

# CHAPTER-1

## MEASURING INSTRUMENTS

### ✓ Measurements: -

It is the process of comparison a standard quantity with the measured quantity.

### ✓ Static characteristics: -

These are used to measure a condition when it is not varying w.r.t time.

### (a) Accuracy: -

→ It is defined as the closeness of any instrument reading towards the true value.

→ It is always express in terms of in accuracy or percentage of error.

### (b) Precision: -

It is defined as the property of reproducibility of any measure value by the instrument.

OR

→ It is also defined as the measure of repeatability of an instruments.

→ An instrument can be a precised one even though it is not accurate.

### (c) Error

It is defined as the difference between measured value and the true value of a quantity.

$$\text{Error} = M.V - T.V$$

### (d) Resolution:-

It is defined as—the smallest or least change in input, which can be detected by the measuring instrument.

### (e) Sensitivity:-

Sensitivity of an instrument is defined as the ratio of magnitude of out-put signal to magnitude of input signal.

It is also defined <sup>OR</sup> as—the ratio between change in ~~output~~ <sup>output</sup> to change in input.

$$\text{Sensitivity} = \frac{\Delta o}{\Delta i}$$

### (f) Tolerance:-

It can be defined as—the allowable or permissible limit by which a measurement can vary.

Ex - If any instrument has tolerance  $\pm 0.002$  & true value is 10V, then 10.1 is not acceptable. The value between 10 - 0.002 to 10 + 0.002 are acceptable.

### Condition of Instrument:-

The instruments can broadly classified into 2 types.

1. Absolute instrument
2. Secondary instrument.

### (a) Absolute instruments:-

- These instrument gives the magnitude of a physical constant of the instrument.
- These are generally not available in market for public use & measurement is very much time consuming.

### (b) Secondary instruments:-

- These instruments are calibrated in comparison with an absolute instrument.
- These instrument can be used to measure a quantity by absorbing the indicating output.

### Deflecting controlling and damping arrangements in indicating type instrument:-

- In indicating type instrument a pointer is present which moves over a calibrated scale.
- In this type of instrument generally 3 types torques are developed.

### (1) Deflecting torque:-

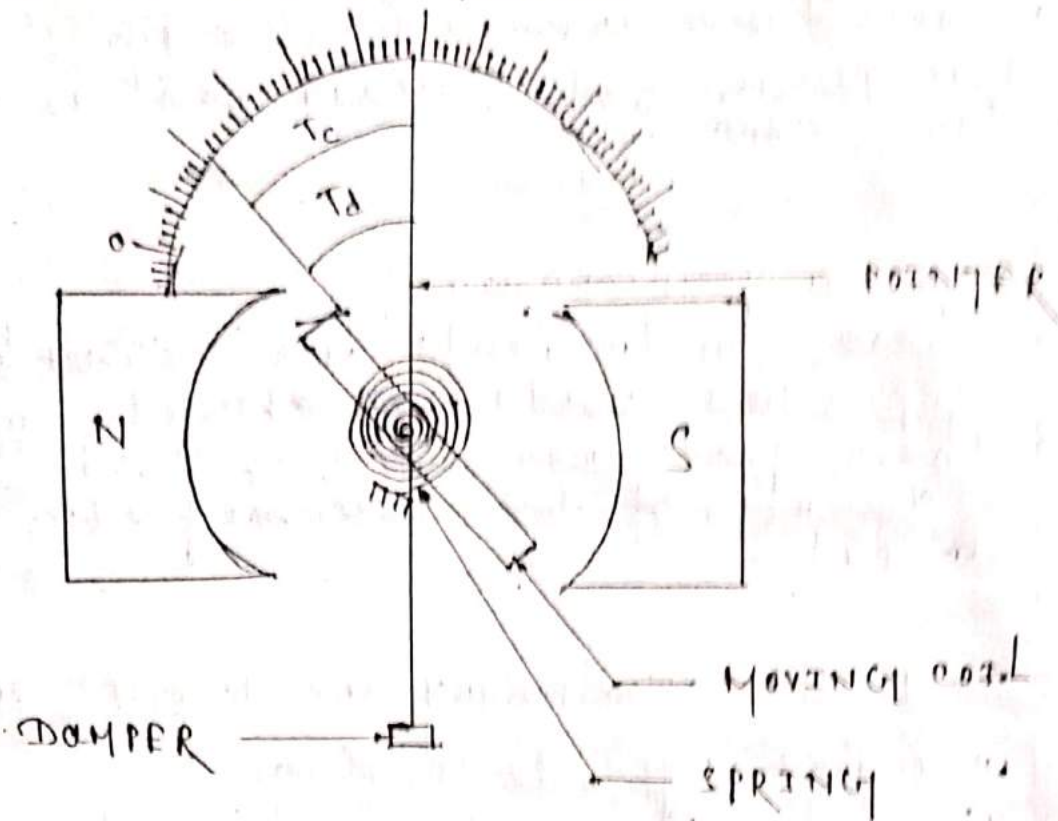
- This torque is used to move the pointer over the scale.
- It is denoted by  $T_d$ .

### (2) Controlling torque:-

- Controlling torque is used to neutralise or cancel the deflecting torque.
- It is denoted by  $T_c$ .
- $T_c \propto$  is proportional to the deflection done by the pointer.

### (3) Damping torque:-

This torque is used to absorb the oscillation of the pointer.



(Fig: PMMC type indicating instrument)

- The above figure shows an indicating type instrument arrangement.
- In this arrangement a moving coil is present in between the 2 poles of permanent magnet.
- When current ( $i$ ) flows through the moving coil then a magnetic field is developed around the coil, which interacts with the magnetic field of permanent magnet.

- Due to this interaction the coil get deflected by an angle ' $\theta$ '.
- one pointer is attached to the coil which also get deflected and shows the measured value on the scale.
- The torque developed or responsible for this deflection is  $T_d$ .
- When the current ( $I$ ) is removed, then a spring attached to the coil is used to bring it back to its original position.
- The torque which is responsible to bring the pointer back is called as controlling torque or ( $T_c$ ).
- one damper is connected to the pointer which is a cylinder and piston arrangement used to absorb the oscillation of the pointer. The torque developed by the damper is called damping torque.

### Calibration of instrument :-

- It is the process of comparison of a particular instrument with known standard instrument.
- It is done to obtain the accuracy and errors in an instrument.
- static characteristics are measured by calibration method.
- the instrument which is used for measurement must be calibrated against some reference instrument of higher accuracy.

## CHAPTER - 2

### ANALOGY AMMETERS & VOLTMETERS

1. PMMC (Permanent magnet moving coil instrument)
2. MI (Moving iron type instrument)
3. Dynamometer type instrument
4. Rectifier type instrument
5. Induction type instrument

→ Ammeter is the instrument which is used to measure current. It is always connected in series in the ckt whose current is to be measured.

→ Voltmeter is the instrument which is used to measure the voltage. It is always connected in parallel with the ckt whose voltage is to be measured.

→ The ~~at~~ basic working principle of ammeter and voltmeter is same.

→ In an ammeter the deflecting torque is produced by the current which is to be measured.

→ In voltmeter the deflecting torque is produced by the current which is proportional to the voltage to be measured.

Errors in instrument (Ammeter & Voltmeter): -

Generally 2 types of errors are common in ammeter & voltmeter which are due to friction and temperature.

## 1- Friction: -

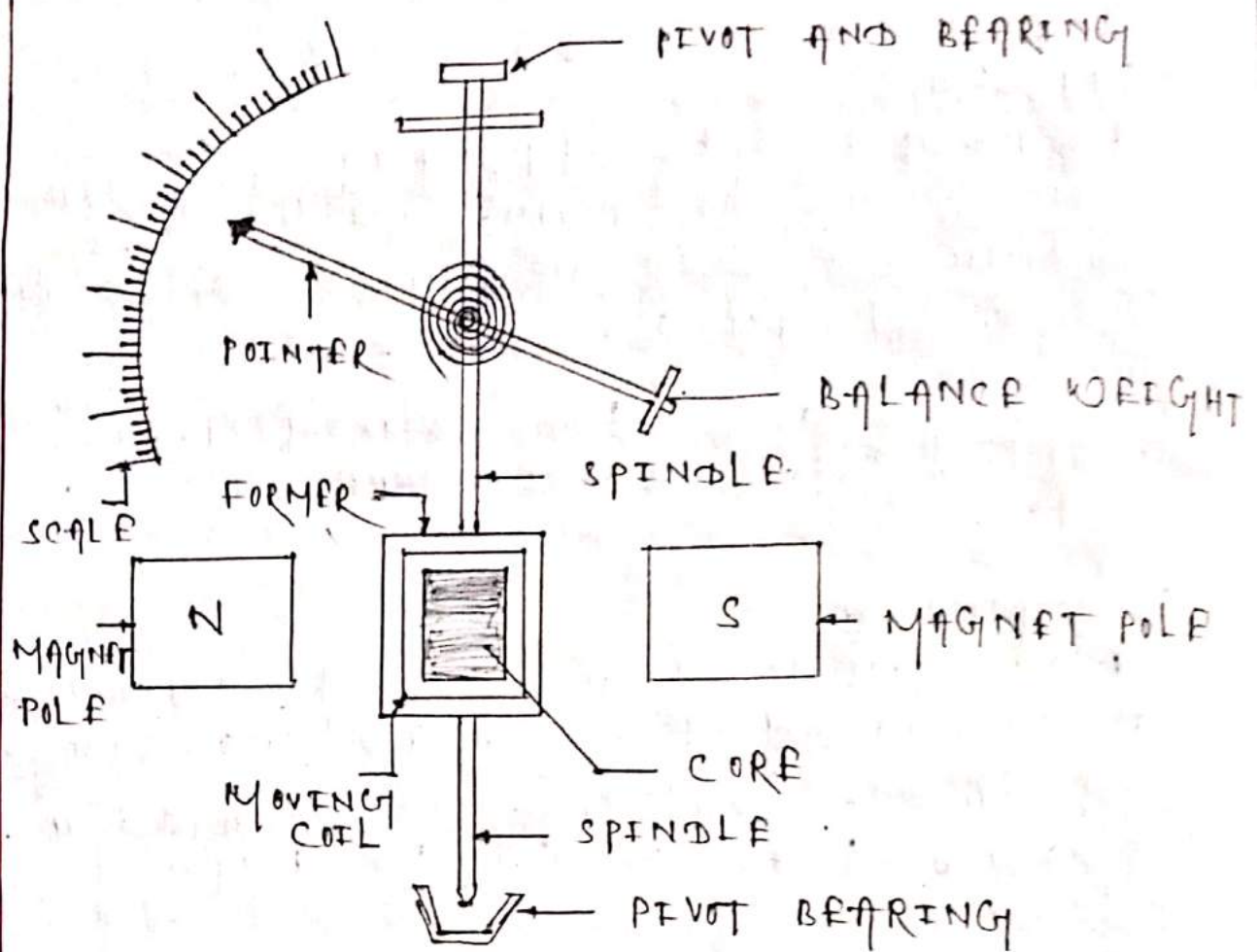
- To reduce the effect of friction, the weight of the moving system must be measured as small as possible with the operating force.
- In other words, the ratio of torque to weight must be very large.

## 2- Temperature: -

- It is possible to reduce the error which is caused by the temperature change.
- The instrument must be mounted in such a position that it will be properly ventilated.
- A swamping resistance of low temperature coefficient material can be connected in series with the coil to reduce the temperature effect on the instrument.

## (A) PMMC instrument: -

- The permanent magnet moving coil instrument consists of a moving coil, permanent magnet system, controlling and damping arrangement and a pointer scale arrangement.
- It can be used for both DC current and voltage measurement.
- The diagram of PMMC type instrument is given below.



