process is an ***isentropic*** process, for which we denote n by k, where $k = c_p/c_v$, the ratio of specific heat at constant pressure to that at constant volume.[8] This ***specific heat ratio*** k is also called the ***adiabatic exponent.*** For expansion with friction n is less than k, and for compression with friction n is greater than k. Values for k are given in Appendix A, Table A.5, and in thermodynamics texts and handbooks. For air and diatomic gases at usual temperatures, we can take k as 1.4.

By combining Eqs. (2.4) and (2.6), we can obtain other useful relations such as

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{\rho_2}{\rho_1}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \qquad (2.7)$$

SAMPLE PROBLEM 2.5 If an artificial atmosphere consists of 20% oxygen and 80% nitrogen by volume, at 14.7 psia and 60°F, what are (*a*) the specific weight and partial pressure of the oxygen and (*b*) the specific weight of the mixture?

Solution

Table A.5: $R\text{ (oxygen)} = 1554\text{ ft}^2/(\text{sec}^2\cdot{}^\circ\text{R}),$

$R\text{ (nitrogen)} = 1773\text{ ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

Eq. (2.5): 100% O_2: $\gamma = \dfrac{32.2(14.7\times 144)}{1554(460+60)} = 0.0843\text{ lb/ft}^3$

Eq. (2.5): 100% N_2: $\gamma = \dfrac{32.2(14.7\times 144)}{1773(520)} = 0.0739\text{ lb/ft}^3$

(*a*) Each ft^3 of mixture contains 0.2 ft^3 of O_2 and 0.8 ft^3 of N_2.

So for 20% O_2, $\gamma = 0.20(0.0843) = 0.01687\text{ lb/ft}^3$ ***ANS***

From Eq. (2.5), for 20% O_2, $p = \dfrac{\gamma RT}{g} = \dfrac{0.01687(1554)520}{32.2}$

$= 423\text{ lb/ft}^2\text{ abs} = 2.94\text{ psia}$ ***ANS***

Note that this = 20%(14.7 psia).

(*b*) For 80% N_2, $\gamma = 0.80(0.0739) = 0.0591\text{ lb/ft}^3$.

Mixture: $\gamma = 0.01687 + 0.0591 = 0.0760\text{ lb/ft}^3$ ***ANS***

[8] Specific heat and other thermodynamic properties of gases are discussed in Sec. 13.1.

Exercises

2.7.1 A gas at 60°C under a pressure of 10000 mb abs has a specific weight of 99 N/m^3. What is the value of R for the gas? What gas might this be? Refer to Appendix A, Table A.5.

2.7.2 A hydrogen-filled balloon of the type used in cosmic-ray studies is to be expanded to its full size, which is a 100-ft-diameter sphere, without stress in the wall at an altitude of 150,000 ft. If the pressure and temperature at this altitude are 0.14 psia and −67°F respectively, find the volume of hydrogen at 14.7 psia and 60°F that should be added on the ground. Neglect the balloon's weight.

2.7.3 Calculate the density, specific weight, and specific volume of air at 120°F and 50 psia.

2.7.4 Calculate the density, specific weight, and specific volume of air at 50°C and 3400 mb abs.

2.7.5 If natural gas has a specific gravity of 0.6 relative to air at 14.7 psia and 68°F, what are its specific weight and specific volume at that same pressure and temperature. What is the value of R for the gas? Solve without using Table A.2.

2.7.6 Given that a sample of dry air at 40°F and 14.7 psia contains 21% oxygen and 78% nitrogen by volume. What is the partial pressure (psia) and specific weight of each gas?

2.7.7 Prove that Eq. (2.7) follows from Eqs. (2.4) and (2.6).

2.8 Compressibility of Perfect Gases

Differentiating Eq. (2.6) gives $npv^{n-1}dv + v^n dp = 0$. Inserting the value of dp from this into $E_v = -(v/dv)\,dp$ from Sec. 2.5 yields

$$E_v = np \tag{2.8}$$

So for an isothermal process of a gas $E_v = p$, and for an isentropic process $E_v = kp$.

Thus, at a pressure of 15 psia, the isothermal modulus of elasticity for a gas is 15 psi, and for air in an isentropic process it is 1.4(15 psi) = 21 psi. Assuming from Table 2.1 a typical value of the modulus of elasticity of cold water to be 320,000 psi, we see that air at 15 psia is 320,000/15 = 21,000 times as compressible as cold water isothermally, or 320,000/21 = 15,000 times as compressible isentropically. This emphasizes the great difference between the compressibility of normal atmospheric air and that of water.

Sample Problem 2.6 (*a*) Calculate the density, specific weight, and specific volume of oxygen at 100°F and 15 psia (pounds per square inch absolute; see Sec. 2.7). (*b*) What would be the temperature and pressure of this gas if we compressed it isentropically to 40% of its original volume? (*c*) If the process described in (*b*) had been isothermal, what would the temperature and pressure have been?

Solution

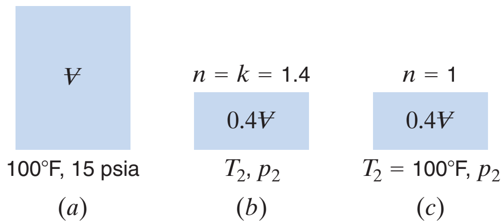


Table A.5 for oxygen (O_2): Molar mass $M = 32.0$, $k = 1.40$

(*a*) Sec. 2.7: $R \approx \frac{R_0}{M} = \frac{49{,}709}{32.0} = 1553 \text{ ft·lb/(slug·°R)}$ (as in Table A.5)

From Eq. (2.4):
$$\rho = \frac{p}{RT} = \frac{15 \times 144 \text{ lb/ft}^2}{[1553 \text{ ft·lb/(slug·°R)}][(460 + 100)\text{°R}]}$$
$$= 0.00248 \text{ slug/ft}^3 \quad \textbf{\textit{ANS}}$$

With $g = 32.2 \text{ ft/sec}^2$, $\gamma = \rho g = 0.00248(32.2) = 0.0800 \text{ lb/ft}^3$ ***ANS***

Eq. (2.2):
$$v = \frac{1}{\rho} = \frac{1}{0.002\,48} = 403 \text{ ft}^3\text{/slug} \quad \textbf{\textit{ANS}}$$

(*b*) Isentropic compression: $v_2 = 40\% \, v_1 = 0.4(403) = 161.1 \text{ ft}^3\text{/slug}$
$$\rho_2 = 1/v_2 = 0.00621 \text{ slug/ft}^3$$

Eq. (2.6) with $n = k$: $pv^k = (15 \times 144)(403)^{1.4} = (p_2 \times 144)(161.1)^{1.4}$
$$p_2 = 54.1 \text{ psia} \quad \textbf{\textit{ANS}}$$

From Eq. (2.4): $p_2 = 54.1 \times 144 \text{ psia} = \rho RT = 0.00621(1553)(460 + T_2)$
$$T_2 = 348\text{°F} \quad \textbf{\textit{ANS}}$$

(*c*) Isothermal compression: $T_2 = T_1 = 100\text{°F}$ ***ANS***

$pv = \text{constant}$: $(15 \times 144)(403) = (p_2 \times 144)(0.4 \times 403)$
$$p_2 = 37.5 \text{ psia} \quad \textbf{\textit{ANS}}$$

SAMPLE PROBLEM 2.7 Calculate the density, specific weight, and specific volume of chlorine gas at 25°C and pressure of 600 kN/m^2 abs (kilonewtons per square meter absolute; see Sec. 2.7). Given the molar mass of chlorine $(Cl_2) = 71$.

