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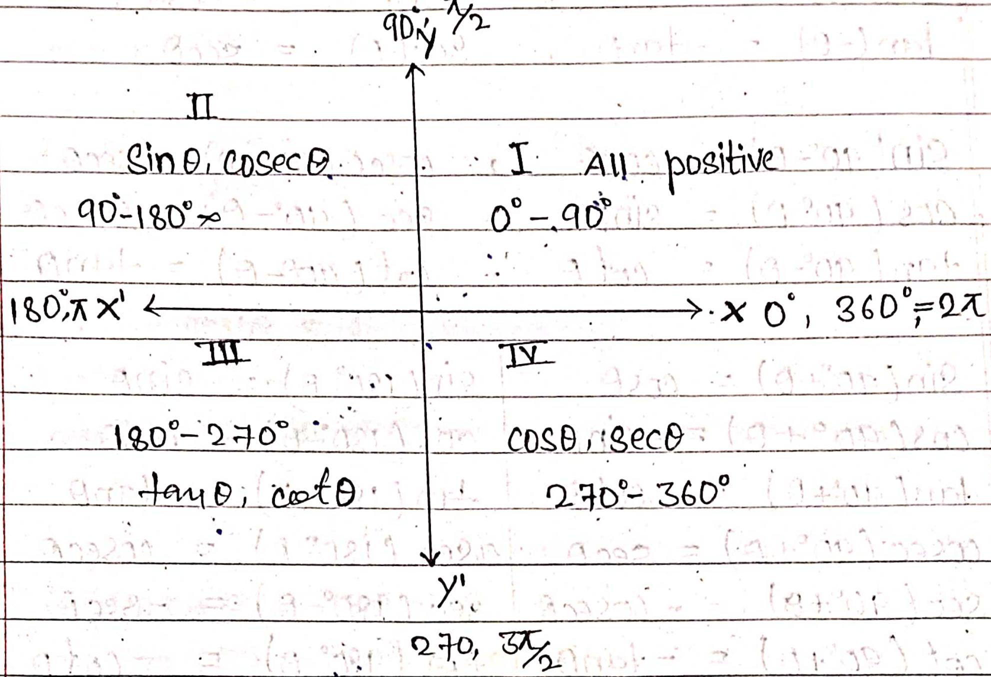
Important Trigonometry formulas:



Angle	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\mathrm{cosec}\,\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

$\sin(-\theta) = -\sin\theta$, $\cot(-\theta) = -\cot\theta$

$\cos(-\theta) = \cos\theta$, $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\,\theta$

$\tan(-\theta) = -\tan\theta$, $\sec(-\theta) = \sec\theta$

$\sin(90°-\theta) = \cos\theta$, $\mathrm{cosec}(90°-\theta) = \sec\theta$

$\cos(90°-\theta) = \sin\theta$, $\sec(90°-\theta) = \mathrm{cosec}\,\theta$

$\tan(90°-\theta) = \cot\theta$, $\cot(90°-\theta) = \tan\theta$

$\sin(90°+\theta) = \cos\theta$	$\sin(180°-\theta) = \sin\theta$
$\cos(90°+\theta) = -\sin\theta$	$\cos(180°-\theta) = \cos\theta$
$\tan(90°+\theta) = -\cot\theta$	$\tan(180°-\theta) = -\tan\theta$
$\mathrm{cosec}(90°+\theta) = \sec\theta$	$\mathrm{cosec}(180°-\theta) = \mathrm{cosec}\,\theta$
$\sec(90°+\theta) = -\mathrm{cosec}\,\theta$	$\sec(180°-\theta) = -\sec\theta$
$\cot(90°+\theta) = -\tan\theta$	$\cot(180°-\theta) = -\cot\theta$

$\sin(180°+\theta) = -\sin\theta$	$\sin(270°-\theta) = -\cos\theta$
$\cos(180°+\theta) = -\cos\theta$	$\cos(270-\theta) = -\sin\theta$
$\tan(180°+\theta) = \tan\theta$	$\tan(270°-\theta) = \cot\theta$
$\mathrm{cosec}(180°+\theta) = -\mathrm{cosec}\,\theta$	$\mathrm{cosec}(270°-\theta) = -\sec\theta$
$\sec(180°+\theta) = -\sec\theta$	$\sec(270°-\theta) = -\mathrm{cosec}\,\theta$
$\cot(180°+\theta) = \cot$	$\cot(270°-\theta) = \tan\theta$

$\sin(270°+\theta) = -\cos\theta$

$\cos(270°+\theta) = \sin\theta$

$\tan(270°+\theta) = -\cot\theta$

$\mathrm{cosec}(270°+\theta) = -\sec\theta$

$\sec(270°+\theta) = \mathrm{cosec}\,\theta$

$\cot(270°+\theta) = -\tan\theta$

$\sin\theta = \frac{Perp}{Hyp}$, $\operatorname{cosec}\theta = \frac{Hyp}{Perp}$

$\cos\theta = \frac{Base}{Hyp}$, $\sec\theta = \frac{Hyp}{Base}$

$\tan\theta = \frac{Perp}{Base}$, $\cot\theta = \frac{Base}{Perp}$

→ $\sin^2\theta + \cos^2\theta = 1$
→ $\sec^2\theta - \tan^2\theta = 1$
→ $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

→ $\sin(n\pi + \theta) = (-1)^n \sin\theta$
→ $\cos(n\pi + \theta) = (-1)^n \cos\theta$
→ $\tan(n\pi + \theta) = \tan\theta$
→ $\sin(\frac{n}{2}\pi + \theta) = (-1)^{\frac{n-1}{2}} \cos\theta$
→ $\cos(\frac{n\pi}{2} + \theta) = (-1)^{\frac{n+1}{2}} \sin\theta$

→ $\tan(\frac{n\pi}{2} + \theta) = -\cot\theta$

→ $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

→ $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

→ $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

→ $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

→ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

→ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\longrightarrow$ $\cot(A+B) = \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$

$\longrightarrow$ $\cot(A-B) = \dfrac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$

$\longrightarrow$ $\sin^2 A - \sin^2 B = (\sin A - \sin B)(\sin A + \sin B)$

$\longrightarrow$ $\cos^2 A - \cos^2 B = (\cos A - \cos B)(\cos A + \cos B)$

$\longrightarrow$ $2 \sin A \cdot \cos A = \sin(A+B) + \sin(A-B)$

$\longrightarrow$ $2 \cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$\longrightarrow$ $2 \cos A \cdot \cos B = \cos(A-B) - \cos(A+B)$

$\longrightarrow$ $2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$\longrightarrow$ $\sin 2A =$ (i) $2 \sin A \cdot \cos A$

(ii) $\dfrac{2 \tan A}{1 + \tan^2 A}$

$\longrightarrow$ $\sin A =$ (i) $2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}$, (ii) $\dfrac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$

$\longrightarrow$ $\cos 2A =$ (i) $\cos^2 A - \sin^2 A$

(ii) $1 - 2 \sin^2 A$

(iii) $2 \cos^2 A - 1$

(iv) $\dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

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Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

→ $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

→ $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

→ $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

→ $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

→ $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

→ $\sin 3A = 3\sin A - 4\sin^3 A$

→ $\cos 3A = 4\cos^3 A - 3\cos A$

$\rightarrow$ $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$\rightarrow$ $(\cos A - \sin A)^2 = 1 - \sin 2A$

$\rightarrow$ $(\cos A + \sin A)^2 = 1 + \sin 2A$

$\rightarrow$ $\tan(\pi/4 + A) = \frac{1 + \tan A}{1 - \tan A}$

$\rightarrow$ $\cos A \cdot \cos(60^\circ + A) \cdot \cos(60 - A) = \frac{1}{4} \cos 3A$

$\rightarrow$ $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$

$\rightarrow$ $\sin A \cdot \sin(60 + A) \cdot \sin(60 - A) = \frac{1}{4} \sin 3A$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, \quad x \geq 0$

Q. If $\tan A + \tan B = p$, $\cot A + \cot B = q$, prove that $\cot(A+B) = \frac{1}{p} - \frac{1}{q}$

Sol: Given that,

$$\tan A + \tan B = p$$
$$\cot A + \cot B = q$$

So,

$$R.H.S = \frac{1}{p} - \frac{1}{q}$$

$$= \frac{1}{\tan A + \tan B} - \frac{1}{\cot A + \cot B}$$

$$= \frac{1}{\frac{1}{\cot A} + \frac{1}{\cot B}} - \frac{1}{\cot A + \cot B}$$

$$= \frac{1}{\frac{\cot B + \cot A}{\cot A \cdot \cot B}} - \frac{1}{\cot A + \cot B}$$

$$= \frac{\cot A \cdot \cot B}{\cot A + \cot B} - \frac{1}{\cot A + \cot B}$$

$$= \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$$

$$= \cot(A+B)$$

$\therefore$ R.H.S = L.H.S

So, $\cot(A+B) = \frac{1}{p} - \frac{1}{q}$ proved.

Q. If $\sin\theta + \operatorname{cosec}\theta = 2$, show that $\sin^n\theta + \operatorname{cosec}^n\theta = 2$ 'n' all positive integers.

Sol:- Given that,

$\therefore \sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \frac{\sin^2\theta + 1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta + 1 - 2\sin\theta = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0$

$\Rightarrow \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1$

from questions,

$\because \sin^n\theta + \operatorname{cosec}^n\theta = 2$

$\Rightarrow (1)^n + \cancel{\operatorname{cosec}}(1)^n = 2$

$\Rightarrow 1^1 + 1^1 = 2$

$\Rightarrow 1 + 1 = 2$

$\Rightarrow 2 = 2$ proved.

Q. Find the value of $\frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$

Sol:- Given that,

$$\because \frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$$

divided by $\cos 15^\circ$

$$= \frac{\frac{\cancel{\cos 15^\circ}}{\cancel{\cos 15^\circ}} + \frac{\sin 15^\circ}{\cos 15^\circ}}{\frac{\cancel{\cos 15^\circ}}{\cancel{\cos 15^\circ}} - \frac{\sin 15^\circ}{\cos 15^\circ}}$$

$$= \frac{1 + \tan 15^\circ}{1 - \tan 15^\circ}$$

$$= \frac{\tan 45^\circ + \tan 15^\circ}{1 - \tan 15^\circ \cdot \tan 45^\circ} \quad [\because \tan 45^\circ = 1]$$

$$= \tan(45^\circ + 15^\circ)$$

$$= \tan 60^\circ$$

$$= \sqrt{3} \quad \text{Ans.}$$

Q. If $A + B = 45^\circ$, prove that

(i) $(1 + \tan A)(1 + \tan B) = 2$

Sol:- Given that,

$$A + B = 45^\circ$$

$$\Rightarrow \tan(A+B) = \tan 45^\circ$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \cdot \tan B$$

Adding '1' on to both sides.

$\Rightarrow 1 + \tan A + \tan B = 1 + 1 - \tan A \cdot \tan B$

$\Rightarrow 1 + \tan A + \tan B + \tan A \cdot \tan B = 2$

$\Rightarrow 1(1 + \tan A) + \tan B(1 + \tan A) = 2$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

proved.

Find $\tan 22\frac{1}{2}^\circ$

Sol:- ~~Let $\tan 22\frac{1}{2}^\circ$~~ Let $A = B = 22\frac{1}{2}^\circ$

Then,

$\because (1 + \tan A)(1 + \tan B) = 2$

$\Rightarrow \left(1 + \tan 22\frac{1}{2}^\circ\right)\left(1 + \tan 22\frac{1}{2}^\circ\right) = 2$

$\Rightarrow \left(1 + \tan 22\frac{1}{2}^\circ\right)^2 = 2$

$\Rightarrow 1 + \tan 22\frac{1}{2}^\circ = \sqrt{2}$

$\Rightarrow \tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$ Ans.

(ii) $(\cot A - 1)(\cot B - 1) = 2$

Sol:- Given that,

$A + B = 45^\circ$

$\because \cot(A + B) = \cot 45^\circ$

$\Rightarrow \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow \cot A \cdot \cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A \cdot \cot B = \cot A + \cot B + 1$

Adding '1' on both sides.