(Fig: PMMC type instrument)

The component of PMMC instrument are describe below:-

### Moving coil:-

- The moving coil is wound with many turns of silk covered copper wire on a rectangular aluminium former.
- The coil moves freely in the field of permanent magnet.

### Magnet system:-

- The coil which is describe above is present within a magnetic field which can achieved by placing permanent magnet.



→ The angle of rotation of the coil may vary with the arrangement of magnetic poles.

→ For example of U-shaped magnets provides maximum  $90^\circ$  rotation, by using concentric magnetic poles the angle of rotation can be extended up to  $250^\circ$  or more.

### Control: -

Here coils are attached by bronze springs which provides the controlling torque. This springs in few cases also carries the current in or out of the coil.

### Damping: -

The damping torque is produced by the movement of aluminium former which absorbs the oscillation.

### Pointer scale: -

→ The pointer is connected to the spindle which moves over the graduated scale.

→ The pointer is of very light weight and the arrangement is done in such a way that it reduces the parallax error.

## Working principle:-

- When a current is to be measured that current is allowed to pass through the moving coil.
- Due to the flowing of current, a magnetic field is developed around the coil former.
- This generated magnetic field interacts the permanent magnetic field and a deflecting torque is produced in the coil.

$$T_d = N B l d I$$

Where,

$N$  = No. of turns in the coil

$B$  = Magnetic flux intensity in  $\text{wb/m}^2$

$l$  = length of the coil

$d$  = diameter of the coil

} Dimension of the coil

$I$  = current flowing through the coil in 'A'.

Since,

$N, B, l,$  &  $d$  are constant for a particular coil & magnet arrangement, so  $T_d \propto I$

- So, the deflecting torque is produced in the coil proportional to the current flowing through it.

- When the pointer get deflected the springs which are connected to the pointer are stretched and controlling torque is developed in the springs.

- If  $T_c$  is the controlling torque then,  
 $T_c = T_d$

→ The dampers are connected to the pointer or damping arrangement is made in the coil to absorb the oscillation of the pointer.

### Ammeter shunt:-

→ The coil winding of a ammeter is very small and light which can carry very small amount of current.

→ When heavy are to be measured then if all the current will flow through the coil, the coil get damaged.

→ To avoid this situation the major part of the current is bypassed through a low resistance called shunt.

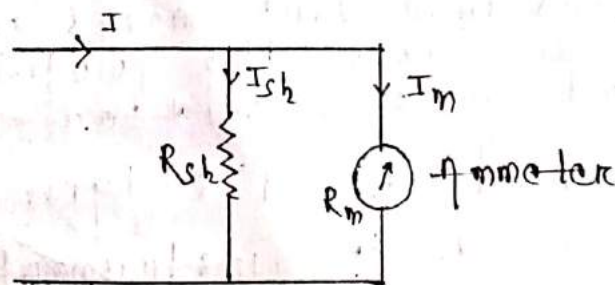


Fig: Basic ammeter

### Advantages:-

→ The scale is uniform.

→ It consumes less power as  $25\text{mW}$  to  $200\text{mW}$ .

→ The torque to weight ratio is very high, so the instrument accuracy is very high.

→ The single instrument can be used for different current and voltage range measurements.

### Disadvantages:-

→ It can only be used for DC measurements.

→ Cost of these instrument are comparatively higher than moving iron instrument.

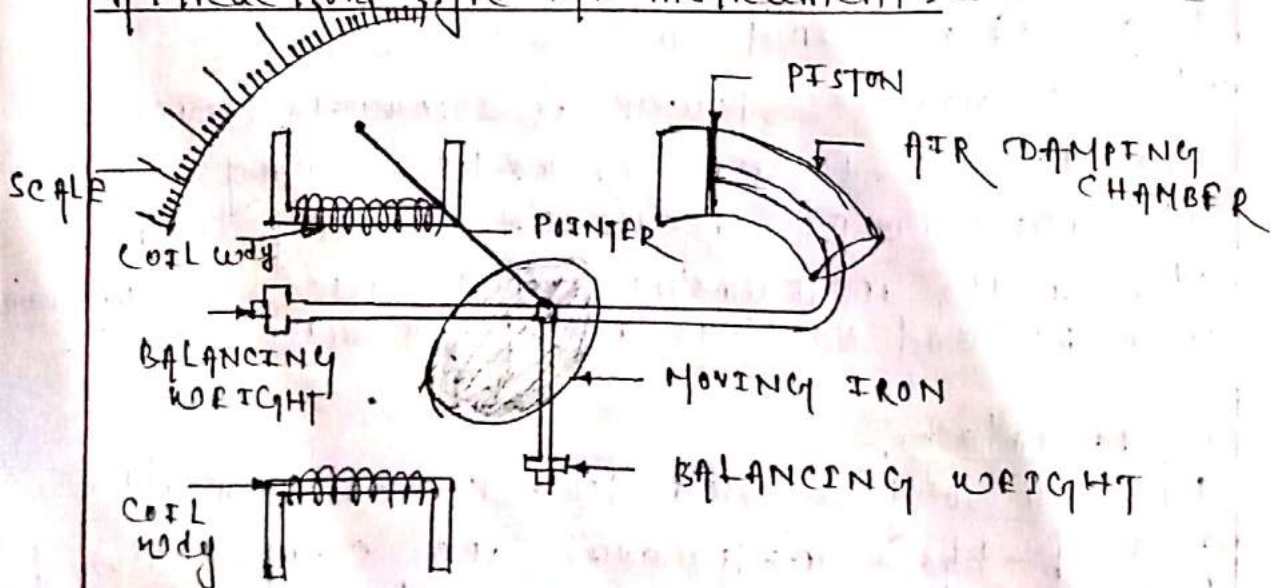
## (B) Moving iron type instrument (MI):-

In this type of instrument a moving iron piece rotates within a current carrying coil. For this reason it is called as moving iron type instrument.

### Working principle:-

- A plate or vane of soft iron is used to make moving element of the system.
- This iron vane is situated in such a way that it can move in a magnetic field produced by in the coil.
- The coil is excited by the current or voltage to be measured.
- When the coil excited it becomes an electro-magnet and iron vane moves in such a way that it also moves the pointer over the graduated scale associated through it.
- MI instrument are of 2 types:-
  1. Attraction type MI instrument
  2. Repulsion type MI instrument.

### Attraction type MI instrument:-



(Fig: Attraction type MI instrument)

→ The above figure shows an attraction-type MT instrument.

→ Here MT is a flat disc, which is mounted within the magnetic field of the coil.

→ When the current flows through the coil, a magnetic field is produced which attracts the moving iron piece and rotates it.

→ The pointer which is connected to the MT also gets deflected due to the movement of moving iron.

→ The controlling torque is provided by gravity control by using balancing weights if the instruments are vertically mounted.

→ Damping is provided by air friction with the help of a light aluminium piston which moves inside a fixed chamber.

### (c) Repulsion-type MT instrument:-

→ In repulsion-type MT instrument 2 iron vanes are present inside the coil where one is fixed and the other one is movable.

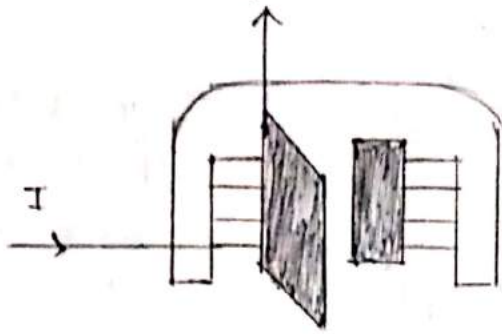
→ When the current flows through the coil, the coil gets magnetised and a force of repulsion acts between two iron vanes which rotate the movable iron vane.

→ There are 2 different designs of repulsion-type MT instrument.

1. Radial vane type

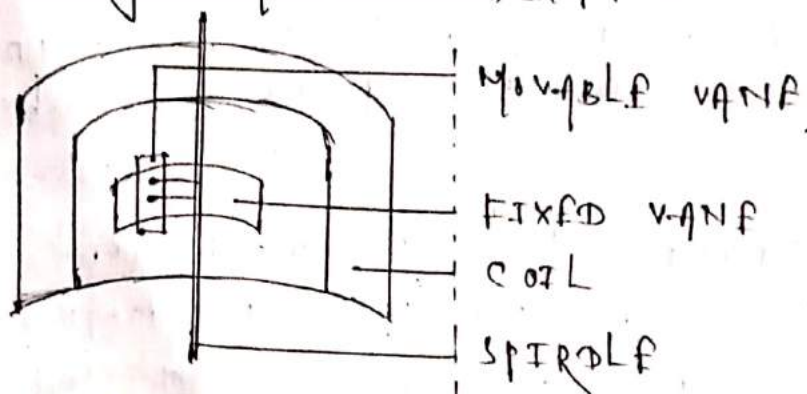
2. Co-axial vane type.

## 1 Radial vane type MT instrument :-



- In this type the vanes are radial strips are iron.
- The strips are within the coil.
- The fixed vane is attached to the coil and the movable vane is attached to the spindle of the instrument.

## 2 Co-axial vane type MT instrument :-



- In this type of instrument the fixed and moving vane are present as a section co-axial cylinder.
- The controlling torque for MT instrument can be provided by springs or gravity control.
- The damping torque is produced by connecting an air friction type piston and cylinder arrangement.

## Errors in MI instrument: -

### Errors with both DC & AC measurement: -

#### (i) Hysteresis error: -

- This error occurs since the value of flux density is different for the same current during ascending and descending values.
- Generally the instrument tends to read higher value during the descending input of voltage & current than the ascending input.

#### (ii) Temperature error: -

- The effect of temperature changes on MI type instrument is very high due to the shell fitting of coil and series resistances and high temperature coefficient spring material.
- Around 0.02-1% of variation occurs per °C temperature changes.

#### (iii) Stray magnetic field: -

- This error occurs due to the magnetic field other than the instrument magnetic field which is present in its environment.
- Due to the stray magnetic field the instrument operating magnetic field get distorted.

## Errors in AC Measurement:

### (i) Frequency Error:

Changes in frequency of AC quantities generally change in the reactance of instrument coil, so its effect of magnetic field strength of the coil.

### (ii) Eddy current error:

These errors are caused by eddy currents induced in the iron parts of the instruments.

### Advantages:

- These instruments can be used for both AC & DC measurement.
- These instruments are very cheap and simple in construction.
- Accuracy of the is very high.
- MT type instrument has very large scale length.
- The degree of deflection is around  $340^\circ$  in circular scale.
- It has very high operating torque, so torque to weight ratio is very high which reduces the friction of the instrument.

### Disadvantages:

- Power consumption is higher for low voltages range measurements.
- Stiffness of the spring reduces with increasing in temperature.
- Frequency change can cause error in AC measurement.
- Stray magnetic field effects of MT type instrument.
- Hysteresis error is also present in this type of instrument.