Solution

Sec. 2.7: $$R = \frac{R_0}{M} = \frac{8312}{71} = 117.1\ \text{N·m/(kg·K)}$$

From Eq. (2.4): $$\rho = \frac{p}{RT} = \frac{600\,000\ \text{N/m}^2}{[117.1\ \text{N·m/(kg·K)}][(273 + 25)\text{K}]}$$

$$= 17.20\ \text{kg/m}^3 \qquad \textit{ANS}$$

With $g = 9.81\ \text{m/s}^2$, $\gamma = \rho g = 17.20(9.81) = 168.7\ \text{N/m}^3$ ***ANS***

Eq. (2.2): $$v = \frac{1}{\rho} = \frac{1}{17.20} = 0.0581\ \text{m}^3/\text{kg} \qquad \textit{ANS}$$

EXERCISES

2.8.1 Methane at 22 psia is compressed isothermally, and nitrogen at 16 psia is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?

2.8.2 Methane at 140 kPa abs is compressed isothermally, and nitrogen at 100 kPa abs is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?

2.8.3 (*a*) If 10 m^3 of nitrogen at 30°C and 125 kPa are expanded isothermally to 25 m^3, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for nitrogen is 1.40.

2.8.4 Helium at 25 psia and 65°F is isentropically compressed to one-fifth of its original volume. What is its final pressure?

2.9 STANDARD ATMOSPHERE

Standard atmospheres were first adopted in the 1920s in the United States and in Europe to satisfy a need for standardization of aircraft instruments and aircraft performance. As knowledge of the atmosphere increased, and man's activities in it rose to ever greater altitudes, such standards have been frequently extended and improved.

The International Civil Aviation Organization adopted its latest ***ICAO Standard Atmosphere*** in 1964, which extends up to 32 km (105,000 ft). The International Standards Organization adopted an ***ISO Standard Atmosphere*** to

50 km (164,000 ft) in 1973, which incorporates the ICAO standard. The United States has adopted the ***U.S. Standard Atmosphere,*** last revised in 1976. This incorporates the ICAO and ISO standards, and extends to at least 86 km (282,000 ft or 53.4 mi); for some quantities it extends as far as 1000 km (621 mi).

Figure 2.2 graphically presents variations of temperature and pressure in the U.S. Standard Atmosphere. In the lowest 11.02 km (36,200 ft), called the ***troposphere,*** the temperature decreases rapidly and linearly at a ***lapse rate*** of −6.489°C/km (−3.560°F/1000 ft). In the next layer, called the ***stratosphere,*** about 9 km (30,000 ft) thick, the temperature remains constant at −56.5°C (−69.7°F). Next, in the ***mesosphere,*** it increases, first slowly and then more rapidly, to a maximum of −2.5°C (27.5°F) at an altitude around 50 km (165,000 ft or 31 mi). Above this, in the upper part of the mesosphere known as the ***ionosphere,*** the temperature again decreases.

The standard absolute pressure[9] behaves very differently from temperature (Fig. 2.2), decreasing quite rapidly and smoothly to almost zero at an altitude of

[9] Absolute pressure is discussed in Secs. 2.7 and 3.4.

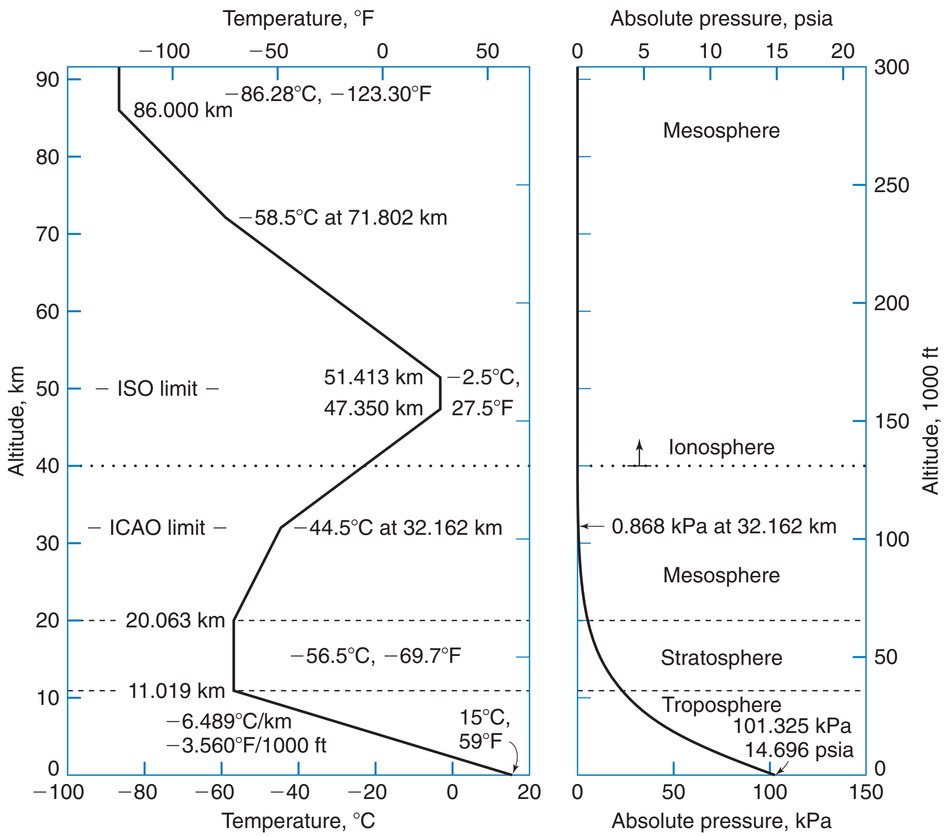


Figure 2.2
The U.S. Standard Atmosphere, temperature, and pressure distributions.

30 km (98,000 ft). The pressure profile was computed from the standard temperatures using methods of fluid statics (Sec. 3.2). The representation of the standard temperature profile by a number of linear functions of elevation (Fig. 2.2) greatly facilitates such computations (see Sample Prob. 3.1*d*).

Temperature, pressure, and other variables from the ICAO Standard Atmosphere, including density and viscosity, are tabulated together with gravitational acceleration out to 30 km and 100,000 ft in Appendix A, Table A.3. Engineers generally use such data in design calculations where the performance of high-altitude aircraft is of interest. The standard atmosphere serves as a good approximation of conditions in the atmosphere; of course the actual conditions vary somewhat with the weather, the seasons, and the latitude.

2.10 Ideal Fluid

An ***ideal*** fluid is usually defined as a fluid in which there is no *friction;* it is ***inviscid*** (its viscosity is zero). Thus the internal forces at any section within it are always normal to the section, even during motion. So these forces are purely pressure forces. Although such a fluid does not exist in reality, many fluids approximate frictionless flow at sufficient distances from solid boundaries, and so we can often conveniently analyze their behaviors by assuming an ideal fluid. As noted in Sec. 2.7, take care to not confuse an ideal fluid with a perfect (ideal) gas.