$\because \cot A \cdot \cot B + 1 = \cot A + \cot B + 1 + 1$

$\Rightarrow \cot A \cdot \cot B + 1 - \cot A - \cot B = 2$

$\Rightarrow \cot A(\cot B - 1) - 1(\cot B - 1) = 2$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

proved

1. $\sin(A+B) + \sin(A-B) = 2\sin A \cdot \cos B$
2. $\sin(A+B) - \sin(A-B) = 2\cos A \cdot \sin B$
3. $\cos(A+B) + \cos(A-B) = 2\cancel{\cos A} \cdot \cancel{\cos B}\ \cancel{\sin A \cdot \sin B}$
4. $\cos(A+B) - \cos(A-B) = 2\sin A \cdot \sin B$

5. $\cos C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

6. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{D+C}{2}\right)$

7. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

8. $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

9. $\sin 2A =$ (a) $2\sin A \cdot \cos A$ (b) $\frac{2\tan A}{1-\tan^2 A}$

Q. Prove that, $\cot\frac{\pi}{8} - \tan\frac{\pi}{8} = 2$

Sol:- L.H.S:

$$\cot\frac{\pi}{8} - \tan\frac{\pi}{8}$$

$$= \frac{\cos\frac{\pi}{8}}{\sin\pi/8} - \frac{\sin\pi/8}{\cos\pi/8}$$

$$= \frac{\cos^2\pi/8 - \sin^2\pi/8}{\sin\pi/8 \cdot \cos\pi/8}$$

$$= \frac{\cos 2 \times \pi/8}{\cos \pi/8 \cdot \cancel{\sec} \pi/8}$$ $[\because \cos 2A = \cos^2 A - \sin^2 A]$

multiply '$\frac{2}{2}$' by,

$$= \frac{\cos 2 \times \pi/8}{\frac{1}{2} \times 2 (\cos \pi/8 \cdot \cos \pi/8)}$$

$$= \frac{\cos \cancel{2} \times \pi/\cancel{8}\, 4}{\frac{1}{2} \times \sin \cancel{2} \times \pi/\cancel{8}\, 4}$$

$$= \frac{\cos \pi/4}{\frac{1}{2} \times \sin \pi/4}$$

$$= \frac{\cancel{1/\sqrt{2}}}{\frac{1}{2} \times \cancel{\frac{1}{\sqrt{2}}}} = \frac{1}{1} \times 2 = 2$$

$\therefore$ L.H.S = R.H.S proved.

Q. $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$

Sol:- R.H.S

$\because \tan 54^\circ$

$= \tan(45^\circ + 9^\circ)$

$$= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \cdot \tan 9^\circ}$$

$$= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} = \frac{1 + \frac{\sin 9^\circ}{\cos 9^\circ}}{1 - \frac{\sin 9^\circ}{\cos 9^\circ}} = \frac{\frac{\cos 9^\circ + \sin 9^\circ}{\cancel{\cos 9^\circ}}}{\frac{\cos 9^\circ - \sin 9^\circ}{\cancel{\cos 9^\circ}}} = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

$\therefore$ R.H.S = L.H.S

$n=1$

1. $\sin(n\pi+\theta) = (-1)^n \sin\theta \Rightarrow \sin(\pi+\theta) = (-1)^1 \sin\theta = -\sin\theta$
2. $\cos(n\pi+\theta) = (-1)^2 \cos\theta$
3. $\tan(n\pi+\theta) = \tan\theta$

$\tan(720+\theta)$

$= \tan(4\pi+\theta) = \tan\theta$

4. $\sin(\frac{n\pi}{2}+\theta) = (-1)^{\frac{n-1}{2}} \cos\theta$,

when 'n' is odd number.

Eg:- Let $n=1,3$

$\sin(\frac{\pi}{2}+\theta) = (-1)^{\frac{1-1}{2}} \cos\theta$

$= (-1)^{0/2} \cos\theta$

$= (-1)^0 \cos\theta$

$= 1 \cdot \cos\theta$

$= \cos\theta$

$\sin(\frac{3\pi}{2}+\theta) = (-1)^{\frac{3-1}{2}} \cos\theta$

$= (-1)^{2/2} \cos\theta$

$= (-1)^1 \cos\theta$

$= -\cos\theta$

5. $\cos(\frac{n\pi}{2}+\theta) = (-1)^{\frac{n+1}{2}} \sin\theta$

Eg:- Let $n=1,5$

$\because \cos(\frac{\pi}{2}+\theta) = (-1)^{\frac{1+1}{2}} \sin\theta$

$= (-1)^{2/2} \sin\theta$

$= (-1)^1 \sin\theta$

$= -1 \cdot \sin\theta$

$= -\sin\theta$

$\because \cos(\frac{5\pi}{2}+\theta) = (-1)^{\frac{5+1}{2}} \sin\theta$

$= (-1)^{6/2} \sin\theta$

$= (-1)^3 \sin\theta$

$= -1 \cdot \sin\theta$

$= -\sin\theta$

6. $\tan(\frac{n\pi}{2}+\theta) = -\cot\theta$

Eg:- Let $n=1$

$\tan(\frac{\pi}{2}+\theta) = -\cot\theta$

Q: $\sin(1185^\circ)$

Sol:- $1185^\circ = 90^\circ \times 13 + 15^\circ$

$\sin\left(\frac{13\pi}{2} + 15^\circ\right) = (-1)^{\frac{13-1}{2}} \sin\theta$

$= (-1)^{12/2} \sin 15^\circ$

$= (-1)^6 \sin 15^\circ$

$= 1 \cdot \sin 15^\circ$

$= \sin 15^\circ$

Q. $\cos 500^\circ$

Sol:- $500^\circ = 90^\circ \times 5 + 50^\circ$

$\cos\left(\frac{5\pi}{2} + 50^\circ\right) = (-1)^{\frac{5+1}{2}} \sin 50^\circ$

$= (-1)^{6/2} \sin 50^\circ$

$= (-1)^3 \sin 50^\circ$

$= -\sin 50^\circ$

Q: $\sec 300^\circ + 80^\circ$

Sol:- $380^\circ = 90^\circ \times 4 + 20^\circ$

$\sec\left(\frac{\cancel{4}^{2}\pi}{\cancel{2}} + 20^\circ\right) = \sec(2\pi + 20^\circ)$

$= \sec 20^\circ$

1. $\sin^2\alpha + \cos^2\alpha = 1$
2. $1 + \tan^2\alpha = \sec^2\alpha$
3. $1 + \cot^2\alpha = \operatorname{cosec}^2\alpha$

Q: Prove the following,

$$\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}}$$

Sol:- R.H.S

$$\therefore \sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\Rightarrow \cos \cancel{2} \times \frac{A}{\cancel{2}} = 2\cos^2\left(\frac{A}{2}\right) - 1 \quad [\because \cos 2A = 2\cos^2 A - 1]$$

$$\Rightarrow \cos A = 2\cos^2 A/_2 - 1$$

$$\Rightarrow \cos A + 1 = 2\cos^2 A/_2$$

$$\Rightarrow 1 + \cos A = 2\cos^2 A/_2$$

$$\because \cos 2(A/_2) = 1 - 2\sin^2(A/_2)$$

$$\Rightarrow \cos A = 1 - 2\sin^2(A/_2)$$

$$\Rightarrow \cos A - 1 = -2\sin^2(A/_2)$$

$$\Rightarrow -(1-\cos A) = -(2\sin^2(A/_2))$$

$$\Rightarrow 1 - \cos A = 2\sin^2 A/_2$$

from question.

$$\therefore \sqrt{\frac{1+\cos A}{1+\cos A}} = \sqrt{\frac{\cancel{2}\sin^2 A}{\cancel{2}\cos^2 A}} = \frac{\sin A}{\cos A} = \tan A$$

so, R.H.S = L.H.S

Q. If $A+B+C = \pi$ and $\cos A = \cos B \cdot \cos C$, show that $\tan B + \tan C = \tan A$

Sol:- Given that,

$$A+B+C = \pi$$

$$\cos A = \cos B \cdot \cos C$$

L.H.S

$\because \tan B + \tan C$

$= \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}$

$= \frac{\sin B \cdot \cos C + \sin C \cdot \cos B}{\cos B \cdot \cos C}$

$= \frac{\sin(B+C)}{\cos B \cdot \cos C}$ $[\because \sin(B+C) = \sin B \cdot \cos C + \cos B \cdot \sin C]$

Then,

$A+B+C=\pi$

$\Rightarrow B+C = \pi - A$

$\because \frac{\sin(\pi - A)}{\cos A}$ $[\because \cos A = \cos B \cdot \cos C]$

$= \frac{\sin A}{\cos A}$

$= \tan A$

So, L.H.S = R.H.S proved.

Q. If $A+B+C=\pi$ and $\cos B \cdot \cos C = \cos A$, then show that $2 \cot B \cdot \cot C = 1$

Sol:- Given that,

$A+B+C=\pi$

$\cos B \cdot \cos C = \cos A$

$\therefore A+B+C=\pi$

$\Rightarrow A = \pi - (B+C)$

$\Rightarrow \cos A = \cos(\pi - (B+C))$

$\Rightarrow \cos B \cdot \cos C = -\cos(B+C)$

$\Rightarrow \cos B \cdot \cos C = -(\cos B \cdot \cos C - \sin B \cdot \sin C)$

$\Rightarrow \cos B \cdot \cos C = -\cos B \cdot \cos C + \sin B \cdot \sin C$

$\Rightarrow \cos B \cdot \cos C + \cos B \cdot \cos C = \sin B \cdot \sin C$

$\Rightarrow 2\cos B \cdot \cos C = 1(\sin B \cdot \sin C)$

$\Rightarrow \dfrac{2\cos B \cdot \cos C}{\sin B \cdot \sin C} = 1$

$\Rightarrow 2\cot B \cdot \cot C = 1$

proved

Q. $\cos(120°+A) \cdot \cos(120°-A) + \cos(120°+A) \cdot \cos A + \cos A + \cos(120°-A) + 3/4 = 0$

Sol:- L.H.S

$\cos(120°+A) \cdot \cos(120°-A) + \cos(120°+A) \cdot \cos A + \cos A + \cos(120°-A) + 3/4$

$= \cos^2 120 - \sin^2 A + \cos A[\cos(120°+A) + \cos(120°-A)] + 3/4$

$= (\cos 120°)^2 - (\sin A)^2 + \cos A[\cos(120°+A) + \cos(120°-A)] + 3/4$

$= \left(-\frac{1}{2}\right)^2 - (\sin A)^2 + \cos A[2\cos 120° \cdot \cos A] + \frac{3}{4}$ $\left[\because 2\cos\theta \cdot \cos\alpha = \cos(\theta+\alpha) + \cos(\theta-\alpha)\right]$

$= \frac{1}{4} - \sin^2 A + \cos A[\cancel{2} \times -\frac{1}{\cancel{2}} \cdot \cos A] + \frac{3}{4}$

$= \frac{1}{4} - \sin^2 A + -\cos^2 A + \frac{3}{4}$

$= \frac{1}{4} - (\sin^2 A + \cos^2 A) + \frac{3}{4}$

$= \frac{1}{4} - 1 + \frac{3}{4} = \frac{1-4+3}{4} = \frac{4-4}{4} = \frac{0}{4} = 0$

$\therefore$ L.H.S = R.H.S proved.