# Electrodynamometer type Instrument (EMMC) :-

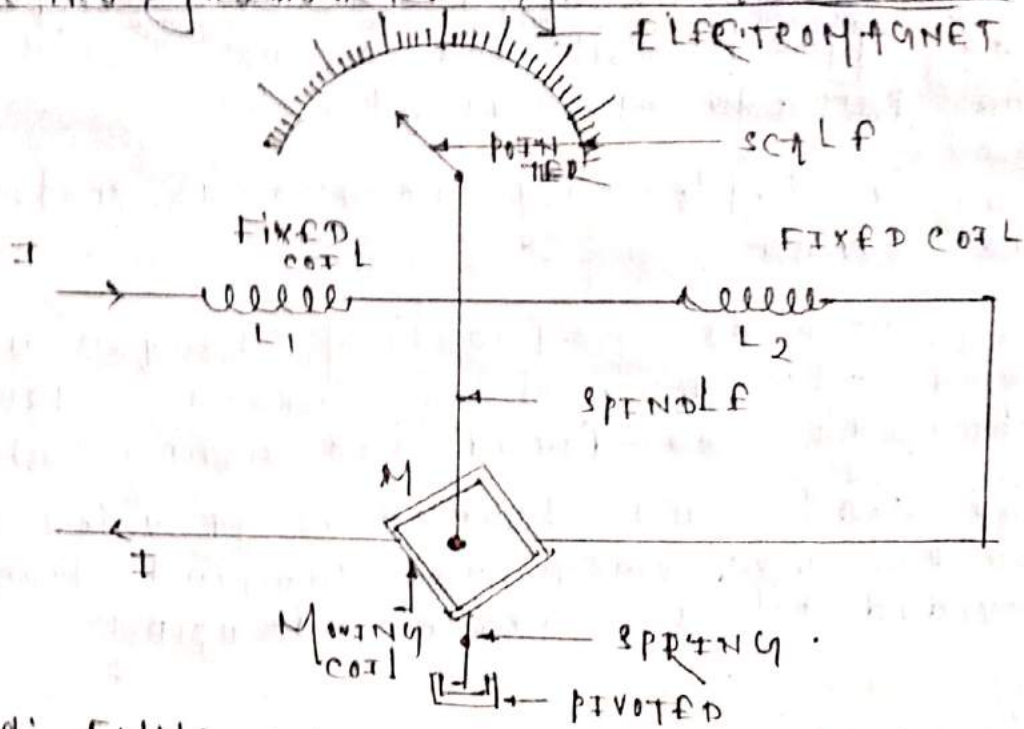


Fig: EMMC type / electro-dynamometer type instrument.

## Construction:-

- The instrument consists of two fix. coil  $L_1$  &  $L_2$  and one moving coil 'M'. The moving coil is placed on a spindle, which is pivoted, for producing controlling torque spring is attached to the spindle.
- pointer is connected in the spindle which moves over the graduated scale.

## Working:-

- The fix coil made of up few tons of copper wire and known as current coil, the current need to be measured is passed through the fixed coil at the starting.
- The moving coil is made of up large number of turns of copper wire and is called as pressure coil.

- When a magnetic field is developed in the fixed coil and moving coil due to the passage of current.
- Then a deflecting torque is produced in the moving coil.
- The provided deflecting torque is proportional to product of current passing through the fixed and moving coil.
- The controlling torque is provided by spring control mechanism and damping torque is provided by a current damping.

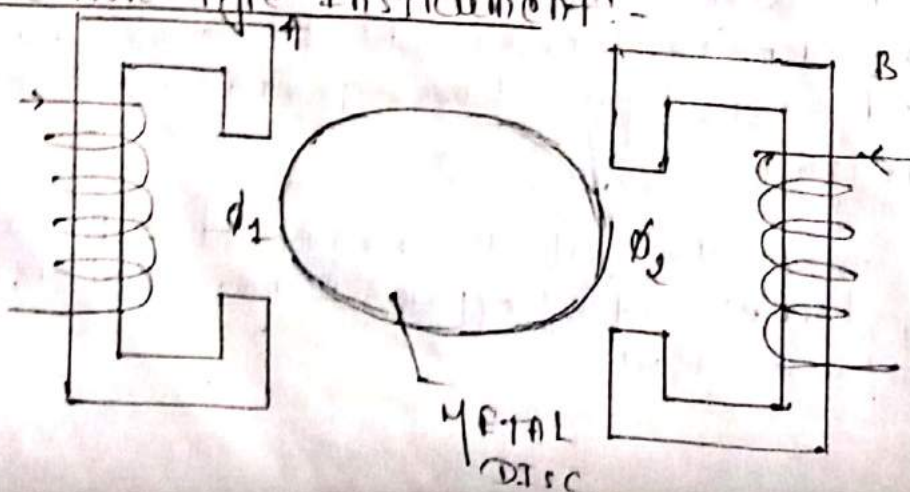
### Advantages:-

- It can measure both AC & DC parameter.
- It is free from hysteresis error.
- Magnetic field strength can be varied & there is no change of magnetic field loss like in case of PMMC type instrument.

### Disadvantages:-

- It has low sensitivity.
- More frictional loss due to heavy moving parts.
- scale is non-uniform.

### Induction type Instrument:-



- The above figure show an induction type instrument which can be used only for the measurement of AC quantities.
- It can be used as ammeter, voltmeter, wattmeter or energy meter.

### Construction:-

- Two electro-magnets and a metal disc are the main parts of this type of instrument, generally the metal disc is made up of aluminium.
- The metal disc is placed in between the two electro-magnets.

### Working:-

- The electro-magnets A & B produces flux  $\phi_1$  &  $\phi_2$  respectively.
- Both the flux  $\phi_1$  &  $\phi_2$  are induced into the metal disc.
- Due to the phase difference between the fluxes a deflecting torque is produced on the disc.
- The deflecting torque ( $T_d$ )  $\propto \phi_{1m} \phi_{2m} \sin \theta$   
where,

$\phi_{1m}$  = maximum flux generated by electro-magnet A.

$\phi_{2m}$  = Maximum flux generated by electro-magnet B.

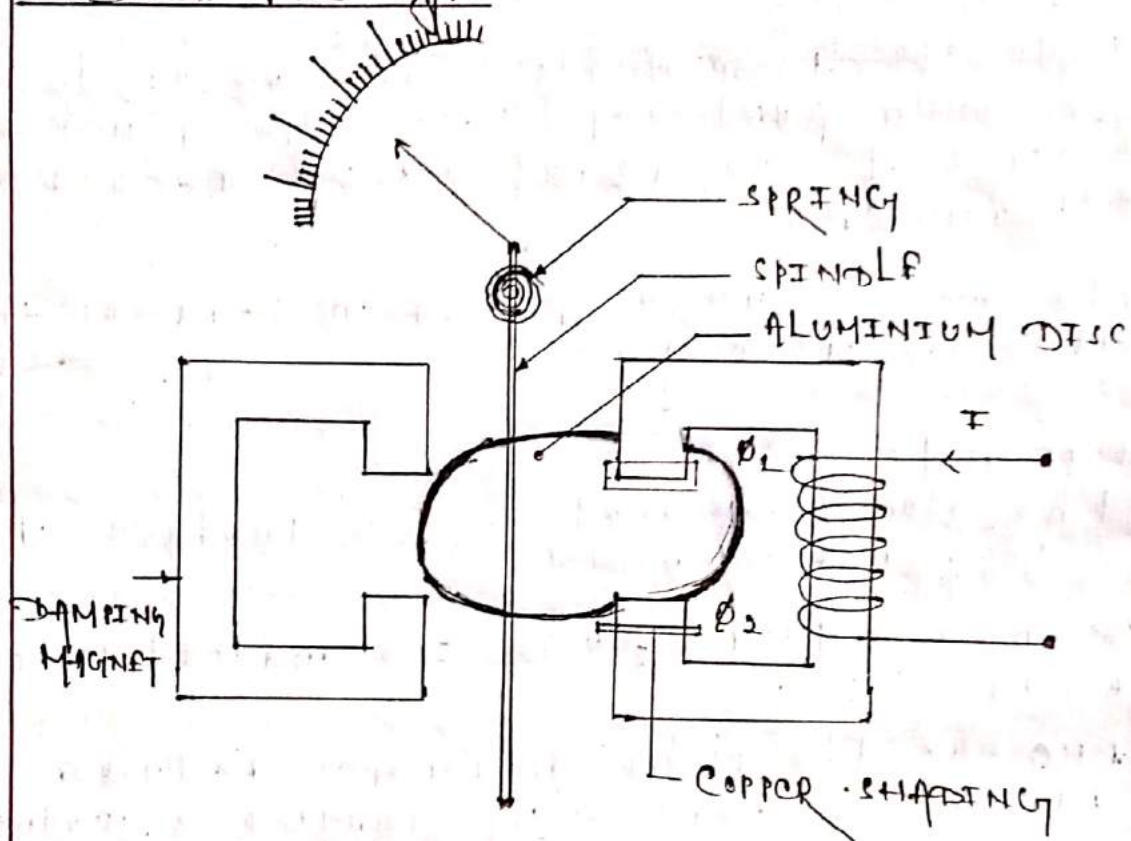
$\theta$  = phase difference between the two fluxes.

- The maximum amount of deflection is produced when the phase difference is  $90^\circ$ .

→ The induction type instrument can be divided into two types.

- (i) shaded pole type
- (ii) split phase type.

(i) shaded pole type :-



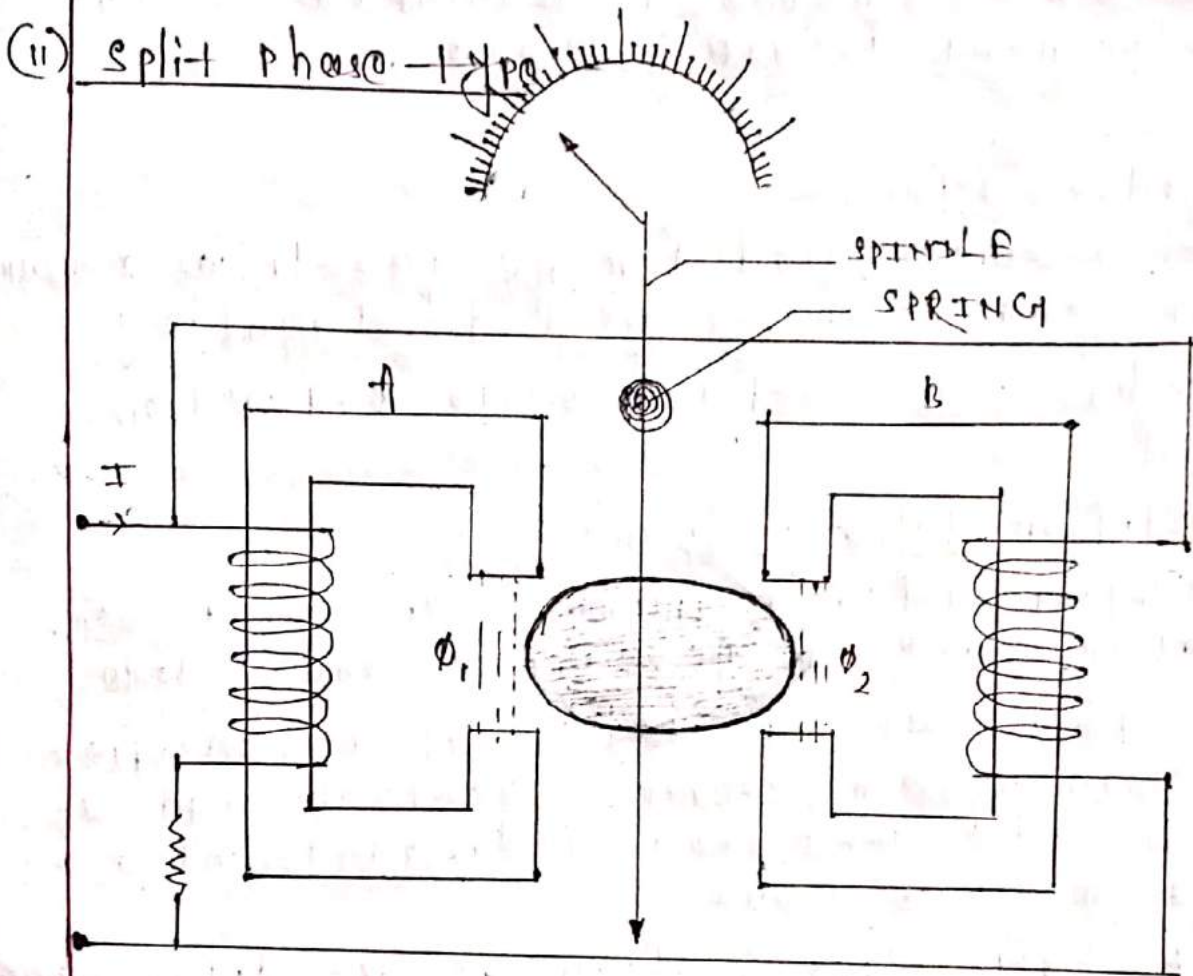
→ It consists of a metal disc which is placed in between the poles of an electro-magnet.