In a ***real*** fluid, either liquid or gas, tangential or shearing forces always develop whenever there is motion relative to a body, thus creating fluid friction, because these forces oppose the motion of one particle past another. These friction forces give rise to a fluid property called viscosity.

2.11 Viscosity

The ***viscosity*** of a fluid is a measure of its resistance to shear or angular deformation. Motor oil, for example, has high viscosity and resistance to shear, is cohesive, and feels "sticky," whereas gasoline has low viscosity. The friction forces in flowing fluid result from the cohesion and momentum interchange between molecules. Figure 2.3 indicates how the viscosities of typical fluids depend on temperature. As the temperature increases, the viscosities of all liquids *decrease,* while the viscosities of all gases *increase*. This is because the force of cohesion, which diminishes with temperature, predominates with liquids, while with gases the predominating factor is the interchange of molecules between the layers of different velocities. Thus a rapidly-moving gas molecule shifting into a slower-moving layer tends to speed up the latter. And a slow-moving molecule entering a faster-moving layer tends to slow down the faster-moving layer. This molecular interchange sets up a shear, or produces a friction force between adjacent layers. At higher temperatures molecular activity increases, so causing the viscosity of gases to increase with temperature.

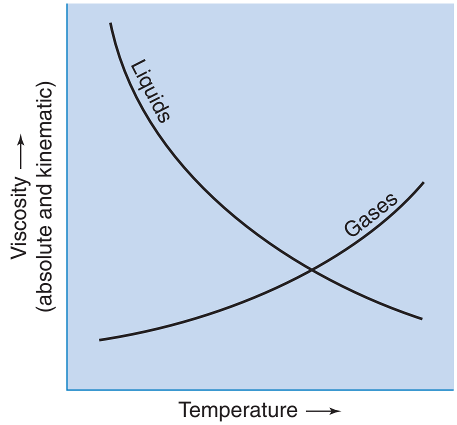


Figure 2.3
Trends in viscosity variation with temperature.

Figures A.1 and A.2 in Appendix A graphically present numerical values of absolute and kinematic viscosities for a variety of liquids and gases, and show how they vary with temperature.

Consider the classic case of two parallel plates (Fig. 2.4), sufficiently large that we can neglect edge conditions, a small distance Y apart, with fluid filling the space between. The lower plate is stationary, while the upper one moves parallel to it with a velocity U due to a force F corresponding to some area A of the moving plate.

At boundaries, particles of fluid adhere to the walls, and so their velocities are zero relative to the wall. This so-called ***no-slip condition*** occurs with all viscous fluids. Thus in Fig. 2.4 the fluid velocities must be U where in contact with the plate at the upper boundary and zero at the lower boundary. We call the form of the velocity variation with distance between these two extremes, as depicted in Fig. 2.4, a ***velocity profile.*** If the separation distance Y is not too great, if the velocity U is not too high, and if there is no net flow of fluid through the space, the velocity profile will be linear, as in Fig. 2.4*a*. If, in addition, there is a small amount of bulk fluid transport between the plates, as could result from pressure-fed lubrication for example, the velocity profile becomes the sum of

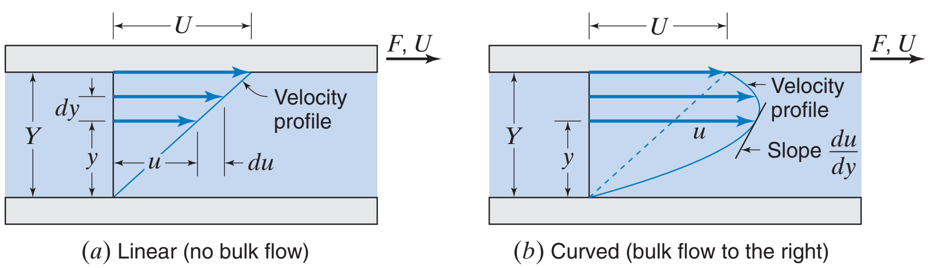


Figure 2.4
Velocity profiles.

the previous linear profile plus a parabolic profile (Fig. 2.4*b*); the parabolic additions to (or subtractions from) the linear profile are zero at the walls (plates) and maximum at the centerline. The behavior of the fluid is much as if it consisted of a series of thin layers, each of which slips a little relative to the next.

For a large class of fluids under the conditions of Fig. 2.4*a*, experiments have shown that

$$F \propto \frac{AU}{Y}$$

We see from similar triangles that we can replace U/Y by the velocity gradient du/dy. If we now introduce a constant of proportionality μ (mu), we can express the shearing stress τ (tau) between any two thin sheets of fluid by

$$\tau = \frac{F}{A} = \mu\frac{U}{Y} = \mu\frac{du}{dy} \qquad (2.9)$$

We call Eq. (2.9) ***Newton's equation of viscosity,*** since Sir Isaac Newton (1642–1727) first suggested it. Although better known for his formulation of the fundamental laws of motion and gravity and for the development of differential calculus, Newton, an English mathematician and natural philosopher, also made many pioneering studies in fluid mechanics. In transposed form, Eq. (2.9) defines the proportionality constant

$$\mu = \frac{\tau}{du/dy} \qquad (2.10)$$

known as the ***coefficient of viscosity,*** the ***absolute viscosity,*** the ***dynamic viscosity*** (since it involves force), or simply the ***viscosity*** of the fluid. We shall use "absolute viscosity" to help differentiate it from another viscosity that we will discuss shortly.

We noted in Sec. 2.1 that the distinction between a solid and a fluid lies in the manner in which each can resist shearing stresses. We will clarify a further distinction among various kinds of fluids and solids by referring to Fig. 2.5. In

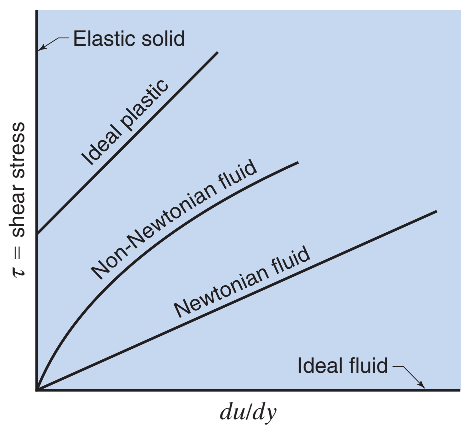


Figure 2.5

the case of a solid, shear stress depends on the *magnitude* of the deformation; but Eq. (2.9) shows that in many fluids the shear stress is proportional to the *time rate* of (angular) deformation.

A fluid for which the constant of proportionality (i.e., the absolute viscosity) does not change with rate of deformation is called a ***Newtonian fluid,*** and this plots as a straight line in Fig. 2.5. The slope of this line is the absolute viscosity, μ. The ideal fluid, with no viscosity (Sec. 2.10), falls on the horizontal axis, while the true elastic solid plots along the vertical axis. A plastic that sustains a certain amount of stress before suffering a plastic flow corresponds to a straight line intersecting the vertical axis at the yield stress. There are certain non-Newtonian fluids[10] in which μ varies with the rate of deformation. These are relatively uncommon in engineering usage, so we will restrict the remainder of this text to the common fluids that under normal conditions obey Newton's equation of viscosity.