α – Alfa / Alpha	θ – theta	κ – Kappa
β – beta	ϕ – phi	μ – mu
γ – gamma	ψ – psi	ν – nu
δ – delta	ξ – xi	π – pi
ε – epsilon	η – eta	ρ – rho
ι – iota	ζ – zeta	σ – sigma
	λ – lamda	

τ – tau

χ – chi

ω – omega

Γ – ten gamma

Δ – delta

~~4~~ Σ – sigma

Illustrative Examples

1. ~~Exo~~ Prove the followings

(i) $\cot\frac{\pi}{8} - \tan\frac{\pi}{8} = 2$

Sol:- L.H.S

$$\cot\frac{\pi}{8} - \tan\frac{\pi}{8}$$

$$= \frac{\cos\frac{\pi}{8}}{\sin\pi/8} - \frac{\sin\frac{\pi}{8}}{\cos\frac{\pi}{8}}$$

$$= \frac{\cos^2\pi/8 - \sin^2\pi/8}{\sin\pi/8 \cdot \cos\pi/8}$$

$$= \frac{\cos^2\pi/8 - \sin^2\pi/8}{\frac{1}{2}\cdot 2\sin\pi/8 \cdot \cos\pi/8} \qquad [\because \sin 2\theta = 2\sin\theta\cdot\cos\theta]$$

$$= \frac{\cos^2 \pi/8 - \sin^2 \pi/8}{\frac{1}{2}\sin 2\times\pi/8} \qquad [\because \cos^2\theta - \sin^2\theta = \cos 2\theta]$$

$$= \frac{\cos 2\times\pi/8}{\frac{1}{2}\sin 2\times\pi/8} = \frac{\cos\pi/4}{\frac{1}{2}.\sin\pi/4} = \frac{\cancel{1/\sqrt{2}}}{\frac{1}{2}.\cancel{1/\sqrt{2}}}$$

$$= \frac{1}{1}\times 2$$

$= 2$ R.H.S (proved)

(ii) $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$

Sol:-

R.H.S $= \tan 54^\circ$

$= \tan(45^\circ + 9^\circ)$

$$= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ . \tan 9^\circ}$$

$$= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ}$$

$$= \frac{1 + \frac{\sin 9^\circ}{\cos 9^\circ}}{1 - \frac{\sin 9^\circ}{\cos 9^\circ}} = \frac{\frac{\cos 9^\circ + \sin 9^\circ}{\cancel{\cos 9^\circ}}}{\frac{\cos 9^\circ - \sin 9^\circ}{\cancel{\cos 9^\circ}}} = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

$=$ L.H.S (proved)

(iii) $\tan 10^\circ + \tan 35^\circ + \tan 10^\circ \cdot \tan 35^\circ = 1$

Sol:- We know that, $\tan 45^\circ = 1$

$\cancel{\tan} = \cancel{45^\circ}$

$\therefore \tan(10 + 35^\circ) = 1$

$\Rightarrow \dfrac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \cdot \tan 35^\circ} = 1$

$\Rightarrow \tan 10^\circ + \tan 35^\circ = 1 - \tan 10^\circ \cdot \tan 35^\circ$

$\Rightarrow \tan 10^\circ + \tan 35^\circ + \tan 10^\circ \cdot \tan 35^\circ = 1$

$\therefore$ L.H.S = R.H.S (proved)

2. Prove the followings:

(i) $\tan \frac{A}{2} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Sol:- R.H.S $= \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

$\left[\because \cos 2A = 1 - 2\sin^2 A\right]$
$\left[\because \cos 2A = 2\cos^2 A - 1\right]$

$= \sqrt{\dfrac{\cancel{2}\sin^2 A/2}{\cancel{2}\cos^2 A/2}}$

$= \sqrt{\dfrac{\sin^2 A/2}{\cos^2 A/2}} = \dfrac{\sin A/2}{\cos A/2} = \tan A/2$

L.H.S (proved)

(iii) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \tan\left(\dfrac{\pi}{4}+\dfrac{A}{2}\right)$

Sol:- $L.H.S = \sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{\sin^2\frac{A}{2}+\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}{\sin^2\frac{A}{2}+\cos^2\frac{A}{2}-2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}}$$

$$= \sqrt{\frac{\left(\sin\frac{A}{2}+\cos A/2\right)^2}{\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)^2}}$$

$$= \frac{\sin\frac{A}{2}+\cos\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}$$

divided by '$\cos\frac{A}{2}$' numerator and denominator.

$$= \frac{\frac{\sin A/2}{\cos A/2}+\frac{\cos A/2}{\cos A/2}}{\frac{\cos A/2}{\cos A/2}-\frac{\sin A/2}{\cos A/2}}$$

$$= \frac{\tan A/2+1}{1-\tan A/2} = \frac{1+\tan A/2}{1-\tan A/2}$$

$$= \frac{\tan\frac{\pi}{4}+\tan\frac{A}{2}}{1-\tan\frac{\pi}{4}\cdot\tan A/2} \quad \left[\because \tan\frac{\pi}{4} = \tan 45 = 1\right]$$

$$= \tan\left(\frac{\pi}{4}-\frac{A}{2}\right)$$

R.H.S (proved)

3. Find the maximum and minimum values of the of the ~~value~~ followings.

(i) $5\sin x + 12\cos x$

Sol:- Let $5 = r\cos\theta$ and $12 = r\sin\theta$

Both sides are square applied.

$$r^2\cos^2\theta = 25$$
$$r^2\sin^2\theta = 144$$

Adding L.H.S to L.H.S and R.H.S to R.H.S

$$\Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 25 + 144$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 169$$
$$\Rightarrow r^2 \times 1 = 169$$
$$\Rightarrow r^2 = 169$$
$$\Rightarrow r = \sqrt{169} = \pm 13$$

$\therefore$ ~~$5\sin x + 12\cos x$~~

$\Rightarrow$ = maximum value $= 13$ and minimum value $= -13$

(ii) $8\cos x - 15\sin x - 2$

Sol:-

4.(i) $\cos^6 A - \sin^6 A = 2\cos A \left(1 - \frac{1}{4}\sin^2 2A\right)$

Sol:- $L.H.S = \cos^6 A - \sin^6 A$

$= (\cos^2 A)^3 - (\sin^2 A)^3$

$[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

$= (\cos^2 A - \sin^2 A)(\cos^4 A + \cos^2 A \cdot \sin^2 A + \sin^4 A)$

$= \cos 2A \{(\cos^2 A)^2 + (\sin^2 A)^2 + \cos^2 A \cdot \sin^2 A\}$ $[\because \cos 2A = \cos^2 A - \sin^2 A]$

$= \cos 2A \{(\cos^2 A + \sin^2 A)^2 - 2\cos^2 A \cdot \sin^2 A + \cos^2 A \cdot \sin^2 A\}$

$= \cos 2A \{1 - \cos^2 A \cdot \sin^2 A\}$

$= \cos 2A \left\{1 - \frac{(\sin 2A)^2}{4}\right\}$ $[\because \sin^2 2A = 4\sin^2 A \cdot \cos^2 A]$

$= \cos 2A \left\{1 - \frac{4\sin^2 A \cdot \cos^2 A}{4}\right\}$

$= \cos 2A \left(1 - \frac{1}{4}\sin^2 2A\right)$

$=$ R.H.S (proved.)

(ii) $\sin^4\theta = \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$.

Sol:- $L.H.S = \sin^4\theta = (\sin^2\theta)^2$

$$= \left(\frac{1-\cos 2\theta}{2}\right)^2 \qquad [\because \cos 2\theta = 1-2\sin^2\theta]$$

$$= \cancel{1-\cos^2 2\theta +}$$

$$= \frac{1+\cos^2 2\theta - 2\cos 2\theta}{4}$$

$$= \frac{1}{4}(1+\cos^2 2\theta - 2\cos 2\theta)$$

$$= \frac{1}{4} + \frac{1}{4}\cos^2 2\theta - \frac{1}{2}\cos 2\theta$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2\theta + \frac{1}{4}\left(\frac{1-\cos 4\theta}{2}\right) \qquad [\because \cos 2\theta = 2\cos^2\theta - 1]$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2\theta + \frac{1}{8} - \frac{1}{8}\cos 4\theta$$

$$= \frac{3}{8} - \frac{1}{2}\cos 2\theta - \frac{1}{8}\cos 4\theta = \text{R.H.S (proved)}.$$

5. Prove that

$$\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} = \cot 2A$$

Sol :- $\text{L.H.S} = \dfrac{1}{\tan 3A - \tan A} - \dfrac{1}{\cot 3A - \cot A}$

$$= \frac{1}{\frac{1}{\cot 3A} - \frac{1}{\cot A}} - \frac{1}{\cot 3A - \cot A}$$

$$= \frac{1}{\frac{\cot A - \cot 3A}{\cot 2A \cdot \cot A}} + \frac{1}{\cot A - \cot 3A}$$

$$= \frac{\cot 3A \cdot \cot A}{\cot A - \cot 3A} + \frac{1}{\cot A - \cot 3A}$$

$$= \frac{\cot 3A \cdot \cot A - 1}{\cot A - \cot 3A}$$

$$= \cot(3A - A)$$

$$= \cot 2A \quad = \text{R.H.S (proved)}$$

6. Proved that,

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$$

Sol:- $\text{L.H.S} = \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \left\{\cos\left(\pi - \frac{3\pi}{8}\right)\right\}^4 + \left\{\cos\left(\pi - \frac{\pi}{8}\right)\right\}^4$$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{\pi}{8}$$

$$= 2\left\{\left(\cos^2\frac{\pi}{8}\right)^2 + \left(\cos^2\frac{3\pi}{8}\right)^2\right\}$$

$$= 2\left\{\left(\frac{1 + \cos 2 \cdot \frac{\pi}{8}}{2}\right)^2 + \left(\frac{1 + \cos 2 \cdot \frac{3\pi}{8}}{2}\right)^2\right\}$$

$$= 2\left\{\left(\frac{1 + \cos 45^\circ}{2}\right)^2 + \left(\frac{1 + \cos\frac{3\pi}{4}}{2}\right)^2\right\}$$

$$= 2\left\{\left(\frac{1 + 1/\sqrt{2}}{2}\right)^2 + \left(\frac{1 - 1/\sqrt{2}}{2}\right)^2\right\}$$

$$= 2\left\{\frac{1 + \frac{1}{2} + 2\frac{1}{\sqrt{2}}}{4} + \frac{1 + \frac{1}{2} - 2\times\frac{1}{\sqrt{2}}}{4}\right\}$$

$$= \cancel{2}\left[\frac{1 + \frac{1}{2} + \frac{\cancel{2}^{\sqrt{2}}}{\cancel{\sqrt{2}}} + 1 + \frac{1}{2} - \frac{\cancel{2}^{\sqrt{2}}}{\cancel{\sqrt{2}}}}{\cancel{4}\, 2}\right]$$

$$= \cancel{\tfrac{3}{2}\times} \frac{3 + \cancel{\sqrt{2}} - \cancel{\sqrt{2}}}{2}$$

$$= \frac{3}{2} \quad \text{R.H.S (proved)}$$

4. Prove that,

a. $\cot 7\frac{1}{2}^\circ = \sqrt{6}+\sqrt{3}+\sqrt{2}+2$

Sol:- Given that,

$L.H.S = \cot 7\frac{1}{2}^\circ$

$= \cot \frac{15^\circ}{2}$

$= \frac{1+\cos 15^\circ}{\sin 15^\circ}$ $\left[\because \cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}\right]$

Then,

$\cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \cancel{\cos 30} \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\because \cancel{co}\sin 15^\circ$

$= \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

from questions,

$$\frac{1+\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2}+\sqrt{3}+1}{\cancel{2\sqrt{2}}} \times \frac{\cancel{2\sqrt{2}}}{\sqrt{3}-1}$$

$$= \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+3+\sqrt{3}+2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3})^2-1}$$

$$= \frac{2\sqrt{6}+2\sqrt{3}+2\sqrt{2}+4}{3-1}$$

$$= \frac{\cancel{2}(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)}{\cancel{2}}$$

$$= \sqrt{6}+\sqrt{3}+\sqrt{2}+2$$ R.H.S (proved)

(b) $\tan 37\frac{1}{2}^\circ = \sqrt{6}+\sqrt{3}-\sqrt{2}-2$

Sol:- Given that,

$$\tan 37\frac{1}{2}^\circ$$

$$\because \tan\frac{75^\circ}{2}$$

$$\because \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

$$\tan\frac{75^\circ}{2} = \frac{1-\cos 75^\circ}{\sin 75^\circ}$$

$$\cos 75^\circ = \cos(45^\circ+30^\circ)$$
$$= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\sin 75^\circ = \sin(45^\circ+30^\circ)$$
$$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

from question,

$$= \frac{1-\cos 75^\circ}{\sin 75^\circ}$$
$$= \frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$
$$= \frac{\frac{2\sqrt{2}-\sqrt{3}-1}{\cancel{2\sqrt{2}}}}{\frac{\sqrt{3}+1}{\cancel{2\sqrt{2}}}}$$
$$= \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(2\sqrt{2}-\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{2\sqrt{6}-3+\sqrt{3}-2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{2\sqrt{6}+2\sqrt{3}-2\sqrt{2}-2}{3-1}$$

$$= \frac{\cancel{2}(\sqrt{6}+\sqrt{3}-\sqrt{2}-2)}{\cancel{2}}$$

$= \sqrt{6}+\sqrt{3}-\sqrt{2}-2$ R.H.S (proved).