→ one damping magnet is present to absorb the oscillation of the disc.

→ The ends of the electro-magnet are covered with copper shading, so that it will generate flux of difference phase differences.

→ When the electro-magnet winding is excited then two fluxes  $\phi_1$  &  $\phi_2$  are induced on the metal disc & deflecting torque is produced.

→ A spindle is attached to the metal disc which get deflected w.r.t the aluminium disc.



→ In this type of induction type instrument two electro-magnets are present to generate two fluxes  $\phi_1$  &  $\phi_2$ .

→ The aluminium disc is placed in between the two electro-magnets, where the flux  $\phi_1$  &  $\phi_2$  are placed.

→ The deflecting torque is produced on the metal disc, due to the phase difference in  $\phi_1$  &  $\phi_2$ .

→ The spindle attached to the metal plate get deflected w.r.t the metal disc.

Advantages:-

- The scale can be extended over 300°
- Less maintenance is required as the instrument is very simple.

Disadvantages:-

- It is only used for ac quantities measurement and cannot used dc quantity.
- It has non-uniform scale deflection.

Rectifier type instrument:-

- Rectifier type instrument are used for measurement of ac current and voltage.
- In this type of instrument a rectifier is used which converts ac to dc and a PMMC type instrument is used to indicate the measured value.
- Both half wave & full wave rectifier can be used for ac to dc conversion.

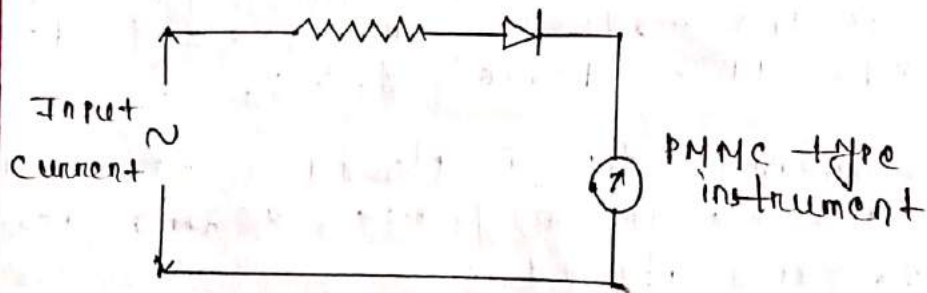


Fig: Rectifier (half wave) type instrument

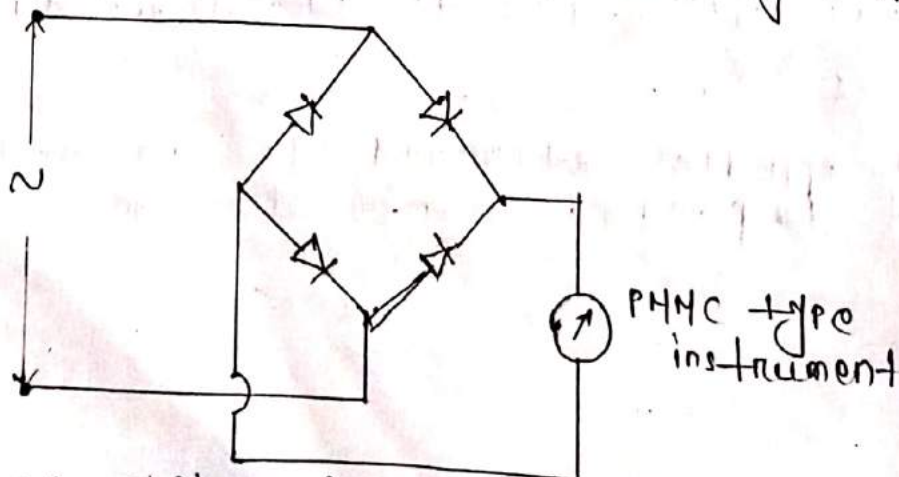


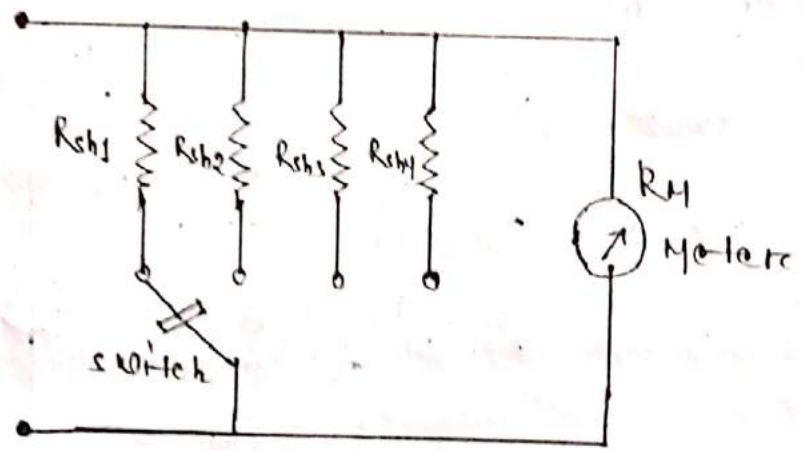
Fig: Rectifier (full wave bridge) type instrument

- PMMC type instrument are the most sensitive instrument than others, but it can only measure dc quantity.
- By connecting resistors to PMMC type instrument, it can be used for ac quantity measurements.

Ammeter shunt:-

Extension of Range of Ammeter:-

- Current range of a dc ammeter may be varied by connecting different values of shunt resistance. This type of dc ammeter where multiple  $R_{sh}$  is present is known as multi range ammeter.



(Fig: Multi range ammeter)

- The above figure shows the diagram of a multi-range ammeter.
- In the above circuit 4 shunt resistances  $R_{sh1}$ ,  $R_{sh2}$ ,  $R_{sh3}$ , &  $R_{sh4}$  are connected in parallel with the meter, which can provide 4 different current ranges  $I_1$ ,  $I_2$ ,  $I_3$  &  $I_4$  respectively.
- Let  $M_1$ ,  $M_2$ ,  $M_3$  &  $M_4$  are the shunt multiplying power for current  $I_1$ ,  $I_2$ ,  $I_3$  &  $I_4$  respectively.

$$m_1 = I_1 / I_m$$

$$m_3 = I_3 / I_m$$

$$m_2 = I_2 / I_m$$

$$m_4 = I_4 / I_m$$

Where,

$I_m$  = full scale deflection current

$$R_{sh1} = \frac{R_m}{(m_1 - 1)}$$

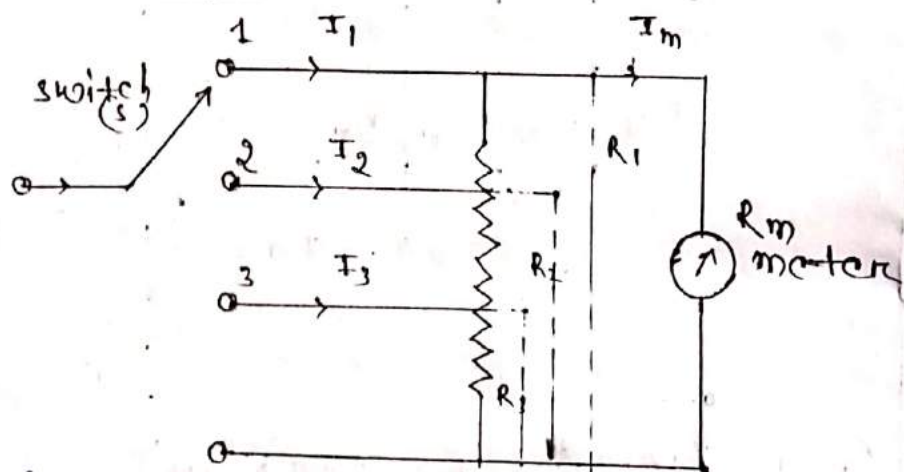
$$R_{sh3} = \frac{R_m}{(m_3 - 1)}$$

$$R_{sh2} = \frac{R_m}{(m_2 - 1)}$$

$$R_{sh4} = \frac{R_m}{(m_4 - 1)}$$

→ This ammeter uses a multi-throw, multi-position make before break switch (S) to select different shunt resistances, this switch is a special switch which is required during the current range changing.

Universal shunt:-



$$I_m R_m = (I_1 - I_m) R_1$$

$$\Rightarrow I_m R_m = I_1 R_1 - I_m R_1$$

$$\Rightarrow I_m R_m + I_m R_1 = I_1 R_1$$

$$\Rightarrow I_m (R_1 + R_m) = I_1 R_1$$

$$\Rightarrow \frac{I_1}{I_m} = \frac{R_1 + R_m}{R_1}$$

$$\Rightarrow m = 1 + \frac{R_m}{R_1}$$

$$\Rightarrow \frac{R_m}{R_1} = m - 1$$

$$\Rightarrow R_1 = \frac{R_m}{m - 1}$$

similarly we can derive  $R_2 = \frac{R_m}{(m_2 - 1)}$

$$R_3 = \frac{R_m}{(m_3 - 1)}$$



### Problem-1

Design a multi-range DC milliammeter using a basic movement with an internal resistance

$R_m = 50 \Omega$  & full scale deflection current

$I_m = I_{mf}$ . The ranges required are 0-10mA,

0-50mA, 0-100mA, 0-500mA.

Given data,

$$R_m = 50 \Omega$$

$$I_m = 1 \text{ mA}$$

0-10mA range

$$I_1 = 10 \text{ mA}$$

$$m_1 = \frac{10}{1} \\ = 10$$

$$R_{sh1} = \frac{R_m}{(m_1 - 1)} = \frac{50}{(10 - 1)} \approx 5.55 \Omega$$

0-100mA range

$$I_3 = 100 \text{ mA}$$

$$m_3 = \frac{100 \text{ mA}}{1 \text{ mA}} = 100$$

$$R_{sh3} = \frac{R_m}{(m_3 - 1)} = \frac{50}{(100 - 1)} \\ = 0.50 \Omega$$

0-50mA range

$$I_2 = 50 \text{ mA}$$

$$m_2 = \frac{50 \text{ mA}}{1 \text{ mA}} = 50$$

~~$R_{sh}$~~

$$R_{sh2} = \frac{R_m}{(m_2 - 1)} = \frac{50}{(50 - 1)} \approx 1.02 \Omega$$

0-500mA range

$$I_4 = 500 \text{ mA}$$

$$m_4 = \frac{500 \text{ mA}}{1 \text{ mA}} = 500$$

$$R_{sh4} = \frac{R_m}{(m_4 - 1)} = \frac{50}{500 - 1} \\ = \frac{50}{499} \\ = \approx 0.1002 \Omega$$

### Problem-2

Design an ignition shunt to provide an ammeter with current ranges 1A, 5A & 10A. A basic meter with an internal resistance with  $50 \Omega$  & a full scale deflection current of 1mA is to be used.