In a ***journal bearing,*** lubricating fluid fills the small annular space between a shaft and its surrounding support. This fluid layer is very similar to the layer between the two parallel plates, except it is curved. There is another more subtle difference, however. For coaxial cylinders (Fig. 2.6) with constant rotative speed ω (omega), the resisting and driving torques are equal. But because the radii at the inner and outer walls are different, it follows that the shear stresses

[10] Typical non-Newtonian fluids include paints, printer's ink, gels and emulsions, sludges and slurries, and certain plastics. An excellent treatment of the subject is given by W. L. Wilkinson in *NonNewtonian Fluids,* Pergamon Press, New York, 1960.

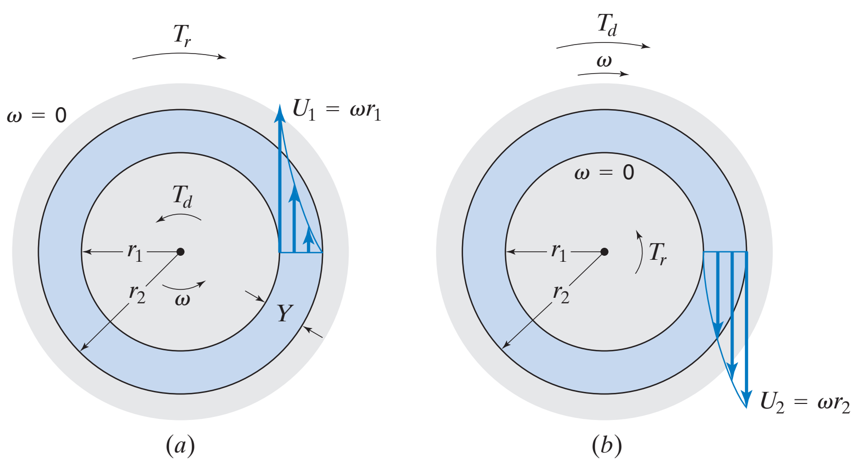


Figure 2.6
Velocity profile, rotating coaxial cylinders with gap completely filled with liquid. (*a*) Inner cylinder rotating. (*b*) Outer cylinder rotating. Z is the dimension at right angles to the plane of the sketch. Resisting torque = driving torque and $\tau \propto (du/dy)$.

$$\tau_1(2\pi r_1 Z)r_1 = \tau_2(2\pi r_2 Z)r_2, \qquad \left(\frac{du}{dy}\right)_1 = \left(\frac{du}{dy}\right)_2 \frac{r_2^2}{r_1^2}$$

and velocity gradients there must also be different (see Fig. 2.6 and equations that accompany it). The shear stress and velocity gradient must vary continuously across the gap, and so the velocity profile must curve. However, as the gap distance $Y \to 0$, $du/dy \to U/Y =$ constant. So, when the gap is very small, we can assume the velocity profile to be a straight line, and we can solve problems in a similar manner as for flat plates.

The dimensions of absolute viscosity are force per unit area divided by velocity gradient. In the British Gravitational (BG) system the dimensions of absolute viscosity are as follows:

$$\text{Dimensions of } \mu = \frac{\text{dimensions of } \tau}{\text{dimensions of } du/dy} = \frac{\text{lb/ft}^2}{\text{fps/ft}} = \text{lb·sec/ft}^2$$

In SI units

$$\text{Dimensions of } \mu = \frac{\text{N/m}^2}{\text{s}^{-1}} = \text{N·s/m}^2$$

A widely used unit for viscosity in the metric system is the ***poise*** (P), named after Jean Louis Poiseuille (1799–1869). A French anatomist, Poiseuille was one of the first investigators of viscosity. The poise = 0.10 N·s/m^2. The ***centipoise*** (cP) (= 0.01 P = 1 mN·s/m^2) is frequently a more convenient unit. It has a further advantage in that the viscosity of water at 68.4°F is 1 cP. Thus the value of the viscosity in centipoises is an indication of the viscosity of the fluid relative to that of water at 68.4°F.

In many problems involving viscosity the absolute viscosity is divided by density. This ratio defines the ***kinematic viscosity*** ν (nu), so called because force is not involved, the only dimensions being length and time, as in kinematics (Sec. 1.1). Thus

$$\nu = \frac{\mu}{\rho} \tag{2.11}$$

We usually measure kinematic viscosity ν in ft^2/sec in the BG system, and in m^2/s in the SI. Previously, in the metric system the common units were cm^2/s, also called the ***stoke*** (St), after Sir George Stokes (1819–1903), an English physicist and pioneering investigator of viscosity. Many found the ***centistoke*** (cSt) (0.01 St = 10^{-6} m^2/s) a more convenient unit to work with.

An important practical distinction between the two viscosities is the following. The absolute viscosity μ of most fluids is virtually independent of pressure for the range that is ordinarily encountered in engineering work; for extremely high pressures, the values are a little higher than those shown in Fig. A.1. The kinematic viscosity ν of gases, however, varies strongly with pressure because of changes in density. Therefore, if we need to determine the kinematic viscosity ν at a nonstandard pressure, we can look up the (pressure-independent) value of μ and calculate ν from Eq. (2.11). This will require knowing the gas density, ρ, which, if necessary, we can calculate using Eq. (2.4).

The *measurement* of viscosity is described in Sec. 11.1.

SAMPLE PROBLEM 2.8 A 1-in-wide space between two horizontal plane surfaces is filled with SAE 30 Western lubricating oil at 80°F. What force is required to drag a very thin plate of 4-ft^2 area through the oil at a velocity of 20 ft/min if the plate is 0.33 in from one surface?

Solution

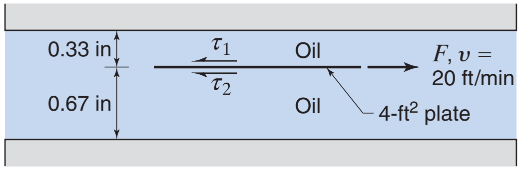


Fig. A.1: $\mu = 0.0063$ lb·sec/ft^2

Eq. (2.9): $\tau_1 = 0.0063 \times (20/60)/(0.33/12) = 0.0764 \text{ lb/ft}^2$

Eq. (2.9): $\tau_2 = 0.0063 \times (20/60)/(0.67/12) = 0.0394 \text{ lb/ft}^2$

From Eq. (2.9): $F_1 = \tau_1 A = 0.0764 \times 4 = 0.305 \text{ lb}$

From Eq. (2.9): $F_2 = \tau_2 A = 0.0394 \times 4 = 0.158 \text{ lb}$

$$\text{Force} = F_1 + F_2 = 0.463 \text{ lb} \qquad \textit{ANS}$$

SAMPLE PROBLEM 2.9 In Fig. S2.9 oil of absolute viscosity μ fills the small gap of thickness Y. (a) Neglecting fluid stress exerted on the circular underside, obtain an expression for the torque T required to rotate the truncated cone at constant speed ω. (b) What is the rate of heat generation, in joules per second, if the oil's absolute viscosity is 0.20 N·s/m^2, $\alpha = 45°$, $a = 45$ mm, $b = 60$ mm, $Y = 0.2$ mm, and the speed of rotation is 90 rpm?