c. $2\cos\frac{\pi}{16} = \sqrt{2+\sqrt{2+\sqrt{2}}}$

Solⁿ:- Given that,

$$2\cos\frac{\pi}{16}$$

$$= \cos\frac{\pi}{8}$$

we know $1+\cos\theta = 2\cos^2\frac{\theta}{2}$

Put $\theta = \frac{\pi}{4}$,

$$\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}$$

$$= 1+\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8}$$

C. Sol^n:- L.H.S

$$2\cos\frac{\pi}{16}$$

$$= \sqrt{\left(2\cos\frac{\pi}{16}\right)^2} = \sqrt{A}$$

$$= \sqrt{4\left(\cos^2\frac{\pi}{16}\right)} \qquad \left[\because 1+\cos\theta = 2\cos^2\frac{\pi}{\theta}\right]$$

$$= \sqrt{2(2\cos^2\pi/16)}$$

$$= \sqrt{2(1+\cos^2\pi/8)}$$

$$= \sqrt{2+2\cos^2\pi/8}$$

$$= \sqrt{2+\sqrt{(2\cos\pi/8)^2}}$$

$$= \sqrt{2+\sqrt{4\cos^2\pi/8}}$$

$$= \sqrt{2+\sqrt{2(2\cos^2\pi/8)}}$$

$$= \sqrt{2+\sqrt{2(1+\cos^2\pi/8}}$$

$$= \sqrt{2+\sqrt{2+2\cos^2\pi/4}} = \sqrt{2+\sqrt{2+\sqrt{(2\cos^2\pi/4)^2}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{4\cos^2\pi/4}}}$$

Q. Find the value of tan 75° and prove that $\tan 75° + \cot 75° = 4$.

Sol^n:- Given that,

$$\tan 75°$$

$$\tan(45°+30°)$$

$$\because \frac{\tan 45° + \tan 30°}{1-\tan 45°\cdot\tan 30°}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-1\cdot\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

~~Then~~ If, $\tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ and then, $\cot 75° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

L.H.S = $\tan 75^\circ + \cot 75^\circ$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3})^2 + (1)^2 + 2\cdot\sqrt{3}\cdot 1 + (\sqrt{3})^2 + (1)^2 - 2\cdot\sqrt{3}\cdot 1}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 1 + \cancel{2\sqrt{3}} + 3 + 1 - \cancel{2\sqrt{3}}}{3-1}$$

$$= \frac{\cancel{8}\,4}{\cancel{2}} = 4$$ R.H.S (proved).

Q. find the value of sin 75°.

Solⁿ:- $\sin(75^\circ)$

$= \sin(45^\circ + 30^\circ)$ [$\because \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$]

$= \sin 45^\circ \cdot \cos 30^\circ + \cancel{\cos 30^\circ \sin 45} \cos 45^\circ \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$ Ans.

(ii) $\cot 75^\circ$

Solⁿ:- $\cot 75^\circ$

$= \cot(45^\circ + 30^\circ)$ [$\because \cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$]

$$= \frac{\cot 45^\circ \cdot \cot 30^\circ - 1}{\cot 30^\circ + \cot 45^\circ} = \frac{1\cdot\sqrt{3} - 1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$ Ans

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(iii) cot 15°

Sol^n:-

$$\cot 15°$$

$$= \cot(45° - 30°)$$

$$= \frac{\cot 45° \cdot \cot 30° + 1}{\cot 30° - \cot 45°}$$

$$= \frac{1 \cdot \sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$ Ans.

(iv) tan 15°

Sol^n:- tan 15°

$$= \tan(45° - 30°)$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \cdot \tan 30°}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3} + 1}{\cancel{\sqrt{3}}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$ Ans.

Q. If $\tan A = 5/6$ and $\tan B = \frac{1}{11}$, show that $A + B = \frac{\pi}{4}$.

Sol^n:- Given that,

$$\tan A = 5/6, \quad \tan B = \frac{1}{11}$$

$$\because A + B = \frac{\pi}{4}$$

$$\Rightarrow \tan(A + B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \tan 45°$$

$$\Rightarrow \frac{5/6 + 1/11}{1 - 5/6 \cdot 1/11} = 1$$

$$\Rightarrow 66$$

$$= \frac{\frac{55+6}{66}}{\frac{66-5}{66}} = \frac{\cancel{61}}{\cancel{66}} \times \frac{\cancel{66}}{\cancel{61}} = 1$$

$\Rightarrow 1 = 1$ R.H.S (proved).

$\Rightarrow$

Q. If $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$, then find the value of $(\alpha+\beta)$.

Solⁿ:- Given that,

$$\tan\alpha = \frac{1}{2}, \quad \tan\beta = \frac{1}{3}$$

$\because (\alpha+\beta)$

$= \tan(\alpha+\beta)$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{3+2}{6}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{6-1}{6}} = \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{5}} = 1$$

So, $\tan(\alpha+\beta) = 1$

$= \tan(\alpha+\beta) = \tan 45^\circ$

$= \tan(\alpha+\beta) = \tan\frac{\pi}{4}$

$\therefore \alpha+\beta = \frac{\pi}{4}$ Ans.

Q. If $\sin\alpha = \frac{15}{17}$ & $\cos\beta = \frac{12}{13}$, where α and β are acqute angles then, find the value of $\sin(\alpha+\beta)$.

Solⁿ:- Given that,

$$\sin\alpha = \frac{15}{17}, \quad \cos\beta = \frac{12}{13}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$= \sqrt{1-\left(\frac{15}{17}\right)^2}$$

$$= \sqrt{1-\frac{225}{289}}$$

$$= \sqrt{\frac{289-225}{289}}$$

$$= \sqrt{\frac{64}{289}} = \frac{8}{17}$$

$$\sin\beta = \sqrt{1-\cos^2\beta}$$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$= \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

So, $\sin(\alpha+\beta)$

$$= \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

$$= \frac{15}{17}\cdot\frac{12}{13} + \frac{8}{17}\cdot\frac{5}{13}$$

$$= \frac{180}{221} + \frac{40}{221} = \frac{220}{221} \quad \text{Ans.}$$

Q. Find the value of $\sin 105^\circ \cdot \cos 105^\circ$.

Sol:- $\sin 105^\circ \times \cos 105^\circ$

$$= \frac{1}{2}\cdot 2\sin 105^\circ\cdot\cos 105^\circ$$

$= \frac{1}{2} \cdot \sin 2(105^\circ)$

$= \frac{1}{2} \cdot \sin 210^\circ$

$= \frac{1}{2} \cdot \sin(180^\circ + 30^\circ)$

$= \frac{1}{2} - \sin 30^\circ$

$= \frac{1}{2} \times -\frac{1}{2} \quad = \frac{-1}{4}$ Ans.

Q. find the value of $\frac{\sqrt{3} - \tan 15^\circ}{1 + \sqrt{3} \cdot \tan 15^\circ}$.

Solⁿ:- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$= \frac{\tan 60^\circ - \tan 15^\circ}{1 + \tan 60^\circ \cdot \tan 15^\circ}$ $[\because \tan 60^\circ = \sqrt{3}]$

$= \tan(60^\circ - 15^\circ)$

$= \tan 45^\circ$

$= 1$

Q. $\frac{1 + \tan A}{1 - \tan A} = \sqrt{3}$

Solⁿ:- $\Rightarrow \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \cdot \tan A} = \sqrt{3}$

$\Rightarrow \tan(45 + A) = \tan 60^\circ$

$\Rightarrow 45^\circ + A = 60^\circ$

$\Rightarrow A = 60^\circ - 45^\circ$

$\Rightarrow A = 15^\circ$

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Q. $\tan x + \tan y = 5$

$\tan x \cdot \tan y = \frac{1}{2}$

then, find the value of $\cot(x+y)$.

Solⁿ:- $\cot(x+y) = \frac{1}{\tan(x+y)}$

$$= \frac{1}{\frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}}$$

$$= \frac{1}{\frac{5}{1-\frac{1}{2}}}$$

$$= \frac{1}{\frac{5}{\frac{2-1}{2}}}$$

$$= \frac{1}{\frac{5}{\frac{1}{2}}}$$

$$= \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

Q. Find the value of $\cos^2 22\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$.

Solⁿ:- $\cos^2 \frac{45^\circ}{2} - \sin^2 \frac{45^\circ}{2}$

$= \cos 2\left(\frac{45}{2}\right)$ $\quad$ [$\because \cos 2A = \cos^2 A - \sin^2 A$

$= \cos 45^\circ$

$= \frac{1}{\sqrt{2}}$

Q. find the value of $\cos 15° \cdot \sin 7\frac{1}{2} \cdot \cos 7\frac{1}{2}°$.

Sol:- $\cos 15° \cdot \sin \frac{15}{2}° \cdot \cos \frac{15}{2}°$

$= \cos 15° \cdot \frac{1}{2} \times 2 \cos \frac{15}{2}° \cdot \cos \frac{15}{2}°$

$= \cos 15° \cdot \frac{1}{2} \cdot \sin 2 \left(\frac{15}{2}°\right)$

$= \cos 15° \cdot \frac{1}{2} \sin 15°$

$= \frac{\frac{1}{2}}{2} \cdot \cancel{2} (2 \cos 15° \cdot \sin 15°)$

$= \frac{1}{\cancel{2}} \cdot \cancel{2} \cos 15° \cdot \sin 15°$

$= 2 \cos 15° \cdot \sin 15°$

$= \sin 2 (15°)$

$= \sin 30°$

$= 1/2$ Ans.

Q. find the minimum value of $\sin \theta \cdot \cos \theta = ?$

EXERCISE – 3(A)

1. Find the value of $\tan^2 45° + \tan^2 30° + \tan^2 60°$

Sol:– Given that,

$$\tan^2 45° + \tan^2 30° + \tan^2 60°$$
$$= (1)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$
$$= 1 + \frac{1}{3} + 3$$
$$= \frac{3+1+9}{3} = \frac{13}{3} \text{ Ans.}$$

2. Simplify

$$\frac{\cos(180+\theta)\cdot\sin(270+\theta)\cdot\tan(180+\theta)}{\cot(270+\theta)\cdot\operatorname{cosec}(180°-\theta)\cdot\sin(-\theta)}$$

Sol:–
$$\frac{\cos(180+\theta)\cdot\sin(270+\theta)\cdot\tan(180+\theta)}{\cot(270+\theta)\cdot\operatorname{cosec}(180°-\theta)\cdot\sin(-\theta)}$$

$$= \frac{-\cos\theta\cdot -\cos\theta\cdot\tan\theta}{-\tan\theta\cdot\operatorname{cosec}\theta\cdot -\sin\theta}$$

$$= \frac{\cos^2\theta\cdot\frac{\sin\theta}{\cos\theta}}{-\tan\cdot -1}$$

$$= \frac{\cos\theta\cdot\sin\theta}{\tan\theta}$$

$$= \frac{\cos\theta\cdot\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \cos\theta\cdot\cancel{\sin\theta}\times\frac{\cos\theta}{\cancel{\sin\theta}}$$

$$= \cos^2\theta \text{ Ans.}$$

3. Prove that

(i) $\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

Sol^n:- R.H.S

$\sin(A+B)\cdot\sin(A-B)$

$= [\sin A\cdot\cos B + \cos A\cdot\sin B][\sin A\cdot\cos B - \cos A\cdot\sin B]$

$= \sin^2 A\cdot\cos^2 B - \cancel{\cos A\cdot\cos B \cdot \cos A\cdot\sin B} + \cancel{\cos A\cdot\sin B\cdot\cos B\cdot\sin A} - \cos^2 A\cdot\sin^2 B$

$= \sin^2 A\cdot\cos^2 B - \cos^2 A\cdot\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\cdot\sin^2 B$

$= \cancel{\sin^2 A\cdot 1 - \sin^2 B - 1 + \sin^2 A\cdot\sin^2 B}$

$= \sin^2 A - \sin^2 A\cdot\sin^2 B - [\sin^2 B - \sin^2 B\cdot\sin^2 A$

$= \sin^2 A - \cancel{\sin^2 A\cdot\sin^2 B} - \sin^2 B + \cancel{\sin^2 B\cdot\sin^2 A}$

$= \sin^2 A - \sin^2 B$

(ii) $\cos^2 A - \sin^2 B = \cos(A+B)\cdot\cos(A-B)$

Sol^n:- R.H.S

$\cos(A+B)\cdot\cos(A-B)$

$= [\cos A\cdot\cos B - \sin A\cdot\sin B][\cos A\cdot\cos B + \sin A\cdot\sin B]$

$= \cos^2 A\cdot\cos^2 B + \cos A\cdot\cos B\cdot\sin A\cdot\sin B - \sin A\cdot\sin B\cdot\cos A\cos B - \sin^2 A\sin^2 B$

$= \cos^2 A\cdot\cos^2 B - \sin^2 A\cdot\sin^2 B$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\cdot\sin^2 B$

$= \cos^2 A - \cancel{\cos^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 B\cdot\cos^2 A}$