Given data

$$R_m = 50 \Omega$$

$$I_m = 1 \text{ mA}$$

ranges = 1 A, 5 A & 10 A

$$I_1 = 1 \text{ A}, I_2 = 5 \text{ A}, I_3 = 10 \text{ A}$$

Sol<sup>n</sup>

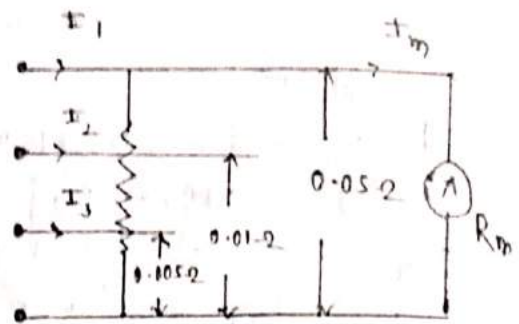
$$m_1 = I_1 / I_m = \frac{1}{10^{-3}} = 1000$$

$$R_{sh1} = \frac{R_m}{(m_1 - 1)} = \frac{50}{999} = 0.05 \Omega$$

$$m_2 = I_2 / I_m = \frac{5}{10^{-3}} = 5000$$

$$R_{sh2} = \frac{R_m}{(m_2 - 1)} = \frac{50}{4999} = 0.01 \Omega$$

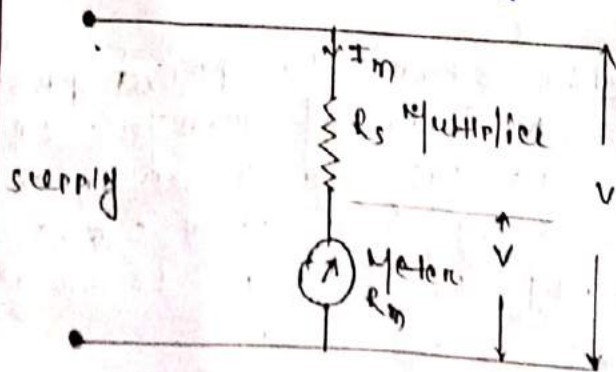
$$R_{sh3} = \frac{R_m}{(m_3 - 1)} = \frac{50}{9999} = 5 \times 10^{-3} = 0.005 \Omega$$



Universal shunt is also known as Ayrton shunt.

### Voltmeter Multiplier:-

- To convert a basic meter movement into voltmeter a series resistance is connected with it. This series resistance is known as multiplier.
- The multiplier limits the current through the meter so that it does not exceed the value of full-scale deflection current and prevents it from getting damage.



In the above diagram a series multiplier  $R_s$  is connected with the meter to extend the voltage range.

Let,

$I_m$  = full scale deflection current of the meter

$R_m$  = Meter internal resistance

$R_s$  = Multiplier series resistance

$V$  = voltage across the meter movement due to current ( $I_m$ ).

$V$  = full range voltage of the instrument after connecting  $R_s$ .

$$V = I_m R_m$$

$$V = I_m (R_s + R_m)$$

$$\Rightarrow \frac{V}{I_m} - R_m = R_s$$

$$\Rightarrow R_s = \frac{V}{I_m} - R_m$$

The multiplying factor for the multiplier

$$m = \frac{V}{V}$$

$$m = \frac{I_m (R_s + R_m)}{I_m R_m}$$

$$\Rightarrow m = \frac{R_s + R_m}{R_m}$$

$$\Rightarrow m = \frac{R_s}{R_m} + 1$$

$$\Rightarrow R_s = (m-1) R_m$$

The multiplier resistance should not change with time and should have small temperature coefficient.

Note :-

Generally manganin & constantan are taken to construct shunt & multipliers.

## Multi-range voltmeter:-

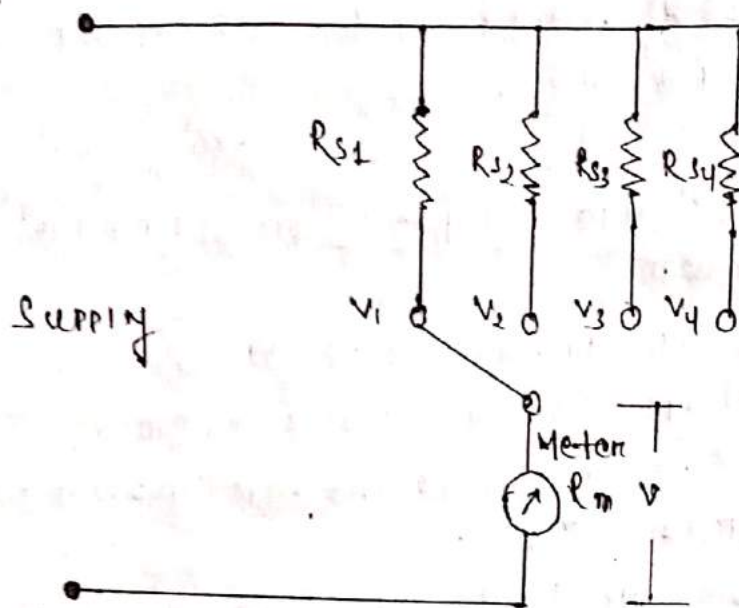


Fig: Multi-range voltmeter

- In multi-range voltmeter, different voltage ranges may be obtained by connecting different no. of multiplier resistor or potential divider arrangement.
- In the above diagram 4 no. of individual multiplier resistors are connected in series with the meter to get different voltage ranges such as  $V_1, V_2, V_3$  &  $V_4$ .
- If  $R_{s1}, R_{s2}, R_{s3}$  &  $R_{s4}$  are the multiplier resistances then,
 
$$R_{s1} = (m_1 - 1) R_m \quad R_{s2} = (m_2 - 1) R_m$$

$$R_{s3} = (m_3 - 1) R_m, \quad R_{s4} = (m_4 - 1) R_m$$
 where,
 
$$m_1 = \frac{V_1}{V}, \quad m_2 = \frac{V_2}{V}, \quad m_3 = \frac{V_3}{V}, \quad m_4 = \frac{V_4}{V}$$
- Voltmeter range can be varied to multiple values also by connecting load to a potential divider arrangement.

## potential divider arrangement:-

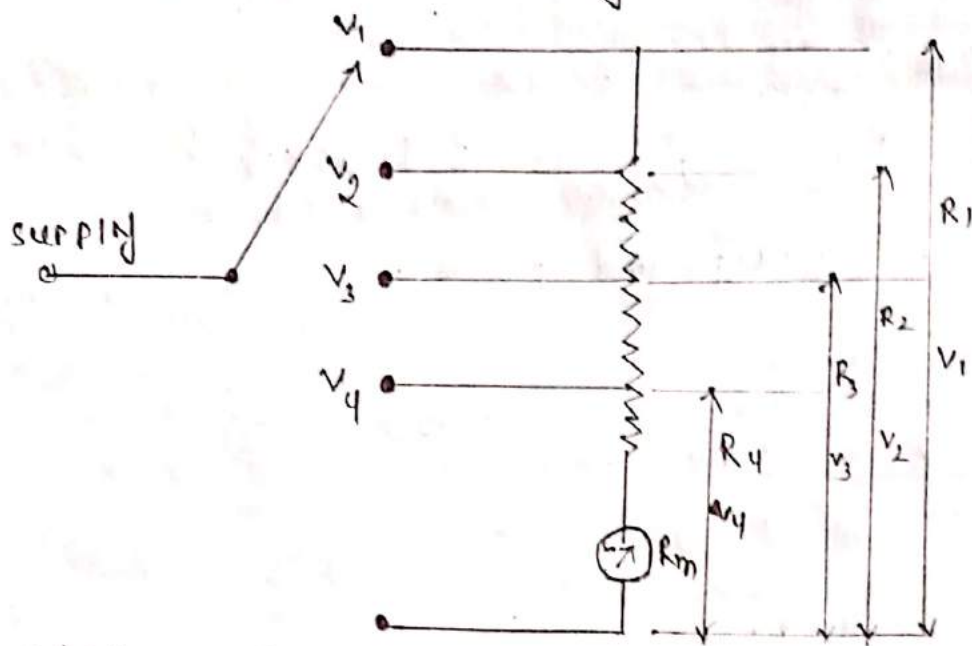


Fig: Multi range voltmeter using potential divider arrangement.

$$R_1 = (m_2 - 1)R_m, \quad R_2 = (m_2 - m_1)R_m$$

$$R_3 = (m_3 - m_2)R_m, \quad R_4 = (m_4 - m_3)R_m$$

### Problem:-

A moving coil instrument gives a full-scale deflection of  $10\text{mA}$ , where the potential difference is terminal  $100\text{mV}$ , calculate

- the shunt resistance for full scale deflection corresponding to  $100\text{Amp}$ .
- the series resistance for full scale reading with  $1000\text{V}$ .

Also calculate the power dissipation in each case.

Given data,

$$I_m = 10\text{mA}$$

$$V = 100\text{mV}$$

$$R_m = \frac{V}{I_m} = 10\Omega$$

$$(a) \quad I = 100 \text{ A}$$

$$I / I_m = \frac{100}{10 \times 10^{-3}} = 10,000$$

$$R_{sh} = \frac{R_m}{m-1} = \frac{10}{(10,000-1)} \approx 1,000 \times 10^{-3}$$

power dissipated  $P = V \times I = 100 \times 100 = 10000$

$$(b) \quad V = 1000 \text{ volt}$$

$$m = \frac{V}{v} = \frac{1000}{100 \times 10^{-3}} = 10,000$$

$$R_s = R_m(m-1) \\ = 10 \times (10,000-1) \\ = 99990 \Omega$$

$$P = V \times I = 1000 \times 10 = 10000$$

# CHAPTER 3

## WATTMETER MEASUREMENT OF POWER

Wattmeter is an instrument which is used to measure the power developed in a circuit across a load.

### Electrodynamometer type Wattmeter:-

Electrodynamometer type wattmeter design is similar to the electrodynamometer instrument which is used for the measurement of current and voltage.

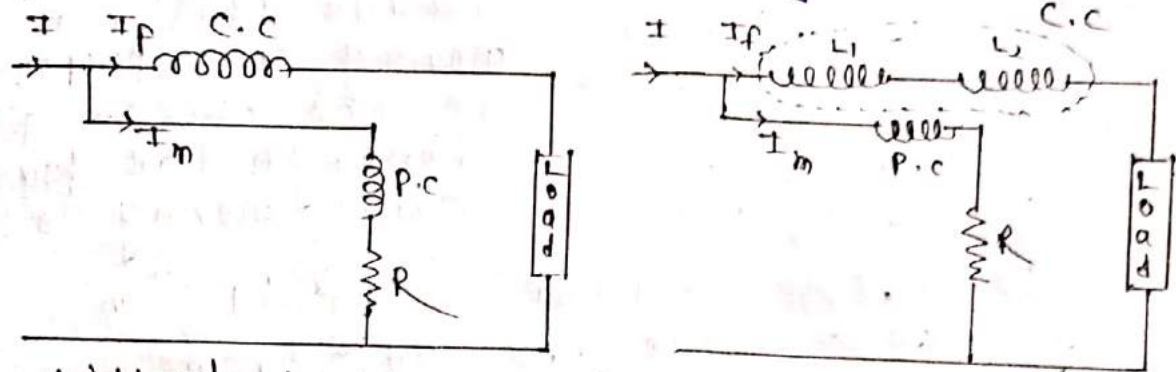


Fig: Electrodynamometer type wattmeter.

→ The electrodynamometer type wattmeter which is used for measurement of power, consists of 2 coils which are:-

1. Fixed coil (current coil)
2. Moving coil (pressure coil)

### Fixed coil:-

→ The fixed coil is connected in series with the load and carry the current in the circuit. therefore it is called as current coil or current coil of wattmeter.

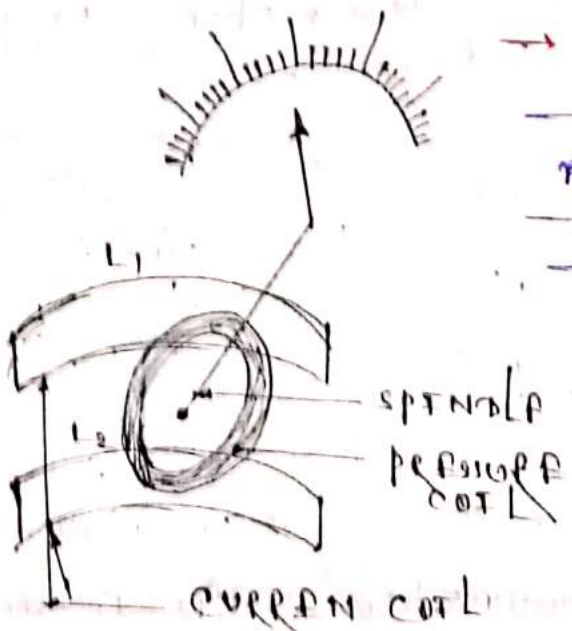
→ This fixed coil is divided into two halves.