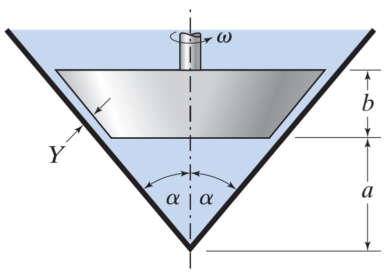


Figure S2.9

Solution

(a) $U = \omega r$; for small gap Y, $\dfrac{du}{dy} = \dfrac{U}{Y} = \dfrac{\omega r}{Y}$

Eq. (2.9): $\tau = \mu \dfrac{du}{dy} = \dfrac{\mu \omega r}{Y}$; $dA = 2\pi r\, ds = \dfrac{2\pi r\, dy}{\cos\alpha}$

From Eq. (2.9): $dF = \tau\, dA = \dfrac{\mu\omega r}{Y}\left(\dfrac{2\pi r\, dy}{\cos\alpha}\right)$

$$dT = r\,dF = \frac{2\pi\mu\omega}{Y\cos\alpha} r^3 dy; \quad r = y\tan\alpha$$

$$dT = \frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha} y^3 dy$$

$$T = \frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\int_a^{a+b} y^3 dy; \quad \frac{y^4}{4}\bigg|_a^{a+b} = \left[\frac{(a+b)^4}{4} - \frac{a^4}{4}\right]$$

$$T = \frac{2\pi\mu\omega\tan^3\alpha}{4Y\cos\alpha}[(a+b)^4 - a^4] \qquad \textbf{\textit{ANS}}$$

(*b*) $[(a+b)^4 - a^4] = (0.105\text{ m})^4 - (0.045\text{ m})^4 = 0.0001175\text{ m}^4$

$$\omega = \left(90\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{radians}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 3\pi\text{ rad/s} = 3\pi\text{ s}^{-1}$$

Heat generation rate = power = $T\omega = \dfrac{2\pi\mu\omega^2\tan^3\alpha}{4Y\cos\alpha}[(a+b)^4 - a^4]$

$$= \frac{2\pi(0.20\text{ N·s/m}^2)(3\pi\text{ s}^{-1})^2(1)^3[0.0001175\text{ m}^4]}{4(2\times10^{-4}\text{ m})\cos 45°}$$

$$= 23.2\text{ N·m/s} = 23.2\text{ J/s} \qquad \textbf{\textit{ANS}}$$

Exercises

2.11.1 At 60°F what is the kinematic viscosity of the gasoline in Fig. A.2, the specific gravity of which is 0.680? Give the answer in both BG and SI units.

2.11.2 To what temperature must the fuel oil with the higher specific gravity in Fig. A.2 be heated in order that its kinematic viscosity may be reduced to three times that of water at 40°F?

2.11.3 Compare the ratio of the absolute viscosities of air and water at 70°F with the ratio of their kinematic viscosities at the same temperature and at 14.7 psia.

2.11.4 A flat plate 200 mm × 750 mm slides on oil ($\mu = 0.85$ N·s/m^2) over a large plane surface (Fig. X2.11.4). What force *F* is required to drag the plate at a velocity *v* of 1.2 m/s, if the thickness *t* of the separating oil film is 0.6 mm?

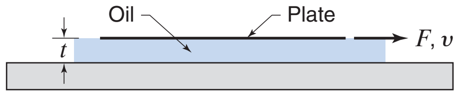


Figure X2.11.4

2.11.5 Refer to Fig. X2.11.4. A flat plate 2 ft × 3 ft slides on oil ($\mu = 0.024$ lb·sec/ft^2) over a large plane surface. What force *F* is required to drag the plate at a velocity *v* of 4 ft/sec, if the thickness *t* of the separating oil film is 0.025 in?

2.11.6 A liquid has an absolute viscosity of 3.2×10^{-4} lb·sec/ft^2. It weighs 56 lb/ft^3. What are its absolute and kinematic viscosities in SI units?

2.11.7 (*a*) What is the ratio of the absolute viscosity of water at a temperature of 70°F to that of water at 200°F? (*b*) What is the ratio of the absolute viscosity of the crude oil in Fig. A.1 ($s = 0.925$) to that of the gasoline ($s = 0.680$), both being at a temperature of 60°F? (*c*) In cooling from 300 to 80°F, what is the ratio of the change of the absolute viscosity of the SAE 30 Western oil to that of the SAE 30 Eastern oil? Refer to Appendix A.

2.11.8 A space 16 mm wide between two large plane surfaces is filled with SAE 30 Western lubricating oil at 35°C (Fig. X2.11.8). What force F is required to drag a very thin plate of 0.4 m^2 area between the surfaces at a speed $v = 0.25$ m/s (*a*) if the plate is equally spaced between the two surfaces, and (*b*) if $t = 5$ mm? Refer to Appendix A.

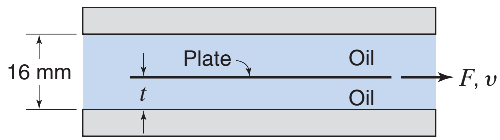


Figure X2.11.8

2.11.9 A journal bearing consists of an 80-mm shaft in an 80.4-mm sleeve 120 mm long, the clearance space (assumed to be uniform) being filled with SAE 30 Western lubricating oil at 40°C (Fig. X2.11.9). Calculate the rate at which heat is generated at the bearing when the shaft turns at 150 rpm. Express the answer in kN·m/s, J/s, Btu/hr, ft·lb/sec, and hp. Refer to Appendix A.

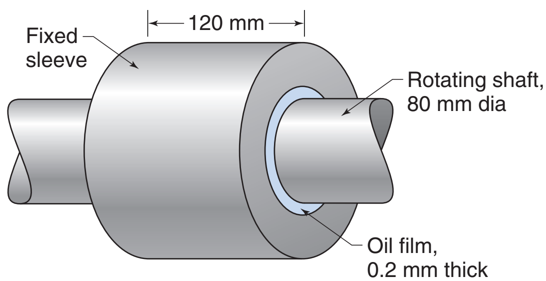


Figure X2.11.9

2.11.10 In using a rotating-cylinder viscometer, a bottom correction must be applied to account for the drag on the flat bottom of the inner cylinder. Calculate the theoretical amount of this torque correction, neglecting centrifugal effects, for a cylinder of diameter d, rotated at a constant angular velocity ω, in a liquid of absolute viscosity μ, with a clearance Δh between the bottom of the inner cylinder and the floor of the outer one.

2.11.11 Assuming a velocity distribution as shown in Fig. X2.11.11, which is a parabola having its vertex 12 in from the boundary, calculate the velocity gradients for $y = 0$, 3, 6, 9, and 12 in. Also calculate the shear stresses in lb/ft^2 at these points if the fluid's absolute viscosity is 600 cP.

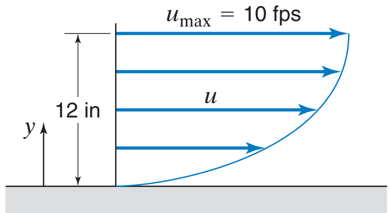


Figure X2.11.11

2.11.12 Air at 50 psia and 60°F is flowing through a pipe. Table A.2 indicates that its kinematic viscosity ν is 0.158×10^{-3} ft^2/sec. (*a*) Why is this ν value incorrect? (*b*) What is the correct value?