$= \cos^2 A - \sin^2 B$

(iii) $\cos A\cdot\cos(60+A)\cdot\cos(60-A) = \frac{1}{4}\cos 3A$

Sol^n:- $\cos A\cdot[\cos 60^\circ\cdot\cos A - \sin 60^\circ\cdot\sin A][\cos 60^\circ\cdot\cos A + \sin 60^\circ\cdot\sin A]$

$= \cos A\cdot[\frac{1}{2}\cdot\cos A - \frac{\sqrt{3}}{2}\cdot\sin A][\frac{1}{2}\cdot\cos A + \frac{\sqrt{3}}{2}\sin A]$

$= \cos A\cdot$

4. Express in terms of acute angles

(i) $\sin 1185°$

Sol^n:- $1185° = 13 \times 90° + 15°$

$= \sin\left(\frac{13\pi}{2} + 15°\right) = (-1)^{\frac{13-1}{2}} \cdot \cos 15°$

#

$= (-1)^{\cancel{12}^{6}/\cancel{2}} \cdot \cos 15°$

$= 1 \cdot \cos 15°$

$= \cos 15°$

(ii) $\operatorname{cosec}(-60°)$

Sol^n:- $-\operatorname{cosec} 60°$

$= -\frac{2}{\sqrt{3}}$

(iii) $\tan 235°$

Sol^n:- $\tan(180° + 55°)$

$= \tan 55°$

(iv) $\tan(-840°)$

Sol^n:- $\tan(0 - 840°)$

$= \cancel{+}\tan 840°$

$= + \tan(4\pi + 120°)$

$= - \tan 120°$

$= \tan(-840)$

$= -\tan 840$

$= -\tan(900 - 60°)$

$= -\tan(5\pi - 60)$

$= -(-\tan 60°)$

$= + \sqrt{3}$ Ans

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5. Prove that

(a.) $\tan 50^\circ = 2\tan 10^\circ + \tan 40^\circ$

Solⁿ:-

$$L.H.S = \tan 50^\circ$$

$$50 = (10+40)$$

$$\Rightarrow \tan 50^\circ = \tan(10^\circ + 40^\circ)$$

$$\Rightarrow \tan 50^\circ = \frac{\tan 10 + \tan 40^\circ}{1 - \tan 10^\circ . \tan 40^\circ}$$

$$\Rightarrow \tan 50^\circ(1 - \tan 10^\circ . \tan 40^\circ) = \tan 10 + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ(1 - \tan 10^\circ . \tan(90 - 50^\circ)) = \tan 10^\circ + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ(1 - \tan 10^\circ . + \cot 50^\circ) = \tan 10^\circ + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ - \tan 10^\circ . \tan 50 . \cot 50^\circ = \tan 10^\circ + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ - \tan 10^\circ . \cancel{\tan 50} \times \frac{1}{\cancel{\tan 50}} = \tan 10^\circ + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ - \tan 10^\circ = \tan 10^\circ + \tan 40^\circ$$

$$\Rightarrow \tan 50^\circ = \tan 10^\circ + \tan 40^\circ + \tan 10^\circ$$

$$\Rightarrow \tan 50^\circ = 2\tan 10^\circ + \tan 40^\circ$$

proved.

b. If $A + B = 45^\circ$, then prove that $(1 + \tan A).(1 + \tan B) = 2$.

Solⁿ:- Given that,

$$A + B = 45^\circ$$

$$\because \tan(A+B) = \tan 45^\circ$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A . \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A . \tan B$$

Adding (1) on both sides.

$\Rightarrow 1 + \tan A + \tan B = 1 + 1 - \tan A \cdot \tan B$

$\Rightarrow 1 + \tan A + \tan B + \tan A \cdot \tan B = 2$

$\Rightarrow 1(1 + \tan A) + \tan B(1 + \tan A) = 2$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

proved.

6. Prove that,

a. $\cos\theta - \sin\theta = \sqrt{2}\,\sin(45° - \theta)$

Soln:-

$$\text{R.H.S} = \sqrt{2}\,\sin(45° - \theta)$$
$$= \sqrt{2}(\sin 45° \cdot \cos\theta - \cos 45° \cdot \sin\theta)$$
$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \cdot \cos\theta - \frac{1}{\sqrt{2}} \cdot \sin\theta\right)$$
$$= \cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}(\cos\theta - \sin\theta)$$
$$= \cos\theta - \sin\theta$$

proved.

b. $\tan 75° - \tan 30° - \tan 75° \cdot \tan 30° = 1$

Soln:-

$$\text{L.H.S} = \tan 75° - \tan 30° - \tan 75° \cdot \tan 30°$$
$$= \tan(45° + 30°) - \tan 30° - \tan(45° + 30°) \cdot \tan 30°$$
$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°} - \tan 30° - \frac{\tan 45° + \tan 30°}{1 - \tan 45 \cdot \tan 30} \times \tan 30°$$
$$= \frac{\tan 45 + \tan 30° - \tan 30°(1 - \tan 30°) - (\tan 45 + \tan 30°)\tan 30°}{1 - \tan 30°}$$
$$= \frac{1 + \cancel{\tan 30°} - \cancel{\tan 30°} + \cancel{\tan^2 30°} - \cancel{\tan 30°} - \cancel{\tan^2 30}}{1 - \tan 30°}$$
$$= \frac{1 - \cancel{\tan 30°}}{1 - \cancel{\tan 30°}} = 1$$

proved.

Q. $\sin 45^\circ \cdot \cos 45^\circ \cdot \tan 45^\circ \cdot \cos 90^\circ = ?$

Sol^n:- $\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 1 \cdot 0$

$= 0$

Q. find the value of $\tan 75^\circ$ and prove that $\tan 75^\circ + \cot 75^\circ = 4$.

Sol^n:- Given that,

$\tan 75^\circ$

$75^\circ = 45^\circ + 30^\circ$

$\because \tan(45^\circ + 30^\circ)$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$

$= \frac{1 + 1/\sqrt{3}}{1 - 1 \cdot 1/\sqrt{3}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

If $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then, $\cot 75^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\because \tan 75^\circ + \cot 75^\circ$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{(\sqrt{3})^2 + 1^2 + \cancel{2\sqrt{3}} + (\sqrt{3})^2 + 1^2 - \cancel{2\sqrt{3}}}{(\sqrt{3})^2 - 1^2}$

$= \frac{3+1+3+1}{2} = \frac{8}{2} = 4$ R.H.S (proved)

Q. $\tan(A+B+C) = ?$

Sol^n:- $\tan((A+B)+C)$

$$= \frac{\tan(A+B) + \tan C}{1 - \tan(A+B)\cdot\tan C}$$

$$= \frac{\frac{\tan A + \tan B}{1 - \tan A\cdot\tan B} + \tan C}{1 - \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B} * \tan C}$$

$$= \frac{\frac{\tan A + \tan B + (1 - \tan A\cdot\tan B)\tan C}{\cancel{1 - \tan A\cdot\tan B}}}{\frac{(1 - \tan A\cdot\tan B - (\tan A + \tan B)\tan C}{\cancel{1 - \tan A\cdot\tan B}}}$$

$$= \frac{\tan A + \tan B + \tan C - \tan A\cdot\tan B\tan C}{1 - \tan A\cdot\tan B - \tan A\cdot\tan C - \tan B\cdot\tan C}$$

$$= \frac{\tan A + \tan B + \tan C - \tan A\cdot\tan B * \tan C}{1 - \tan A\cdot\tan B - \tan B\cdot\tan C - \tan A\cdot\tan C}$$

Q. Prove that,

$$\frac{\sin(A-B)}{\cos A\cdot\cos B} + \frac{\sin(B-C)}{\cos B\cdot\cos C} + \frac{\sin(C-A)}{\cos C\cdot\cos A} = 0$$

Sol^n:- $\frac{\sin(A-B)}{\cos A\cdot\cos B}$

$$= \frac{\sin A\cdot\cos B - \cos A\cdot\sin B}{\cos A\cdot\cos B}$$

$$= \frac{\sin A\cdot\cancel{\cos B}}{\cos A\cdot\cancel{\cos B}} - \frac{\cancel{\cos A}\cdot\sin B}{\cancel{\cos A}\cdot\cos B}$$

$$= \tan A - \tan B \quad \text{------} \quad (1)$$

Similarly,

$$\frac{\sin(A-B)}{\cos A\cdot\cos B} = \tan A - \tan B$$

then, $\frac{\sin(B-C)}{\cos B \cdot \cos C} = \tan B - \tan C$ ——— ②

$\frac{\sin(C-A)}{\cos C \cdot \cos A} = \tan C - \tan A$ ——— ③

we get,

$eq^n(1) + ② + ③$

$= \cancel{\tan A} - \cancel{\tan B} + \cancel{\tan B} - \cancel{\tan C} + \cancel{\tan C} - \cancel{\tan A}$

$= 0$ R.H.S (proved).

Q. If $A+B+C = \pi$ and $\cos A = \cos B \cdot \cos C$, then show that $\tan B + \tan C = \tan A$

Sol^n:- L.H.S $= \tan B + \tan C$

$= \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}$

$= \frac{\sin B \cdot \cos C + \cos B \cdot \sin C}{\cos B \cdot \cos C}$

$= \frac{\sin(B+C)}{\cos B \cdot \cos C}$ $[\because \sin(B+C) = \sin B \cdot \cos C + \cos B \cdot \sin C]$

given that

$\because A+B+C = \pi$

$\Rightarrow B+C = \pi - A$

Puting the value of eq^n ①.

$= \frac{\sin(\pi - A)}{\cos A}$ $[\because \cos B \cdot \cos C = \cos A]$

$= \frac{\sin A}{\cos A}$

$= \tan A$ R.H.S (proved)

Q. Prove that $2\sin 105^\circ \cdot \sin 15^\circ = \frac{1}{2}$

Sol^n:- $L.H.S = 2\sin 105^\circ \cdot \sin 15^\circ$

$= 2\sin(90^\circ + 15^\circ) \cdot \sin 15^\circ$

$= 2\cos 15^\circ \cdot \sin 15^\circ$

$= \sin 2(15^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$ R.H.S (proved)

Q. find the value of $\sin 105^\circ \cdot \cos 105^\circ = ?$

Sol^n:-

$\sin 105^\circ \cdot \cos 15^\circ$

$= 2 \times \frac{1}{2} \sin(90 + 15^\circ) \cdot \cos(15^\circ + 90^\circ)$

$= \frac{1}{2} \times 2\cos 15^\circ \cdot \sin 15^\circ$

$= \frac{1}{2} \sin 2(15^\circ)$

$= \frac{1}{2} \times \sin 30^\circ = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ Ans.

Q. Prove that $\tan A/2 = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

Sol^n:- $R.H.S = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

$= \sqrt{\frac{1 - [\cos^2(\frac{A}{2}) - \sin^2(A/2)]}{1 + \cos^2 A/2 - \sin^2 A/2}}$

$= \sqrt{\frac{\sin^2 A/2 + \cancel{\cos^2 A/2} - \cancel{\cos^2 A/2} + \sin^2 A/2}{\cancel{\sin^2 A/2} + \cos^2 A/2 + \cos^2 A/2 - \cancel{\sin^2 A/2}}}$

$$= \sqrt{\frac{\cancel{2}\sin^2 A/2}{\cancel{2}\cos^2 A/2}} = \frac{\sin A/2}{\cos A/2} = \tan\frac{A}{2} \quad \text{L.H.S (proved)}$$

Q. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \tan\left(\frac{\pi}{4}+\frac{A}{2}\right)$

Sol^n:- $\text{L.H.S} = \sqrt{\frac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{\sin^2 A/2 + \cos^2 A/2 + \cancel{2}\cos A/2 \times \sin A/2}{\sin^2 A/2 + \cos^2 A/2 - \cancel{2}\sin A/2 \cdot \cos A/2}}$$

$$= \sqrt{\frac{(\sin A/2 + \cos A/2)^2}{(\cos A/2 - \sin A/2)^2}} \quad [\because (a\pm b)^2 = a^2+b^2 \pm 2ab]$$

$$= \frac{\sin A/2 + \cos A/2}{\cos A/2 - \sin A/2}$$

Dividing $\cos A/2$ in both numerator and denominator.

$$= \frac{\frac{\sin A/2 + \cos A/2}{\cos A/2}}{\frac{\cos A/2 - \sin A/2}{\cos A/2}}$$

$$= \frac{\frac{\sin A/2}{\cos A/2} + \frac{\cos A/2}{\cos A/2}}{\frac{\cos A/2 - \sin A/2}{\cos A/2}}$$

$$= \frac{\frac{\sin A/2}{\cos A/2} + \frac{\cancel{\cos A/2}}{\cancel{\cos A/2}}}{\frac{\cancel{\cos A/2}}{\cancel{\cos A/2}} - \frac{\sin A/2}{\cos A/2}}$$

$$= \frac{\tan A/2 + 1}{1 - \tan A/2} = \frac{\tan A/2 + \tan\frac{\pi}{4}}{\tan\frac{\pi}{4} - \tan\frac{\pi}{4}\cdot\tan\frac{A}{2}} = \tan\left(\frac{\pi}{4}+\frac{A}{2}\right)$$

R.H.S

Q. If $a = b\cos C$, then which angle is a right angle?

Sol^n:- Given that,

$\because a = b\cos C$

$\Rightarrow a = \cancel{b}\left(\frac{a^2+b^2-c^2}{2a\cancel{b}}\right)$

$\Rightarrow 2a^2 = a^2+b^2-c^2$

$\Rightarrow \cancel{2a^2+b^2-a^2+c^2=0}$

$\Rightarrow 2a^2-b^2-a^2+c^2=0$

$\Rightarrow a^2-b^2+c^2=0$

$\Rightarrow b^2 = a^2+c^2$

So, here, $\angle B = 90°$

i.e. B is the right angle.