→ The wires present in this coil are laminated & are very heavy wires which can carry a large amount of current.

Moving coil:-

the moving coil is connected across the load, so that the current proportional to the voltage drop can flow through the coil. since the current proportional to the voltage is flowing through it, it is also called voltage coil or pressure coil.

the moving coil is connected mounted on a spindle, so that it can move freely.



the deflecting torque develop in the moving coil is proportional to the current flowing through the fixed coil & moving coil.

$$T_d \propto I_f \cdot I_m$$

$$T_d \propto I_f \frac{V}{R_m}$$

$$T_d \propto \text{power}$$

If ' $\phi$ ' is the angle between the voltage & current in the current coil show we can say that current lags the voltage by an angle ' $\phi$ '.

if 'M' mutual inductance between the coils, then we write

$$T_d = \frac{VI_f \cos \phi}{R_m} \frac{dM}{d\phi}$$



## Errors in dynamometer type instrument:-

- The deflection of the pointer is dependent on the power factor of the load, so the pointer deflection may vary produce an error while measuring low power factor loads.
- So another type of arrangement is made to measure this type of load powers which is known as LPF wattmeter.

## Working:-

- When the current passes through the fixed coil & moving coil, then due to the interaction of generated magnetic field, the moving coil turns about its axis.
- The pointer is connected to the spindle of moving coil which also get deflected due to the moving coil moment.
- Spring controls are connected to the moving coil spindle to provide the controlling torque.
- Air friction damping is used to absorb the oscillation of the instrument.

## Note:-

This dynamometer type wattmeter is used for measurement of power in circuit having high power factor.

## Low power factor type dynamometer wattmeter:-

- ordinary electro-dynamometer wattmeter can't measure the power in circuit, having low power factor without any error.
- This happens because the deflecting torque in the moving system is small for low power factor and error introduced in the pressure coil is very large at low power factor.

→ so some special features are incorporated in an electrodynamic type wattmeter to make it low power factor type. These features are: -

(i) Pressure coil current: -

→ The pressure coil current in low power factor wattmeter is generally 40 times the value employed for high power factor wattmeter, so resistance of the pressure coil is decreased to allow the current flow through it.

→ If the pressure coil current will increase than more amount of deflecting torque is produced.

(ii) Compensation of pressure current coil: -

The power loss in the pressure coil is compensated by connecting a compensating coil with the pressure coil in series.

(iii) Compensation for inductance of pressure coil: -

→ Due to low power factor, the value of phase angle  $\phi$  is large and it gives high error.

→ This error can be compensated by connecting a capacitor across a part of series resistance in pressure coil.

(iv) Low power factor type wattmeters are design with small controlling torque: -

By incorporating the above features the low power factor type wattmeter can be presented as follows.

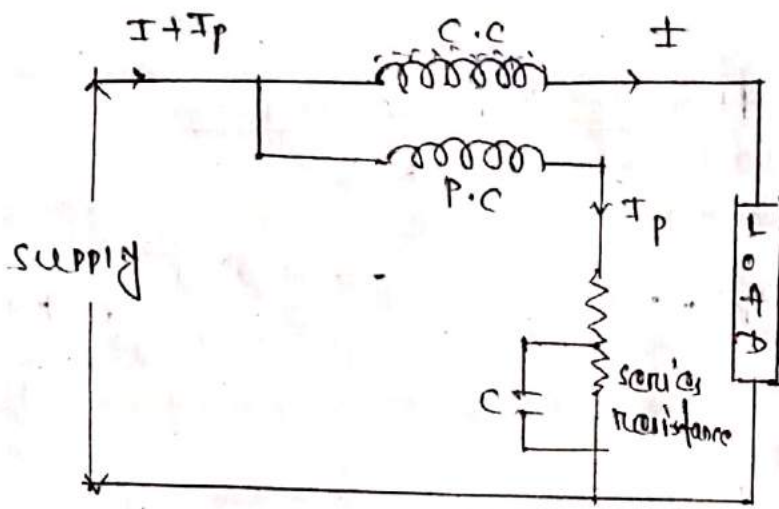
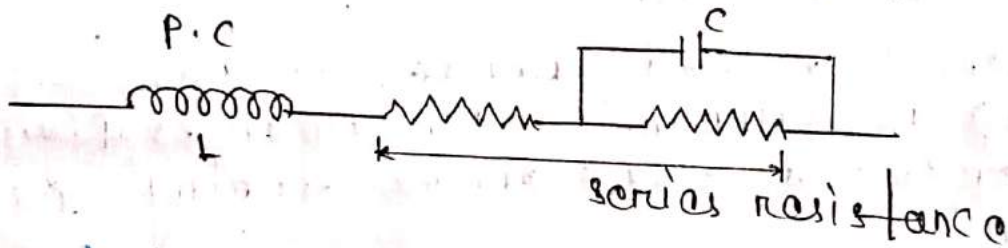


Fig: L.P.F. type dynamometer wattmeter

### Errors in dynamometer type instrument:-

#### (i) Error due to pressure coil inductance:-

- Due to the pressure coil inductance the wattmeter reading can vary while measuring low power factor (lagging power factor).
- This error can be compensated by means of connecting capacitor in parallel with a portion of pressure coil series resistance.



#### (ii) Error due to pressure coil capacitance:-

- The pressure coil circuit may possess capacitance which affects the wattmeter reading to some extent, this effect is not present by measuring at low frequency but its effect increases with increase in frequency.

### (iii) Error caused due to connection!:-

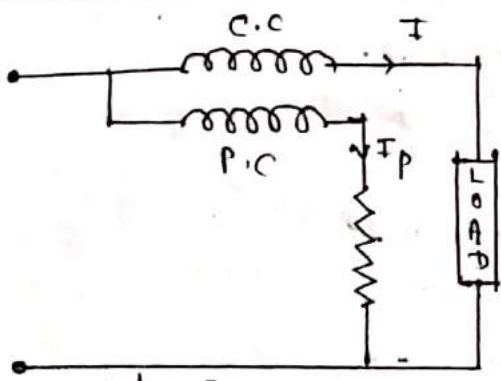


Fig (a)

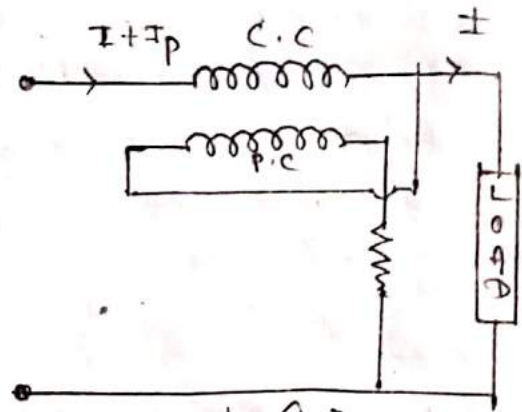
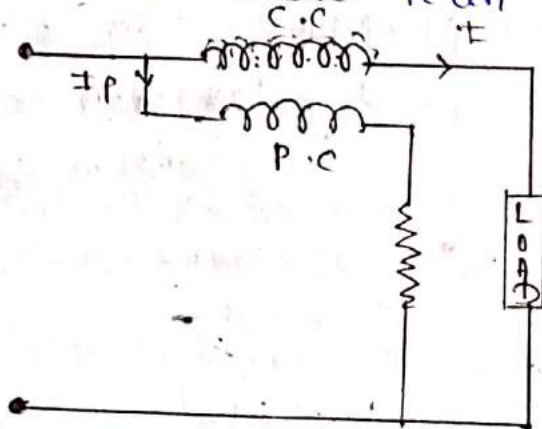


Fig (b)

- Wattmeter can be connected two alternating ways as shown in the figure a & b
- In fig(a) the pressure coil is connected in the supply side, so the voltage drop across the pressure coil is equal to the voltage drop across the load & voltage drop across the current coil.
- so the power indicated by the wattmeter = power consumed by the load + power loss in current coil.
- when the load current is small, the voltage drop in current coil is small so figure (a) introduced small error during small load current.
- In fig(b) the coil current coil carry the current for pressure coil & for the load. so the wattmeter reads the power consumed in the load + power loss in the pressure coil.
- power indicated by the wattmeter = power consumed by load + power loss in pressure coil.
- so when the load current is high but the pressure coil current is less than fig(b) arrangement is used.

→ In fig (b) current coil carries  $(I + I_p)$ , so the magnetic field corresponding to this current is also produced. To compensate this magnetic field a compensating coil is connected with series in the pressure coil which is may identical with the coil.

→ After using the compensating coil the fig (b) can be read run as .



#### (iv) Eddy current error:-

→ Eddy currents are induced in the solid metal parts of the instrument & also within the conductor present in the instrument.

→ This current produce its own field and alter the magnitude & phase of the current coil & pressure coil field, which cause error in the instrument.

#### (v) Stray magnetic field error:-

→ The electro dynamometer wattmeter has a relatively weak magnetic field which is responsible for the generation of  $T_d$ .

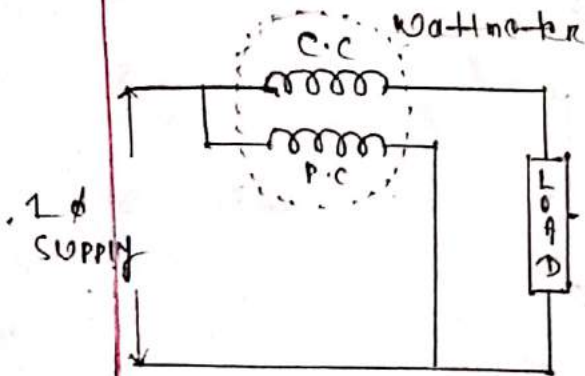
→ This magnetic field is affected by the outside stray magnetic field & causes error.

→ This error can be reduced by proper shielding of the instrument against the stray magnetic field.

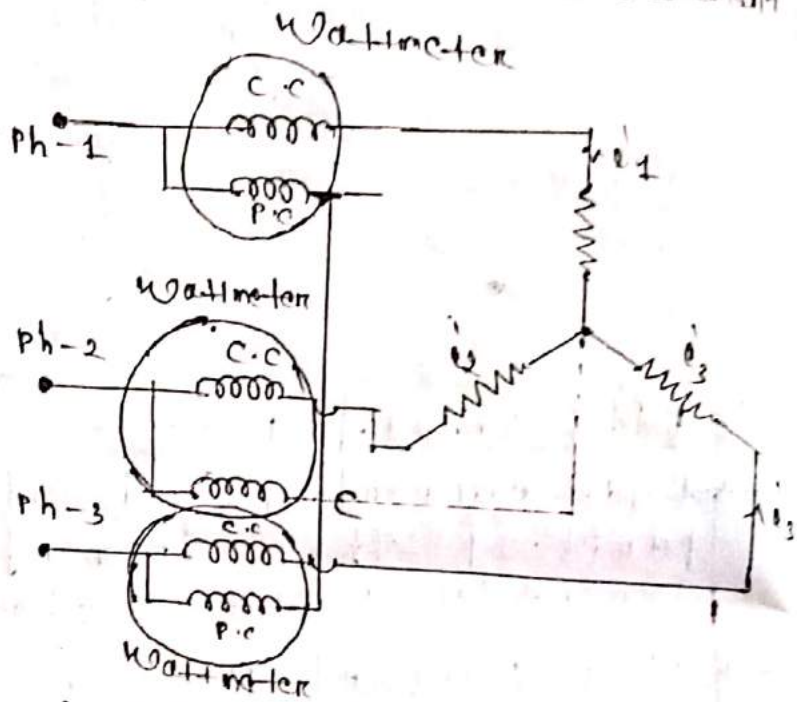
(vi) Temperature Error:-

- The indication of the wattmeter is affected by changes in room temperature.
- This happens because any change in room temperature changes the stiffness of the spring & resistance of the pressure coil.
- To avoid this error low resistance temperature coefficient material should be taken to prepare the spring & resistor.