2.12 Surface Tension

Liquids have cohesion and adhesion, both of which are forms of molecular attraction. ***Cohesion*** enables a liquid to resist tensile stress, while ***adhesion*** enables it to adhere to another body.[11] At the interface between a liquid and a gas, i.e., at the liquid surface, and at the interface between two ***immiscible*** (not mixable) liquids, the out-of-balance attraction force between molecules forms an imaginary surface film which exerts a tension force in the surface. This liquid property is known as ***surface tension.*** Because this tension acts in a surface, we compare such forces by measuring the tension force per unit length of surface. When a second fluid is not specified at the interface, it is understood that the liquid surface is in contact with air. The surface tensions of various liquids cover a wide range, and they decrease slightly with increasing temperature. Values of the surface tension for water between the freezing and boiling points vary from 0.005 18 to 0.004 04 lb/ft (0.0756 to 0.0589 N/m); Table A.1 of Appendix A contains more typical values. Table A.4 includes values for other liquids. ***Capillarity*** is the property of exerting forces on fluids by fine tubes or porous media; it is due to both cohesion and adhesion. When the cohesion is of less effect than the adhesion, the liquid will wet a solid surface it touches and rise at the point of contact; if cohesion predominates, the liquid surface will depress at the point of contact. For example, capillarity makes water rise in a glass tube, while mercury depresses below the true level, as shown in the insert in Fig. 2.7, which is drawn to scale and reproduced actual size. We call the curved liquid surface that develops in a tube a ***meniscus.***

A cross section through capillary rise in a tube looks like Fig. 2.8. From free-body considerations, equating the lifting force created by surface tension to

[11] In 1877 Osborne Reynolds demonstrated that a $\frac{1}{4}$-in-diameter column of mercury could withstand a tensile stress (negative pressure, below atmospheric) of 3 atm (44 psi or 304 kPa) for a time, but that it would separate upon external jarring of the tube. Liquid tensile stress (said to be as high as 400 atm) accounts for the rise of water in the very small channels of xylem tissue in tall trees. For practical engineering purposes, however, we assume liquids are incapable of resisting any direct tensile stress.

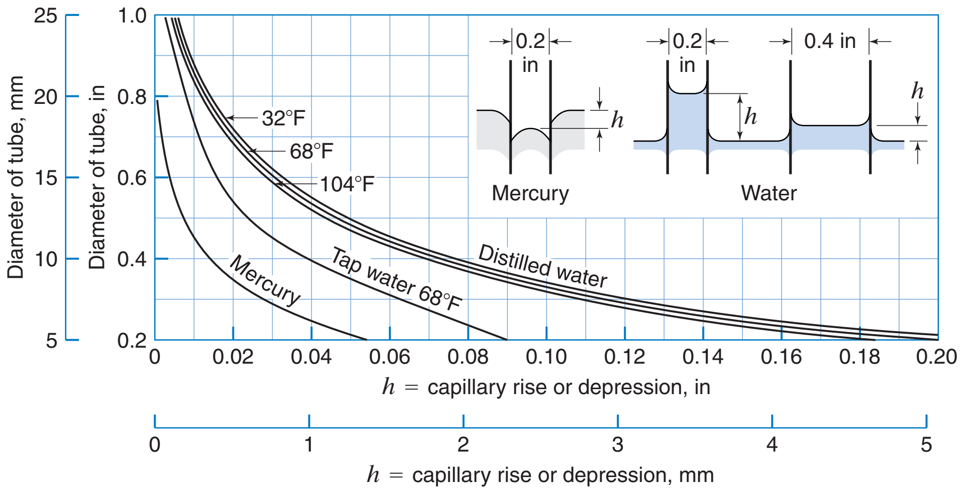


Figure 2.7
Capillarity in clean circular glass tubes, for liquid in contact with air.

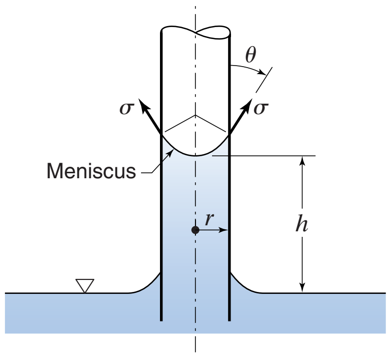


Figure 2.8
Capillary rise.

the gravity force,

$$2\pi r\sigma \cos\theta = \pi r^2 h\gamma$$

so

$$h = \frac{2\sigma \cos\theta}{\gamma r} \tag{2.12}$$

where σ = surface tension (sigma) in units of force per unit length
θ = wetting angle (theta)
γ = specific weight of liquid
r = radius of tube
h = capillary rise[12]

[12] Measurements to a meniscus are usually taken to the point on the centerline.

We can use this expression to compute the *approximate* capillary rise or depression in a tube. If the tube is clean, $\theta = 0°$ for water and about 140° for mercury. Note that the meniscus (Figs. 2.7 and 2.8) lifts a small volume of liquid, near the tube walls, in addition to the volume $\pi r^2 h$ used in Eq. (2.12). For larger tube diameters, with smaller capillary rise heights, this small additional volume can become a large fraction of $\pi r^2 h$. So Eq. (2.12) overestimates the amount of capillary rise or depression, particularly for larger diameter tubes. The curves of Fig. 2.7 are for water or mercury in contact with air; if mercury is in contact with water, the surface tension effect is slightly less than when in contact with air. For tube diameters larger than $\frac{1}{2}$ in (12 mm), capillary effects are negligible.

Surface tension effects are generally negligible in most engineering situations. However, they can be important in problems involving capillary rise, such as in the soil water zone; without capillarity most forms of vegetable life would perish. When we use small tubes to measure fluid properties, such as pressures, we must take the readings while aware of the surface tension effects; a true reading would occur if surface tension effects were zero. These effects are also important in hydraulic model studies when the model is small, in the breakup of liquid jets, and in the formation of drops and bubbles. The formation of drops is extremely complex to analyze, but is, for example, of critical concern in the design of inkjet printers, a multi-billion-dollar business.

SAMPLE PROBLEM 2.10 Water at 10°C stands in a clean glass tube of 2-mm diameter at a height of 35 mm. What is the true static height?

Solution

Table A.1 at 10°C: $\gamma = 9804\ \text{N/m}^3,\ \sigma = 0.0742\ \text{N/m}.$

Sec. 2.12 for clean glass tube: $\theta = 0°.$

Eq. (2.12): $$h = \frac{2\sigma}{\gamma r} = \frac{2(0.0742\ \text{N/m})}{(9804\ \text{N/m}^3)0.001\ \text{m}}$$

$$= 0.015\,14\ \text{m} = 15.14\ \text{mm}$$

Sec. 2.12: True static height = 35.00 − 15.14 = 19.86 mm ***ANS***

EXERCISES

2.12.1 Tap water at 68°F stands in a glass tube of 0.32-in diameter at a height of 4.50 in. What is the true static height?

2.12.2 Distilled water at 20°C stands in a glass tube of 6.0-mm diameter at a height of 18.0 mm. What is the true static height?

2.12.3 Use Eq. (2.12) to compute the capillary depression of mercury at 68°F ($\theta = 140°$) to be expected in a 0.05-in-diameter tube.