Q: In a triangle ABC if $a=18, b=24, c=30$

$\cos B = ?$

Sol^n:- Given that,

$a=18, \quad b=24, \quad c=30$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(18)^2+(30)^2-(24)^2}{2\cdot 18\cdot 30}$$

$$= \frac{324+900-576}{1080}$$

$$= \frac{1224-576}{1080}$$

$$= \frac{648}{1080}$$

Q. If $a\cos B = b\cos A$, then find $\cos B$.

Sol^n:- Given that,

$$\because a\cos B = b\cos A$$
$$\Rightarrow a\cos B + a\cos B = b\cos A + a\cos B$$
$$\Rightarrow 2a\cos B = c \qquad [\because c = b\cos A + a\cos B]$$
$$\Rightarrow \cos B = \frac{c}{2a}$$

Ans.

Q. If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then prove that ABC is an equilateral triangle.

Sol^n:- Given that,

$$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$$

$$= \frac{\frac{b^2+c^2-a^2}{2bc}}{a} = \frac{\frac{a^2+c^2-b^2}{2ac}}{b} = \frac{\frac{a^2+b^2-c^2}{2ab}}{c}$$

$$= \frac{b^2+c^2-a^2}{\cancel{2abc}} = \frac{a^2+c^2-b^2}{\cancel{2abc}} = \frac{a^2+b^2-c^2}{\cancel{2abc}}$$

$$\Rightarrow b^2 \cancel{+c^2} - a^2 = a^2 \cancel{+c^2} - b^2$$
$$\Rightarrow b^2 - a^2 - a^2 + b^2 = 0$$
$$\Rightarrow 2b^2 - 2a^2 = 0$$
$$\Rightarrow 2(b^2 - a^2) = 0$$
$$\Rightarrow b^2 - a^2 = 0$$
$$\Rightarrow b^2 = a^2$$
$$\Rightarrow b = a$$

Similiary,

$$\Rightarrow \cancel{a^2} + c^2 - b^2 = \cancel{a^2} + b^2 - c^2$$
$$\Rightarrow +c^2 - b^2 - b^2 + c^2 = 0$$
$$\Rightarrow 2c^2 - 2b^2 = 0$$
$$\Rightarrow 2(c^2 - b^2) = 0 \quad \Rightarrow c^2 = b^2$$

Therefore, $a = b = c$

So, ABC is an equilateral triangle.

7. Prove that,

(a) $\sin 20° \cdot \sin 40° \cdot \sin 60° \cdot \sin 80° = \frac{3}{16}$

Sol^n:-

$$\sin 20° \cdot \sin 40° \cdot \sin 60° \cdot \sin 80°$$
$$= \sin 20° \cdot \sin(60° - 20°) \cdot \sin 60° \cdot \sin(60° + 20°)$$
$$= \sin 60° \cdot \frac{1}{4} \sin 3(20°)$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{4} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}$$

(b.) $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) = 4 \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\beta+\gamma}{2} \cdot \sin\frac{\gamma+\alpha}{2}$

Sol^n:- L.H.S

$$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$$
$$= (\sin\alpha + \sin\beta) + \sin\gamma - \sin(\alpha+\beta+\gamma)$$
$$= 2\sin\left(\frac{\alpha+\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right) + 2\cos\left(\frac{\gamma+\alpha+\beta+\gamma}{2}\right) \cdot \sin\left(\frac{\gamma-\alpha-\beta-\gamma}{2}\right)$$
$$= 2\sin\left(\frac{\alpha+\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right) - 2\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right) \cdot \sin\left(\frac{\alpha+\beta}{2}\right)$$
$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$
$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[2\sin\left(\frac{\alpha-\gamma}{2}\right) \cdot \sin\left(\frac{\beta-\gamma}{2}\right)\right]$$
$$= 4\sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\gamma}{2}\right) \cdot \sin\left(\frac{\beta-\gamma}{2}\right)$$

R.H.S

Determinant

Prove without expansion :

(i) $\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix} = 0$

Sol:- Let $\Delta = \begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$

$= -\begin{vmatrix} 0 & x & y \\ x & 0 & z \\ y & -z & 0 \end{vmatrix}$ (interchanging the rows and columns)

$= -\Delta$

i.e. $\Delta = -\Delta$

$\Rightarrow \Delta + \Delta = 0$

$\Rightarrow 2\Delta = 0$

$\Rightarrow \Delta = 2 \times 0 = 0$

Hence, $\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix} = 0$

(ii) $\begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix} = \begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix}$

Sol:- L.H.S $\begin{vmatrix} -b & 1 & a \\ -c & 1 & b \\ -a & 1 & c \end{vmatrix} = -\begin{vmatrix} b & 1 & a \\ c & 1 & b \\ a & 1 & c \end{vmatrix}$ (interchanging the columns)

$$= \begin{vmatrix} 0 & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix} \quad \text{R.H.S (proved)}$$

(iii) $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = (x+2)(x-1)^2$

Sol:- L.H.S

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} x+2 & 1 & 1 \\ x+2 & x & 1 \\ x+2 & 1 & x \end{vmatrix} \quad C_1 \to C_1 + C_2 + C_3$$

$$= x+2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

Replacing $R_2 \to (R_2 - R_1)$ and $R_3 \to (R_3 - R_1)$

$$= x+2 \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

$$= x+2 \{1(x-1)(x-1) - 0\}$$

$$= (x+2)(x-1)^2 \quad \text{proved.}$$

Q. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

Solⁿ:- $$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} \quad C_1 \to C_1+C_2+C_3$$

$$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

$$= x+a+b+c \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} \quad \begin{matrix} R_1 \to R_1-R_2 \\ R_2 \to R_2-R_3 \end{matrix}$$

$$= x+a+b+c \begin{vmatrix} 0 & -x & 0 \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix}$$

$$= x+a+b+c \left[1(x^2-0) \right]$$

$$= x+a+b+c(x^2)$$
$$= x^2(x+a+b+c) \quad \text{R.H.S (proved).}$$

Q. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

Solⁿ:- $$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

Taking a, b, & c common from R_1, R_2 & R_3 respecectively.

we get

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= abc \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= abc \left[1 (a-b)(b^2-c^2) - (b-c)(a^2-b^2) \right]$$

$$= abc \{ (a-b)(b-c)(b+c) - (b-c) \cdot (a-b)(a+b) \}$$

$$= abc (a-b)(b-c) \{ \cancel{b} + c - a - \cancel{b} \}$$

$$= abc (a-b)(b-c)(c-a)$$ R.H.S proved.

Q. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x+y+z)(x-y)(y-z)(z-x)$

Soln:- L.H.S

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = \begin{vmatrix} x-y & y-z & z \\ (x-y)(x+y) & (y-z)(y+z) & z^2 \\ y+z & z+x & x+y \end{vmatrix}$$

Taking common $(x-y)$ & $(y-z)$ from C_1 & C_2 respectively.

we get,

$$(x-y)(y-z)\begin{vmatrix} 1 & 1 & z \\ x+y & (y+z) & z^2 \\ -1 & -1 & x+y \end{vmatrix} \quad R_1 \to R_1 + R_3$$

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & z+x+y \\ x+y & y+z & z^2 \\ -1 & -1 & x+y \end{vmatrix}$$

$$= (x-y)(y-z)\{(z+x+y)(-x-\cancel{y}+\cancel{y}+z)\}$$

$$= (x-y)(y-z)\{(z+x+y)(z-x)\}$$

$$= (z+x+y)(x-y)(y-z)(z-x) \quad \text{R.H.S (proved.)}$$

Q. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol^n:-

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

$= (a-b)(b-c)\{1(b+c-a-b)\}$

$= (a-b)(b-c)\{\cancel{b}+c-a-\cancel{b}\}$

$= (a-b)(b-c)(c-a)$ R.H.S proved.

Q. Prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Solⁿ: L.H.S $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$= \begin{vmatrix} x & 0 & 1 \\ -y & y & 1 \\ 0 & -z & 1+z \end{vmatrix}$

$= x\{yz+0\} + x(y+yz+z)$

$= yz + xy + xyz + xz$

$= xyz\left(\frac{1}{x}+\frac{1}{z}+1+\frac{1}{y}\right)$

$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ R.H.S (proved)

Q. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Sol^n:- $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

$$= \begin{vmatrix} b & c+a & a+b \\ q & r+p & p+q \\ y & z+x & x+y \end{vmatrix} + \begin{vmatrix} c & c+a & a+b \\ r & r+p & p+q \\ z & z+x & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b & c+a & b \\ q & r+p & q \\ y & z+x & y \end{vmatrix} + \begin{vmatrix} b & c+a & a \\ q & r+p & p \\ y & z+x & x \end{vmatrix} + \begin{vmatrix} c & c & a+b \\ r & r & p+q \\ z & z & x+y \end{vmatrix} + \begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b & c+a & a \\ q & r+p & p \\ y & z+x & x \end{vmatrix} + \begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b & c & a \\ q & r & p \\ y & z & x \end{vmatrix} + \begin{vmatrix} b & a & a \\ q & p & p \\ y & x & x \end{vmatrix} + \begin{vmatrix} c & a & a \\ r & p & p \\ z & x & x \end{vmatrix} + \begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$$

$$= \begin{vmatrix} b & c & a \\ q & r & p \\ y & z & x \end{vmatrix} + \begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$$ Replacing C_1 by C_3 & C_1 by C_2

$$= \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$ proved.