1- $\phi$  load power measurement

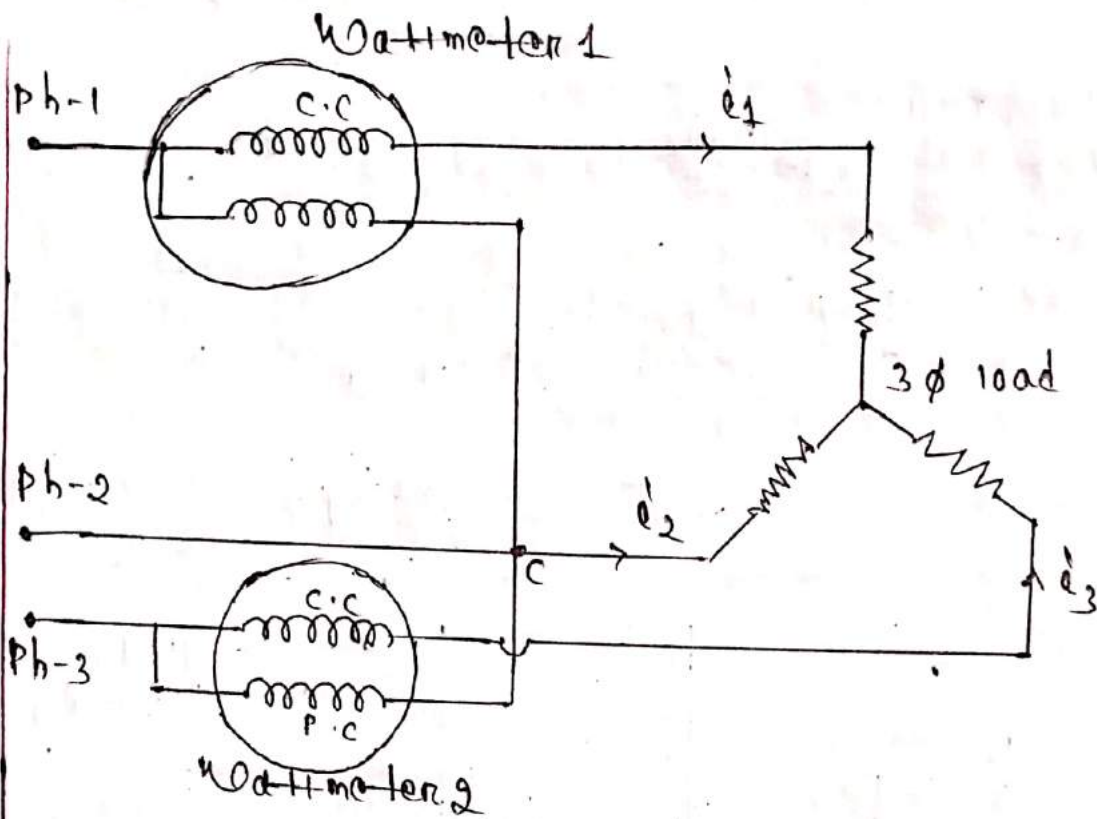


3- $\phi$  load power measurement



3-wattmeter connection

- The pressure coil of all the 3 wattmeter are connected to a common point 'c'.
- The current coils of the wattmeters are connected to the phase line. If each wattmeter read power then the total power consume by the load can be given by  $P = w_1 + w_2 + w_3$



2 wattmeter connection.

→ According to Blondel's theorem, if the common point of the pressure coil is connected to one of the load lines then we will require  $n-1$  no. of wattmeters for the measurement of power.

Where,

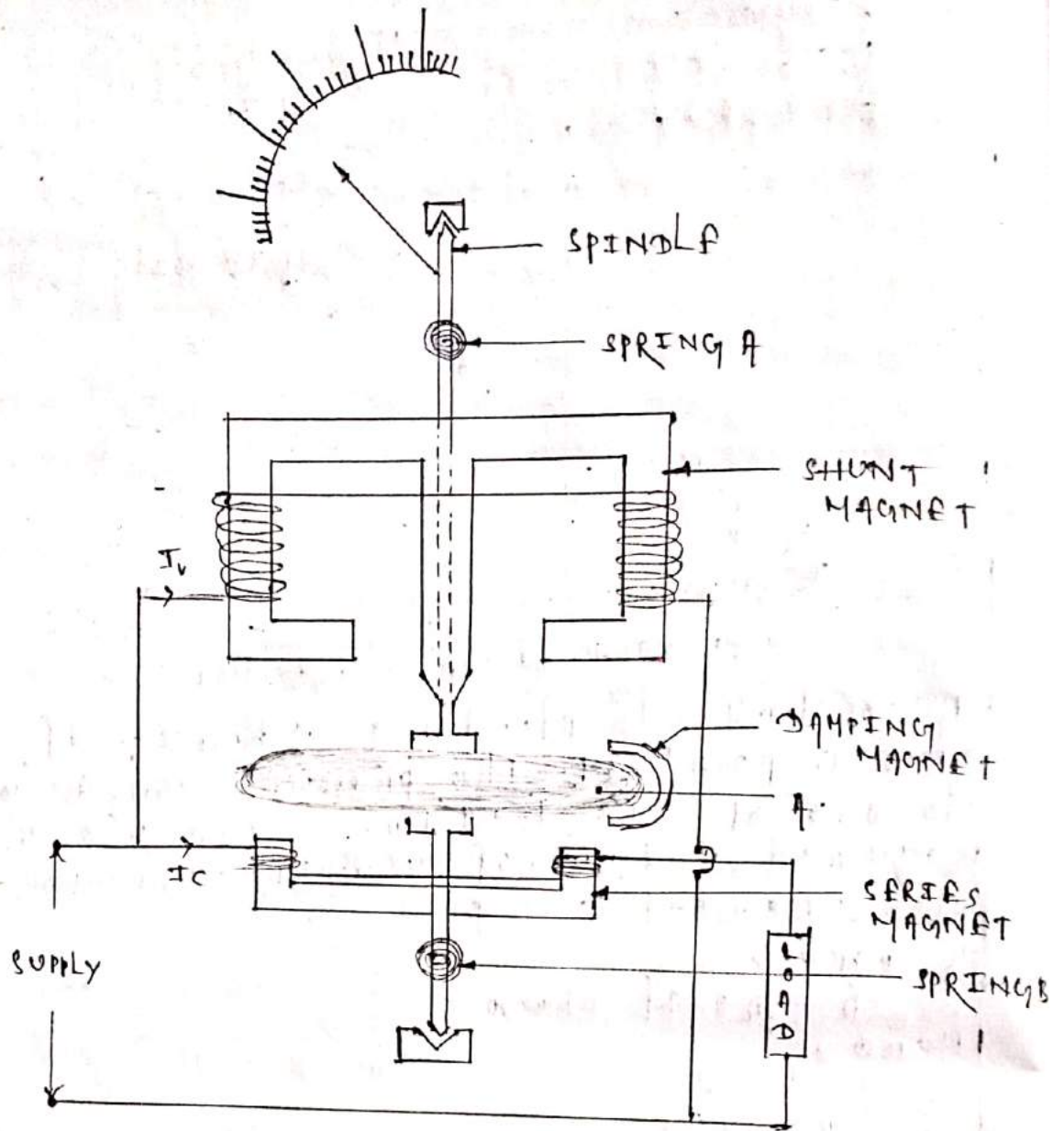
$n = \text{no. of phase}$

Here,

$$n - 1 = 2$$

$$P = W_1 + W_2$$

Induction type wattmeter:-



- The induction type wattmeter is used to measure only the ac power consumed by the load.
- It works on the same principle of as a induction type voltmeter & ammeter instrument.

### Construction:-

- The induction type wattmeter consists of two laminated electro-magnets, known as shunt magnet or series magnet.
- The series magnet is connected in series with the load and is excited by the load current.



- The shunt magnet is connected across the load & is excited by the current proportional to voltage across the load.
- An aluminium disc is mounted in between the two electro-magnets in such a way that if cuts the fluxes produced by both the magnets.
- Spring  $\eta$  &  $k$  are connected to the spindle to provide the controlling torque.
- A damping magnet can be connected to provide the damping torque.

### Working:-

- The two fluxes generated by the electro-magnet induce & eddy current in the aluminium disc.
- Due to the interaction between the fluxes & eddy current field a deflecting torque is produced on the disc.
- Due to the deflecting torque the spindle also rotates along with the aluminium disc & moves the pointer over the scale.
- The deflecting torque produced  $T_d \propto \cos \theta$  = power factor of the load  
 $V$  = supply voltage  
 $I$  = circuit current

### Advantages:-

- The damping torque is produced is very effective.
- The induction type instrument has a very large scale range.

### Disadvantages:-

- Change in atmospheric temperature can vary the resistance of the moving aluminium disc, which affects the deflecting torque.

## Chapter - 4

### Energy Meter & Measurement of energy

Energy is defined as the power consumed by the load over a particular interval of time.

$$\text{Energy} = \text{power} \times \text{time}$$

$$E = P \times t$$

$$E = \int_0^t P dt$$

$$E = \int_0^t V I dt$$

→ If the voltage is measured in voltage & current 'I' is measured in ampere-time 't' in second, then the energy consumed can be expressed in watt-second unit.

→ 1 watt-second = 1 Joule

→ If the time interval 't' is in hour then the energy can be expressed, watt-hour unit.

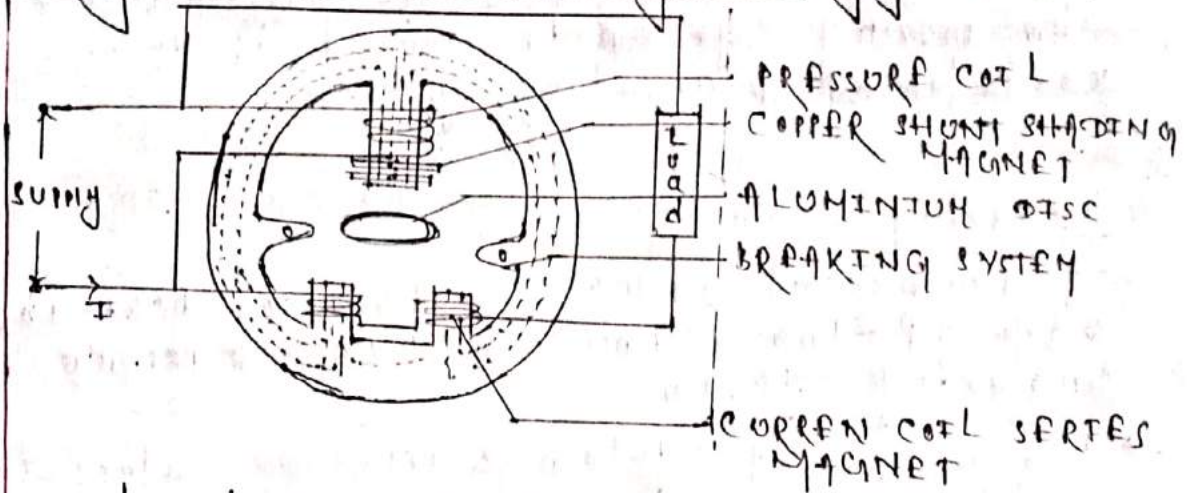
→ 1000 watt-hour is also called as 1 unit in domestic energy meters.

$$\Rightarrow 1 \text{ kWh} = 1 \text{ unit}$$

→ Energy meter is the device or instrument which is used to measure the energy consumed by the load.