2.12.4 Compute the capillary rise in mm of pure water at 10°C expected in an 0.8-mm-diameter tube.

2.12.5 Use Eq. (2.12) to compute the capillary rise of water to be expected in a 0.28-in-diameter tube. Assume pure water at 68°F. Compare the result with Fig. 2.7.

2.13 VAPOR PRESSURE OF LIQUIDS

All liquids tend to evaporate or vaporize, which they do by projecting molecules into the space above their surfaces. If this is a confined space, the partial pressure exerted by the molecules increases until the rate at which molecules reenter the liquid is equal to the rate at which they leave. For this equilibrium condition, we call the vapor pressure the ***saturation pressure.***

Molecular activity increases with increasing temperature and decreasing pressure, and so the saturation pressure does the same. At any given temperature, if the pressure on the liquid surface falls below the saturation pressure, a rapid rate of evaporation results, known as ***boiling.*** Thus we can refer to the saturation pressure as the ***boiling pressure*** for a given temperature, and it is of practical importance for liquids.[13]

We call the rapid vaporization and recondensation of liquid as it briefly passes through a region of low absolute pressure ***cavitation.*** This phenomenon is often very damaging, and so we must avoid it; we shall discuss it in more detail in Sec. 5.10.

Table 2.3 calls attention to the wide variation in saturation vapor pressure of various liquids; Appendix A, Table A.4 contains more values. The very low vapor pressure of mercury makes it particularly suitable for use in barometers. Values for the vapor pressure of water at different temperatures are in Appendix A, Table A.1.

TABLE 2.3 **Saturation vapor pressure of selected liquids at 68°F (20°C)**

	psia	N/m² abs	mb abs
Mercury	0.000025	0.17	0.0017
Water	0.34	2340	23.4
Carbon tetrachloride	1.90	13100	131
Gasoline	8.0	55200	552

[13] Values of the saturation pressure for water for temperatures from 32 to 705.4°F can be found in J. H. Keenan, *Thermodynamic Properties of Water including Vapor, Liquid and Solid States,* John Wiley & Sons, Inc., New York, 1969, and in other steam tables. There are similar vapor tables published for ammonia, carbon dioxide, sulfur dioxide, and other vapors of engineering interest.

Sample Problem 2.11 At approximately what temperature will water boil if the elevation is 10,000 ft?

Solution

From Appendix A, Table A.3, the pressure of the standard atmosphere at 10,000-ft elevation is 10.11 psia. From Appendix A, Table A.1, the saturation vapor pressure p_v of water is 10.11 psia at about 193°F (by interpolation). Hence the water at 10,000 ft will boil at about 193°F. ***ANS***

Compared with the boiling temperature of 212°F at sea level, this explains why it takes longer to cook at high elevations.

Exercises

2.13.1 At what pressure in millibars absolute will 70°C water boil?

2.13.2 At approximately what temperature will water boil in Mexico City (elevation 7400 ft)? Refer to Appendix A.

Problems

2.1 If the specific weight of a gas is 12.40 N/m^3, what is its specific volume in m^3/kg?

2.2 A gas sample weighs 0.108 lb/ft^3 at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 0.075 lb/ft^3?

2.3 If a certain liquid weighs 8600 N/m^3, what are the values of its density, specific volume, and specific gravity relative to water at 15°C? Use Appendix A.

2.4 Find the change in volume of 15.00 lb of water at ordinary atmospheric pressure for the following conditions: (*a*) reducing the temperature by 50°F from 200°F to 150°F; (*b*) reducing the temperature by 50°F from 150°F to 100°F; (*c*) reducing the temperature by 50°F from 100°F to 50°F. Calculate each and note the trend in the changes in volume.

2.5 Initially when 1000.00 mL of water at 10°C are poured into a glass cylinder, the height of the water column is 1000.0 mm. The water and its container are heated to 70°C. Assuming no evaporation, what then will be the depth of the water column if the coefficient of thermal expansion for the glass is 3.8×10^{-6} mm/mm per °C?

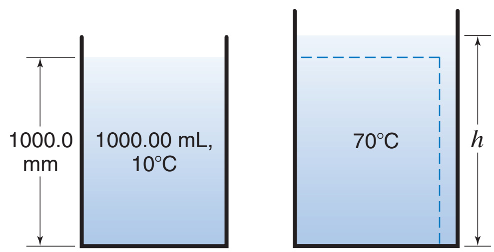


Figure P2.5

2.6 At a depth of 4 miles in the ocean the pressure is 9520 psi. Assume that the specific weight at the surface is 64.00 lb/ft^3 and that the average volume modulus is 320,000 psi for that pressure range. (*a*) What will be the change in specific volume between that at the surface and at that depth? (*b*) What will be the specific volume at that depth? (*c*) What will be the specific weight at that depth? (*d*) What is the percentage change in the specific volume? (*e*) What is the percentage change in the specific weight?

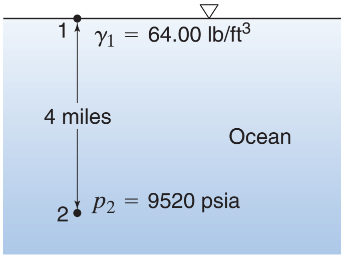


Figure P2.6

2.7 Water at 68°F is in a long, rigid cylinder of inside diameter 0.600 in. A plunger applies pressure to the water. If, with zero force, the initial length of the water column is 25.00 in, what will its length be if a force of 420 lb is applied to the plunger. Assume no leakage and no friction.

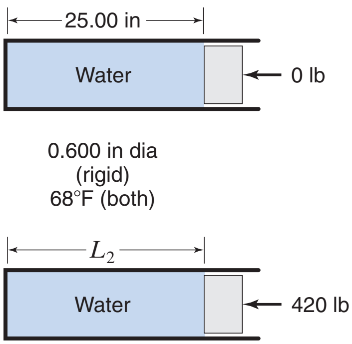


Figure P2.7

2.8 Find the change in volume of 10 m^3 of water for the following situations: (*a*) a temperature increase from 60°C to 70°C with constant atmospheric pressure, (*b*) a pressure increase from zero to 10 MN/m^2 with temperature remaining constant at 60°C, (*c*) a temperature decrease from 60°C to 50°C combined with a pressure increase of 10 MN/m^2.

2.9 A heavy closed steel chamber is filled with water at 40°F and atmospheric pressure. If the temperature of the water and the chamber is raised to 80°F, what will be the new pressure of the water? The coefficient of thermal expansion of the steel is 6.6 × 10^{-6} in/in per °F; assume the chamber is unaffected by the water pressure. Use Table A.1 and Fig. 2.1.

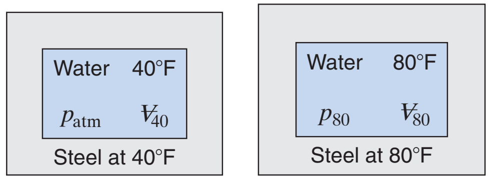


Figure P2.9

2.10 Repeat Exer. 2.6.4 for the case where the tank is made of a material that has a coefficient of thermal expansion of 4.6 × 10^{-6} mm/mm per °C.