Q. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

Solⁿ:- $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ $R_1 \to R_1 - R_2 - R_3$

$= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$= 2\begin{vmatrix} 0 & -c & -b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$= 2\{c(ab+b^2-bc) - b(bc-c^2-ac)\}$

$= 2\{abc + \cancel{b^2c} - \cancel{bc^2} - \cancel{b^2c} - \cancel{bc^2} + abc\}$

$= 2\{2abc\}$

$= 4abc$ R.H.S (proved)

Expand the following determinants.

a. $\begin{vmatrix} 1 & \omega \\ -\omega & \omega \end{vmatrix}$

Sol^n:- $\omega + \omega^2$

b. $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

Sol^n:- $\cos^2\theta + \sin^2\theta$

$= 1$

d. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Sol^n:- $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega^2 \end{vmatrix}$ $C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega^2+\omega & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega^2 \end{vmatrix}$

we know that, the value of $1+\omega+\omega^2=0$

so, the value of determinant ~~value of~~ is 0.

c. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Sol^n:- $1(1-0)$

$= 1$

(e.) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$

Sol^n:- $1(6-6) - 1(6-6) + 1(6-6)$

$= 0 - 0 + 0$

$= 0$

2. Solve the determinants

a: $\begin{vmatrix} 4 & x+1 \\ 2 & x \end{vmatrix} = 5$

Soln:- $4x-(+2x+2) = 5$

$\Rightarrow 4x-2x-2 = 5$

$\Rightarrow 2x = 5+2$

$\Rightarrow x = 7/2 = 03.5$

b. $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$

Soln:- $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix}$

$\Rightarrow 7(7x-6)-6(14-2x)+x(6-x^2) = 0$

$\Rightarrow 49x-42-84+12x+6x-x^3 = 0$

$\Rightarrow 49x+12x+6x-x^3-126 = 0$

$\Rightarrow 67x-x^3 = 126$

$\Rightarrow 0 = x^3-67x+126$

$\Rightarrow x^3-67x+126 = 0$

Suppose $x = 2$

$\therefore 2^3 - 67 \cdot 2 + 126 = 0$

$\Rightarrow 8 - 134 + 126 = 0$

$\Rightarrow -126 + 126 = 0$

$\Rightarrow 0 = 0$

$\therefore$ ~~$(x-2)$ is~~

$\therefore$ $x^3 - 67x + 126$ is divisible by $(x-2)$

So,

$$
\begin{array}{r|l}
x-2 & x^3 - 67x + 126 \; (x^2 + 2x - 63 \\
 & \cancel{x^3} - 2x^2 \\
 & - \quad + \\
\hline
 & \cancel{2x^2} - 67x + 126 \\
 & \cancel{2x^2} - 4x \\
 & - \quad + \\
\hline
 & -\cancel{63x} + \cancel{126} \\
 & -\cancel{63x} + \cancel{126} \\
 & + \quad - \\
\hline
 & 0
\end{array}
$$

$\therefore$ $x^2 + 2x - 63 = 0$

$\Rightarrow$ $x^2 + 9x - 7x - 63 = 0$

$\Rightarrow$ $x(x+9) - 7(x+9) = 0$

$\Rightarrow$ $(x+9)(x-7) = 0$

$\therefore$ $x + 9 = 0$ | $x - 7 = 0$

$x = -9$ | $x = 7$

therefore The Value of $x = 2, 7, -9$

Ans

c. $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

Sol^n:- ~~$x+a\left[(x+c)(x+b)-a^2\right]-b\left[b(x+b)-ac\right]+c\left[ab-c(x+c)\right]=0$~~

$$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0 \qquad C_1 \to C_1+C_2+C_3$$

$$\Rightarrow \begin{vmatrix} x+a+b+c & x+a+b+c & x+a+b+c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$$

$$\Rightarrow x+a+b+c \begin{vmatrix} 1 & 1 & 1 \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0 \qquad C_1 \to C_1-C_2 \ \& \ C_2 \to C_2-C_3$$

$$\Rightarrow x+a+b+c \begin{vmatrix} 0 & 0 & 1 \\ b-x-c & x+c-a & a \\ c-a & a-x-b & x+b \end{vmatrix} = 0$$

$$\Rightarrow x+a+b+c \left\{ 1\left(b-x-c)(a-x-b) - (c-a)(x+c-a)\right\} = 0$$

$$\Rightarrow x+a+b+c \left\{ b(a-x-b) - x(a-x-b) - c(a-x-b) - c(x+c-a) + a(x+c-a) \right\} = 0$$

$$\Rightarrow x+a+b+c \left\{ ab - \cancel{bx} - b^2 - \cancel{ax} + x^2 + \cancel{bx} - \cancel{ac} + \cancel{cx} + bc - \cancel{cx} - c^2 + \cancel{ac} + \cancel{ax} + ac - a^2 \right\} = 0$$

$$\Rightarrow \quad ab - b^2 + x^2 + bc - c^2 + ac - a^2 = 0/(x+a+b+c)$$

$$\Rightarrow \quad x^2 - a^2 - b^2 - c^2 + ab + bc + ac = 0$$

$$\Rightarrow \quad x^2 = a^2 + b^2 + c^2 - ab - bc - ac$$

$$\Rightarrow \quad x = \sqrt{(a+b+c)^2} = a+b+c \text{ or } \sqrt{a^2+b^2+c^2-bc-ab-ca}$$

Ans.

d. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

Sol:- $1\{(1-1-x) - 1(1-1) + 1+x((1+x)^2 - 1\} = 0$

$\Rightarrow \cancel{-x} + 0 + (1+x)^3 - 1 \cancel{+x} = 0$

$\Rightarrow (1+x)^3 = 1$

$\Rightarrow 1+x = \sqrt[3]{1}$

$\Rightarrow \cancel{1}+x = \cancel{1}$

$\Rightarrow x = 0$

Ans.

3. Find minors & co-factors of the determinants

$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{vmatrix}$

Sol:-

minors	co-factors
$M_{11} = -10$	$C_{11} = (-1)^2 \cdot -10 = -10$
$M_{12} = 1$	$C_{12} = (-1)^3 \cdot 1 = -1$
$M_{13} = 7$	$C_{13} = (-1)^4 \cdot 7 = 7$
$M_{21} = 0$	$C_{21} = (-1)^3 \cdot 0 = 0$
$M_{22} = 1$	$C_{22} = (-1)^4 \cdot 1 = 1$
$M_{23} = 2$	$C_{23} = (-1)^5 \cdot 2 = -2$
$M_{31} = 5$	$C_{31} = (-1)^4 \cdot 5 = 5$
$M_{32} = 1$	$C_{32} = (-1)^5 \cdot 1 = -1$
$M_{33} = -3$	$C_{33} = (-1)^6 \cdot -3 = -3$

4. Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Soln:- L.H.S

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$C_1 \to C_1 - C_2$
$C_2 \to C_2 - C_3$

$$\begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix}$$

$$= a\left(b(1+c)+c\right) + 1\left(bc - 0\right)$$

$$= a\left[b + bc + c\right] + bc$$

$$= ab + abc + ac + bc$$

$$= abc\left(\frac{1}{c} + 1 + \frac{1}{b} + \frac{1}{a}\right)$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$ R.H.S (proved).

5. Prove that without expanding.

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

Soln:-
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$abc\begin{vmatrix}1 & 1 & a\\ 1 & 1 & b\\ 1 & 1 & c\end{vmatrix} - abc\begin{vmatrix}1 & 1 & bc\\ 1 & 1 & ca\\ 1 & 1 & ab\end{vmatrix}$$

We know that,

If two rows and two columns of a determinant are equal, the value of determinant is zero.

$$\because abc \times 0 - abc \times 0$$
$$= 0 - 0$$
$$= 0 \quad \text{R.H.S (proved)}$$

7. Prove that $\begin{vmatrix}1 & 1 & 1\\ b+c & c+a & a+b\\ b^2+c^2 & c^2+a^2 & a^2+b^2\end{vmatrix} = (b-c)(c-a)(a-b)$

Soln:- L.H.S

$$\begin{vmatrix}1 & 1 & 1\\ b+c & c+a & a+b\\ b^2+c^2 & c^2+a^2 & a^2+b^2\end{vmatrix} \quad \begin{matrix}C_1 \to C_1 - C_2\\ C_2 \to C_2 - C_3\end{matrix}$$

$$= \begin{vmatrix}0 & 0 & 1\\ b-a & c-b & a+b\\ b^2-a^2 & c^2-b^2 & a^2+b^2\end{vmatrix}$$

$$= \begin{vmatrix}0\times(b-a) & 0\times(c-b) & 1\\ (b-a) & (c-b) & a+b\\ (b+a)(b-a) & (c+b)(c-b) & a^2+b^2\end{vmatrix}$$

$$= (b-a)(c-b)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & a+b \\ (b+a) & (c+b) & a^2+b^2 \end{vmatrix}$$

$$= (b-a)(c-b) \times 1 \begin{vmatrix} 1 & 1 \\ (b+a) & (c+b) \end{vmatrix}$$

$$= (b-a)(c-b) \times (c+\cancel{b}-\cancel{b}-a)$$

$$= (b-a)(c-b)(c-a)$$

$$= -(a-b)\{-(b-c)\}(c-a)$$

$$= (a-b)(b-c)(c-a)$$

R.H.S

H.P

8. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

Sol^n:- L.H.S

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & bc & c^2+1 \end{vmatrix}$$

$$= abc \begin{vmatrix} a+\frac{1}{a} & a & a \\ b & b+\frac{1}{b} & b \\ c & c & c+\frac{1}{c} \end{vmatrix}$$

$$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$ $R_1 \to R_1+R_2+R_3$

$$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$= 1+a^2+b^2+c^2 \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} \quad C_1 \to C_1 - C_2$$

$$C_2 \to C_2 - C_3$$

$$= 1+a^2+b^2+c^2 \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & b^2 \\ 0 & -1 & c^2+1 \end{vmatrix}$$

$$= 1+a^2+b^2+c^2 \left\{ 1(1-0) \right\}$$

$$= 1+a^2+b^2+c^2 \quad \text{R.H.S} \quad \text{(proved)}$$

10. Prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Sol:- $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Replacing C_1 by $C_1 + C_2$

$$= \begin{vmatrix} a-c & b-c & c-a \\ b-a & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix}$$

$$= \begin{vmatrix} -(c-a) & b-c & c-a \\ -(a-b) & c-a & a-b \\ -(b-c) & a-b & b-c \end{vmatrix}$$

$$= -1 \begin{vmatrix} c-a & b-c & c-a \\ a-b & c-a & a-b \\ b-c & a-b & b-c \end{vmatrix}$$

If two rows or columns are same, the value of a determinant is zero.

$= -1 \times 0$

$= 0$ R.H.S (proved)

11. Solve by using cramer's rule

a. $2x - 3y = 7$

$3x + 2y = 3$

Solⁿ:- $\Delta = \begin{vmatrix} 2 & -3 \\ 3 & +2 \end{vmatrix} = 13$

$\Delta x = \begin{vmatrix} 7 & -3 \\ 3 & 2 \end{vmatrix} = 23$

$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 3 \end{vmatrix} = -15$

$x = \frac{\Delta x}{\Delta} = \frac{23}{13}$

$y = \frac{\Delta y}{\Delta} = \frac{-15}{13}$

b. $3x + 2y + 6z = 1$

$2x - 3y + 4z = 3$

$4x - 3y + 7z = 4$

Solⁿ:- $\Delta = \begin{vmatrix} 3 & 2 & 6 \\ 2 & -3 & 4 \\ 4 & -3 & 7 \end{vmatrix}$

$\Rightarrow \Delta = 3\{-21+12\} - 2\{14-16\} + 6\{-6+12\}$

$\Rightarrow \Delta = 3\times(-9) - 2\times(-2) + 6\times 6$

$\Rightarrow \Delta = -27 + 4 + 36 = 13$

$$\Delta x = \begin{vmatrix} 1 & 2 & 6 \\ 3 & -3 & 4 \\ 4 & -3 & 7 \end{vmatrix}$$

$$\Delta x = 1(-21+12) - 2(21-16) + 6(-9+12)$$
$$= -9 - 10 + \cancel{36}\ 18$$
$$= +18 - 19$$
$$= -1$$

$$\Delta y = \begin{vmatrix} 3 & 1 & 6 \\ 2 & 3 & 4 \\ 4 & 4 & 7 \end{vmatrix}$$

$$\Delta y = 3(21-16) - 1(14-16) + 6(8-12)$$
$$= 15 + 2 - \cancel{78}\ 24$$
$$= -7$$

$$\Delta z = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -3 & 3 \\ 4 & -3 & 4 \end{vmatrix}$$

$$\Delta z = 3(-12+9) - 2(8-12) + 1(-6+12)$$
$$= -9 + 8 + 6$$
$$= 5$$

$$x = \frac{\Delta x}{\Delta} = -1/13$$

$$y = \frac{\Delta y}{\Delta} = -7/13$$

$$z = \frac{\Delta z}{\Delta} = 5/13$$ Ans

[Matrices]

A set of mn elements arranged in a rectangular array having 'm' ~~elem~~ rows and 'n' columns. It is denoted by

$$A = [a_{ij}] \text{ or } A = (a_{ij}) \text{ or } \|A\| \text{ or } [A]$$

where, 'i' is the ith rows and 'j' is the jth columns.

Example:- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$

The order of the matrix is $m \times n$.

ROW MATRICES

A matrix having only one row and any number of columns is called a row matrix.

Example:- $A = [a_{11} \; a_{12} \; a_{13} \cdots a_{1n}]_{1 \times n}$

COLUMN MATRIX

A matrix having only one column and any number of rows is called a column matrix.