2.11 (*a*) Calculate the density, specific weight, and specific volume of oxygen at 20°C and 50 kN/m^2 abs. (*b*) If the oxygen is enclosed in a rigid container of constant volume, what will be the pressure if the temperature is reduced to −100°C?

2.12 (*a*) If water vapor in the atmosphere has a partial pressure of 0.50 psia and the temperature is 90°F, what is its specific weight? (*b*) If the barometer reads 14.50 psia, what is the partial pressure of the (dry) air, and what is its specific weight? (*c*) What is the specific weight of the atmosphere (air plus the water vapor present)?

2.13 (*a*) If water vapor in the atmosphere has a partial pressure of 3500 Pa and the temperature is 30°C, what is its specific weight? (*b*) If the barometer reads 102 kPa abs, what is the partial pressure of the (dry) air, and what is its specific weight? (*c*) What is the specific weight of the atmosphere (air plus the water vapor present)?

2.14 If the specific weight of water vapor in the atmosphere is 0.00065 lb/ft^3 and that of the (dry) air is 0.074 lb/ft^3 when the temperature is 70°F, (*a*) what are the partial pressures of the water vapor and the dry air in psia, (*b*) what is the specific weight of the atmosphere (air and water vapor), and (*c*) what is the barometric pressure in psia?

2.15 If an artificial atmosphere consists of 20% oxygen and 80% nitrogen by volume, at 101.32 kN/m^2 abs and 20°C, what are (*a*) the specific weight and partial pressure of the oxygen, (*b*) the specific weight and partial pressure of the nitrogen, and (*c*) the specific weight of the mixture?

2.16 When the ambient air is at 70°F, 14.7 psia, and contains 21% oxygen by volume, 4.5 lb of air are pumped into a scuba tank, capacity 0.75 ft^3. (*a*) What volume of ambient air was compressed? (*b*) When the filled tank has cooled to ambient conditions, what is the (gage) pressure of the air in the tank? (*c*) What is the partial pressure (psia) and specific weight of the ambient oxygen? (*d*) What weight of oxygen was put in the tank? (*e*) What is the partial pressure (psia) and specific weight of the oxygen in the tank?

2.17 (*a*) If 10 ft^3 of carbon dioxide at 50°F and 15 psia is compressed isothermally to 2 ft^3, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for carbon dioxide is 1.28.

2.18 (*a*) If 350 L of carbon dioxide at 20°C and 120 kN/m^2 abs is compressed isothermally to 50 L, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic? The isentropic exponent k for carbon dioxide is 1.28.

2.19 Helium at 180 kN/m^2 abs and 20°C is isentropically compressed to one-fifth of its original volume. What is its final pressure?

2.20 The absolute viscosity of a certain gas is 0.0234 cP while its kinematic viscosity is 181 cSt, both measured at 1013 mb abs and 100°C. Calculate its approximate molar mass, and suggest what gas it may be.

2.21 A hydraulic lift of the type commonly used for greasing automobiles consists of a 10.000-in-diameter ram that slides in a 10.006-in-diameter cylinder (Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.0038 ft^2/sec and specific gravity of 0.83. If the rate of travel of the ram v is 0.5 fps, find the frictional resistance, F when 6 ft of the ram is engaged in the cylinder.

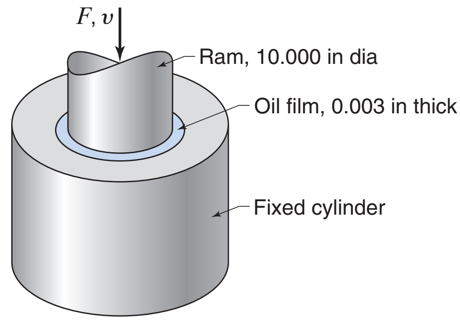


Figure P2.21

2.22 A hydraulic lift of the type commonly used for greasing automobiles consists of a 280.00-mm-diameter ram that slides in a 280.18-mm-diameter cylinder (similar to Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.00042 m^2/s and specific gravity of 0.86. If the rate of travel of the ram is 0.22 m/s, find the frictional resistance when 2 m of the ram is engaged in the cylinder.

2.23 A journal bearing consists of an 8.00-in shaft in an 8.01-in sleeve 10 in long, the clearance space (assumed to be uniform) being filled with SAE 30 Eastern lubricating oil at 100°F. Calculate the rate at which heat is generated at the bearing when the shaft turns at 100 rpm. Refer to Appendix A. Express the answer in Btu/hr.

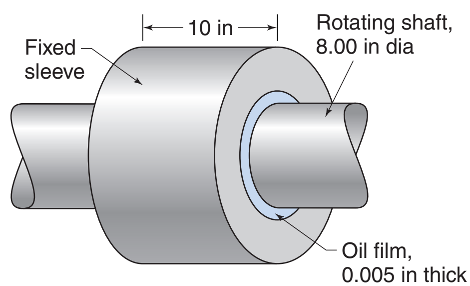


Figure P2.23

2.24 Repeat Prob. 2.23 for the case where the sleeve has a diameter of 8.50 in. Compute as accurately as possible the velocity gradient in the fluid at the shaft and sleeve.

2.25 A disk spins within an oil-filled enclosure, having 2.4-mm clearance from flat surfaces each side of the disk. The disk surface extends from radius 12 to 86 mm. What torque is required to drive the disk at 660 rpm if the oil's absolute viscosity is 0.12 $N{\cdot}s/m^2$?

2.26 It is desired to apply the general case of Sample Prob. 2.9 to the extreme cases of a journal bearing ($\alpha = 0$) and an end bearing ($\alpha = 90°$). But when $\alpha = 0$, $r = \tan \alpha = 0$, so $T = 0$; when $\alpha = 90°$, contact area $= \infty$ due to b, so $T = \infty$. Therefore devise an alternative general derivation that will also provide solutions to these two extreme cases.

2.27 Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed. Its pressure is now 200 kPa abs and its temperature is 20°C. Table A.2 indicates that its kinematic viscosity ν is 15×10^{-6} m^2/s. (*a*) Why is this ν incorrect? (*b*) What is the correct value?

2.28 Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed isentropically. Its pressure is now 194.5 kPa abs and its temperature is 80°C. Table A.2 indicates that its kinematic viscosity ν is 20.9×10^{-6} m^2/s. (*a*) Why is this ν incorrect? (*b*) What is the correct value? (*c*) What would the correct value be if the compression were isothermal instead?

2.29 Pure water at 50°F stands in a glass tube of 0.04-in diameter at a height of 6.78 in. Compute the true static height.

2.30 (*a*) Derive an expression for capillary rise (or depression) between two vertical parallel plates. (*b*) How much would you expect 10°C water to rise (in mm) if the clean glass plates are separated by 1.2 mm?

2.31 By how much does the pressure inside a 2-mm-diameter air bubble in 15°C water exceed the pressure in the surrounding water?

2.32 Determine the excess pressure inside an 0.5-in-diameter soap bubble floating in air, given the surface tension of the soap solution is 0.0035 lb/ft.

2.33 Water at 170°F in a beaker is placed within an airtight container. Air is gradually pumped out of the container. What reduction below standard atmospheric pressure of 14.7 psia must be achieved before the water boils?

2.34 At approximately what temperature will water boil on top of Mount Kilimanjaro (elevation 5895 m)? Refer to Appendix A.