Eg:- $B = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$

C. $x+y+z=3$

$2x+3y+4z=9$

$x+2y-4z=-1$

Sol^n:- $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix}$

$\Delta = 1(-12-8) - 1(-8-4) + 1(4-3)$

$= -20 + 12 + 1$

$= -7$

$\Delta x = \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 4 \\ -1 & 2 & -4 \end{vmatrix} = 3(-12-8) - 1(-36+4) + 1(18+3)$

$= -60 + 32 + 21$

$= -7$

$\Delta y = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 9 & 4 \\ 1 & -1 & -4 \end{vmatrix} = 1(-36+4) - 3(-8-4) + 1(-2-9)$

$= -32 + 36 - 11$

$= -7$

$\Delta z = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & 9 \\ 1 & 2 & -1 \end{vmatrix} = 1(-3-18) - 1(-2-9) + 3(4-3)$

$= -21 + 11 + 3$

$= -7$

$x = \frac{\Delta x}{\Delta} = \frac{\cancel{-7}}{\cancel{-7}} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{\cancel{-7}}{\cancel{-7}} = 1$

$z = \frac{\Delta z}{\Delta} = \frac{\cancel{-7}}{\cancel{-7}} = 1$ Ans.

Zero matrix or Null matrix
If all the elements of a matrix is zero., then, it is called zero or Null matrix.

Eg:- $D = [0]_{1\times1}$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{2\times4}$$

Square Matrix
In a matrix if the number of rows are equal to number of columns then it is called square matrix.

Eg:-

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

Unit matrix
A square matrix is said to be unit matrix if the elements of its main diagonal (i.e. left to right) is 1. and the orther elements are zero,

Eg:- $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

Singular Matrix

Square matrix is said to be a singular matrix if its determinant is zero. (i.e. $|A| = 0$)

Eg :- $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

Non-singular Matrix

If a square matrix is said to be a Non-singular matrix if its determinant value is not zero. (i.e. $|A| \neq 0$)

Eg :- $P = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$

Equality of two matrix.

Two matrix of A and B are said to be equal if and only if

① The order of A ~~and~~ = the order of B

⑪ Each element of A = the corresponding element of B.

Example :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Q. For what value of a the given two matrix are equal.

$A = \begin{bmatrix} 2 & a \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

Ans for any value of a. the matrix are not equal.

Addition of matrix

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, then $A+B = ?$

$$A+B = \begin{bmatrix} a+p & b+q \\ c+r & d+s \end{bmatrix}$$

Note:- Matrix A and B should be equal in order.

PROPORTIES OF MATRICES ADDITION.

1. Matrix addition is commutative, if A and B are two matrices then $A+B = B+A$

if $A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}$$

$$B+A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}$$

2. Matrix addition is associated
if A, B and C are three matrices then, $(A+B)+C = A+(B+C)$.
if

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$(A+B)+C = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 5 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 2 & 2 \\ 4 & 5 \end{bmatrix}$$

3. The ~~identify~~ ~~inden~~ identity matrix for addition is the zero matrix or null matrix.
(i.e. $A+0=A$)

Eg :- $A = \begin{bmatrix} 2 & 3 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & 3 \end{bmatrix}$

4. If A is a matrix negative of A is given by $-A$ and $A+(-A)=0$

5. The substraction of two matrices A and B of the same order is defined as $A-B = A+(-B)$.

Matrix Multiplication

The product of two matrices A and B of the (where the number of columns in A = the number of rows in B) is the matrix AB whose element in the ith row and jth columns is the sum of products formed by multiplying each element in the ith row of A and the corresponding elements in the jth column of B.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} \quad \text{and} \quad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}_{2\times 2}$$

$$AB = \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix}$$

Q. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$, then find $AB = ?$

Sol^n:- $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

Q. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then find $AB = ?$

Sol^n:- $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Q. If $A = \begin{bmatrix} 2 & 5 \\ -6 & 7 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then find $AB = ?$

Sol^n:- $AB = \begin{bmatrix} 2 & 5 \\ -6 & 7 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -25 & 8 \end{bmatrix}$

Q. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, find $AB = ?$

Sol^n:- $AB = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2+0+1 & 0+0+0 & 1+0+1 \\ 2-1-1 & 0-1+0 & 1-1+1 \\ 2+1-1 & 0+1+0 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Q. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}$, then find $AB = ?$

Solⁿ:-

$$AB = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1-1 & 4+1-1 \\ 1+0+1 & 2+0+1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix}$$

Q. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 5 & -7 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & 5 \\ 9 & 8 & 7 \end{bmatrix}$, find $AB = ?$

Solⁿ:- Since, Number of columns in A are not equal to the number of rows in B.
So, matrix multiplication is not possible.

Note:- The product of a scalar of m with a matrix A is denoted by MA, is the matrix each of whose elements is m times the corresponding elements of A.

Example:- If $A = \begin{bmatrix} 2 & 5 \\ 6 & 4 \end{bmatrix}$ & $m = 2$

then, $mA = 2\begin{bmatrix} 2 & 5 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 12 & 8 \end{bmatrix}$

Note:- 1. The matrix products A.A is defined only when A is a square matrix.

Note:- 2. The rule to remember a matrix products is

$\{(m \times n) \text{ matrix}\}\{(n \times p) \text{ matrix}\} = (m \times p)$ matrix.

*. Propostities of matrix Multiplication *.

1. Matrix multiplication is not necessarily commutative
   (i.e. $AB \neq BA$)

2. Matrix multiplication is associative
   i.e. $(AB)C = A(BC)$

3. If we multiply a unit matrix with a matrix A both of having equal order and square also then the result will be that matrix A

Eg :- let $A = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $AI = ?$

$$AI = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$$ Ans.

Note:- $\boxed{AI = A = IA}$

4. let A and B are two matrices such that the product AB is defined.

then $A=0$ or $B=0$ or $A=B=0$ always implies $AB=0$. Conversely.

$AB=0$ does not always imply that $A=0$ or $B=0$ or $A=0=B$.

5. The cancellation law does not hold for matrix multiplication. i.e. $CA = CB$ does not necessarily imply $A=B$.

6. The distributive laws hold for matrix

$$2(x+y) = 2x+2y$$

$$\text{or } 2(3+5) = 2\times3+2\times5$$

then, $\boxed{A(B+C) = AB+AC}$

MULTIPLICATION INVERSE OF A SQUARE MATRIX

→ If A and B are two matrices of order n such that

$$AB = I = BA$$

where, I = Identity matrix of order n

→ Then B is called the multiplicative inverse of A.

→ Similiarly, A is called the multiplicative inverse of B.

→ It is written as A^{-1} or B^{-1}

Note:-1 The zero matrix has no multiplicative inverse.

Note:-2: The unit matrix is the multiplicative inverse of its self.

TRASPOSE

Transpose of a $m \times n$ matrix A is the matrix of order $n \times m$ obtained by interchanging the rows and column of A.

→ It is denoted by A' or A^T

If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 4 & 1 \\ 3 & -1 & 1 \end{bmatrix}$

Minor & Co-factor

The minor of an element a_{ij} of a matrix is obtained by delete the ith row and jth column from the matrix and is denoted by M_{ij}

Eg :- $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

the minor of an element $a_{11} = M_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$ Ans

Co-factor of a matrix.

The co-factor of an element a_{ij} of a matrix A is $(-1)^{i+j} M_{ij}$ and denoted by C_{ij}

If $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then, the co-factor of A is

$M_{11} = 1$ $\quad M_{21} = 1$ $\quad M_{31} = 1$

$M_{12} = 0$ $\quad M_{22} = 2$ $\quad M_{32} = 2$

$M_{13} = -1$ $\quad M_{23} = 1$ $\quad M_{33} = 3$

$C_{11} = 1$

$C_{12} = 0$

$C_{13} = -1$

$C_{21} = -1$

$C_{22} = 2$

$C_{23} = -1$

$C_{31} = 1$

$C_{32} = -2$

$C_{33} = 3$

so, co-factor of A is

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & -1 \\ 1 & -2 & 3 \end{bmatrix}$$

Adjoint of a matrix

The adjoint of a matrix A is the transpose of the matrix obtained replacing each element of a_{ij} in Capital A by its cofactor. The adjoint of A is denoted by adj A.

Note:- $\neq$ Inverse of a matrix

$$A^{-1} = \frac{\text{Adj} A}{|A|}$$

where A is a non-singular matrix

Q. find the adjoint and inverse of the matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$$

Sol^n:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$

$C_{11} = -1$

$C_{12} = 3$

$C_{13} = 8$

$C_{21} = 8$

$C_{22} = -5$

$C_{23} = -6$

$C_{31} = -3$

$C_{32} = 9$

$C_{33} = -5$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -1 & 3 & 8 \\ 8 & +5 & -6 \\ -3 & 9 & -5 \end{bmatrix}$$

$$\text{Adj} A = \begin{bmatrix} -1 & 8 & -3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{bmatrix}$$

inverse of a matrix A

$\because A^{-1} = \frac{Adj A}{|A|}$

so, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{vmatrix} = 1(-1-0) - 2(-3+0) + 3(6+2)$

$= -1 + 6 + 24$

$= 29$

$Adj A = \begin{vmatrix} -1 & 8 & 3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{vmatrix}$

$A^{-1} = \frac{1}{29} \begin{bmatrix} -1 & 8 & 3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{bmatrix}$

$= \begin{bmatrix} -1/29 & 8/29 & 3/29 \\ 3/29 & 5/29 & 9/29 \\ 8/29 & -6/29 & -5/29 \end{bmatrix}$ Ans.

Q. find the inverse of the matrix.

(a) $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

Solⁿ:- $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$C_{11} = -1$, $C_{21} = -2$

$C_{12} = -3$, $C_{22} = 4$

So, cofactor of matrix A is

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ -2 & 4 \end{bmatrix}$$

$$\text{Adj} A = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = -4 - 6 = -10$$

$$A^{-1} = \frac{\text{Adj} A}{|A|} = \frac{-1}{10} \begin{bmatrix} -1 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/10 & 3/10 \\ \frac{01}{5} & -2/5 \end{bmatrix} \text{ Ans.}$$

(ii) $$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Soln:- $$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$C_{11} = 1$

$C_{12} = -2$

$C_{13} = 2$

$C_{21} = -1$

$C_{22} = 3$

$C_{23} = 3$

$C_{31} = 0$

$C_{32} = -4$

$C_{33} = -3$

So, cofactor of matrix A is

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$$

$$\text{Adj} A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 2 & -4 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3+4) + 3(2-0) + 4(-2+0)$$

$$= 3 + 6 - 8$$

$$= 9 - 8$$

$$= 1$$

$$A^{-1} = \frac{Adj\ A}{|A|} = 1 \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ 2 & -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ 2 & -4 & -3 \end{bmatrix}$$ Ans

Q. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$, then find $A^2 - 12A - I_2 = 0$ where, I_2 is the identity matrix of order 2.

Sol:- Given,

$$A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

we have,

$$A^2 = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix}$$

$$= \begin{bmatrix} 61 & 36 \\ 114 & 85 \end{bmatrix}$$

$$12A = 12 \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} = \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= A^2 - 12A - I_2$$

$$= \begin{bmatrix} 61 & 36 \\ 114 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 114 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 114-114-0 & 85-84-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Thus, it is verified $A^2 - 12A - I_2 = 0$

Ans.

Q. Find the inverse of the following matrix.

(i) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Sol:- let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

then,

$$|A| = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= 0 - 1(1-9) + 2(1-6)$$
$$= 8 + (-10)$$
$$= -2$$

So, it has inverse

Adj A -

we have $\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$C_{11} = -1$

$C_{12} = +8$

$C_{13} = -5$

$C_{21} = +1$

$C_{22} = -6$

$C_{23} = +3$

$C_{31} = -1$

$C_{32} = 2$

$C_{33} = -1$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

then, $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$= \frac{-1}{2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & +3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

Ans.

So, co-factor of matrix A

$$\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

(iii) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

then,

$$|A| = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$= 3(-3+4) + 3(2-0) + 4(-2-0)$

$= 3 + 6 - 08$

$= \cancel{12-12} \quad 1$

$= \cancel{0}$

cofactor of A

$C_{11} = 1$

$C_{12} = -2$

$C_{13} = -2$

$C_{21} = -1$

$C_{22} = 3$

$C_{23} = 03$

$C_{31} = +0$

$C_{32} = -4$

$C_{33} = -3$

$$\text{Adj } A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Q. Solve by matrix Method

$x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 2$

Solⁿ:- The given system of equation is of the from $AX = B$.

where, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$\because \cancel{AB = B}\ AX = B$
$\Rightarrow X = A^{-1}B$

To find A^{-1}

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

So,

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) + 1(2+3) + 1(2-1)$
$= 4 + 5 + 1$
$= 10$

cofactor of A

$C_{11} = 4$ $\quad C_{21} = 2$ $\quad C_{31} = 2$
$C_{12} = 5$ $\quad C_{22} = 0$ $\quad C_{32} = -5$
$C_{13} = 1$ $\quad C_{23} = -2$ $\quad C_{33} = 3$