→ Generally induction type energy meters are universally used for measurement of energy in domestic & industrial AC circuit.

## single phase induction type energy meter:-



### Construction:-

- There are 4 main parts in the energy meter system which are:-
1. Driving system
  2. Moving system
  3. Braking system
  4. ~~Rest~~ Registering or counting system.

#### (i) Driving system:-

- The driving system consists of 2 electro-magnets.
- One of the coil or electro-magnet is excited by the load current & is called as current coil or series magnet.
- Another coil or electro-magnet is excited by the current proportional to the voltage across the load. This coil is connected across the load & is known as pressure coil or shunt magnet.
- Copper shadings may be provided to adjust the flux produced by the shunt magnet.

#### (ii) Moving system:-

- Moving system consists of an aluminium disc mounted on a alloy shaft. This disc is present within the air gap between the series & shunt magnet.

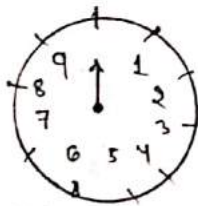
→ A pinion is connected to the shaft on the moving system to connect it to the registering or counting system.

### (iii) Braking system:-

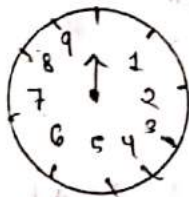
- A permanent magnet is positioned near the edge of the aluminium disc to provide the braking system.
- The braking system is used to control the speed of the moving system.
- By adjusting the position of permanent magnet braking torque can be adjusted.

### (iv) Registering / counting system:-

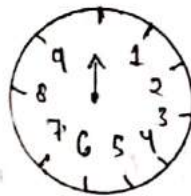
100kWh



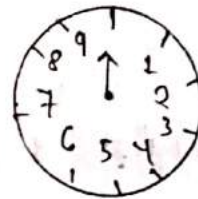
10kWh



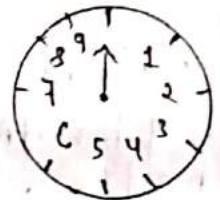
1kWh



1/10 kWh

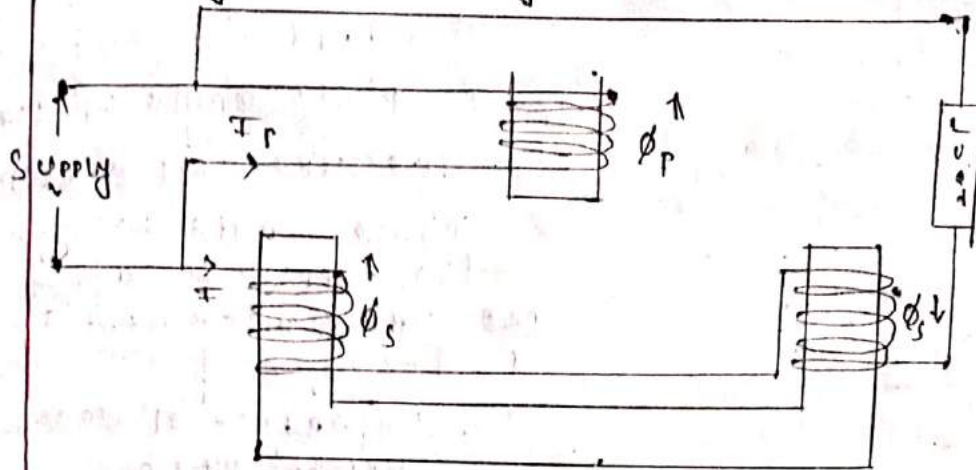


1/100 kWh

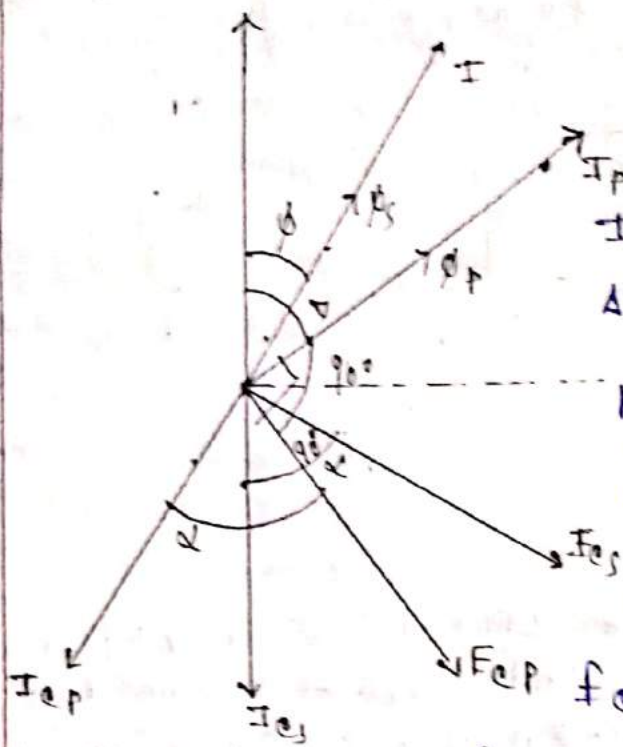


- The function of a registering & counting system is to record continuously a number which is proportional to the revolution of the moving system.
- By using a series of reduction gears the pinion of the shaft drives 5 or 6 no. of pointers to display the value.
- The above registering gear arrangement is also called as cyclo-meter register.

## Theory & operating principle:-



- The above diagram shows the functional driving system of induction-type meter.
- In the above diagram we can see the supply voltage is applied across the pressure coil.
- The pressure winding is highly inductive as it has very large no. of turns.
- $I_p$  current flows in the pressure coil which is proportional to the supply voltage & this current lags the voltage by few degrees less than  $90^\circ$ .
- This  $I_p$  current produces a flux  $\phi_p$  which goes across the aluminium disc & responsible for the production of driving torque.
- $\phi_p$  is in phase with  $I_p$  & the value is proportional to the current  $I_p$ .
- The load current ' $I$ ' flow through the current coil & produces flux  $\phi_s$ . This  $\phi_s$  flux is proportional to load current ' $I$ ' & is in phase with it.
- $\phi_p$  produces an eddy current  $I_{ep}$  &  $\phi_s$  produces an eddy current  $I_{es}$  in the aluminium disc.
- $\phi_s$  interacts with  $I_{ep}$  eddy current &  $\phi_p$  interacts with  $I_{es}$  two produce to different torques. The net torque is the difference between the above two maintain torques.



$V$  = supply voltage  
 $I$  = load current  
 $\phi$  = phase angle of load  
 $I_p$  = pressure coil current  
 $\Delta$  = phase angle between the supply voltage & pressure coil current  $I_p$   
 $f$  = frequency  
 $Z$  = impedance of eddy current path  
 $\alpha$  = phase angle of eddy current paths.  
 $e_{ep}$  = emf induced due to flux  $\phi_p$   
 $e_{es}$  = emf induced due to flux  $\phi_s$

$E_{ep} = V \sin \Delta$ ,  $I_{ep}$  = Eddy current due to flux  $\phi_p$   
 $I_{es}$  = Eddy current due to flux  $\phi_s$

Fig. 1. Phasor diagram of single phase induction type energy meter.

→ The total torque is difference between two torques which can be given by

$$T_d = d \phi_p \phi_s \frac{f}{2} \sin \beta \cos \alpha \quad (\because \text{where } \beta \text{ is angle between two fluxes})$$

$$\Rightarrow T_d = d \phi_p \phi_s \frac{f}{2} \sin (\Delta - \phi) \cos \alpha$$

$$\Rightarrow T_d = K_1 \phi_p \phi_s \frac{f}{2} \sin (\Delta - \phi) \cos \alpha$$

Again  $V \sin \Delta = I \cos \phi$

$$T_d = K_2 V I \frac{f}{2} \sin (\Delta - \phi) \cos \alpha$$

→ Then if for a particular instrument  $f, Z$  &  $\alpha$  are constant, so we can write

$$T_d = K_3 V I \sin (\Delta - \phi)$$

If  $\Delta = 90^\circ$

$$T_d = K_3 V I \sin (90 - \phi)$$

$$T_d = K_3 V I \cos \phi$$

$$\Rightarrow \boxed{T_d = K_3 V \text{ Power}} \quad \text{--- eqn (1)}$$

→ If 'N' is the steady speed then the braking torque can be given by.

$$T_B = K_4 \times N \quad \text{--- eqn (ii)}$$

→ At steady speed the driving torque must be equal to the braking torque.

$$T_d = T_B$$

$$\Rightarrow K_3 \times \text{power} = K_4 \times N$$

$$\Rightarrow N = \frac{K_3}{K_4} \times \text{power}$$

$$\Rightarrow N = K \times \text{power}$$

→ The total no. of revolution during a particular time interval can be given by,

$$\int N dt = \int K \times \text{power} dt$$

$$\Rightarrow \int N dt = K \int (\text{power}) dt$$

$$\Rightarrow \int N dt = K \times \text{energy}$$

→ total no. of revolution of the aluminium disc & energy consumed by the load.

Lag adjustment of energy meter device!

→ We have assume  $\Delta = 90^\circ$ , so that speed of rotation 'N' will be proportional to power.

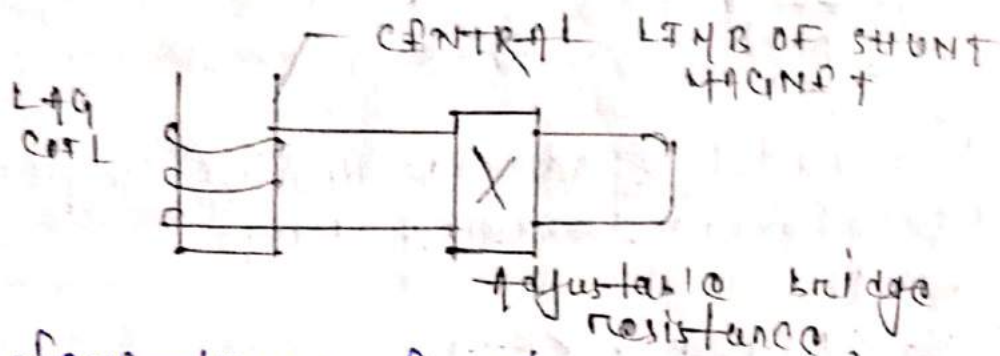
→ In this condition  $\phi$  lags the supply voltage 'V' by  $90^\circ$ .

→ To achieve this the pressure coil winding should be design that it is highly inductive & has a low resistance.

→ This can be obtained by interducing a lag coil which is located on the central limb of the shunt magnet.

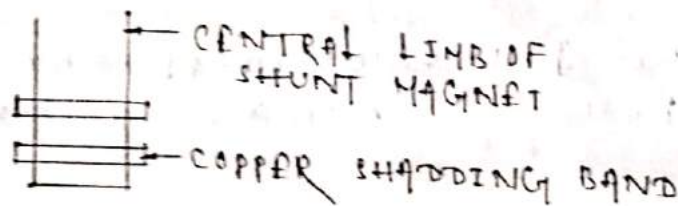
→ The mmf of the lag coil can be exhausted by the making following arrangements.

## (i) Adjustable Resistance:-



- A few turns of wires are placed around the central limb of the shunt magnet and the circuit is closed through a low adjustable bridge resistance.
- This resistance value can be altered to adjust the phase angle of flux of wmt supply voltage  $V$ .

## (ii) Shading Bands:-



- In this arrangement copper shading bands are placed around the central limb of shunt magnet instead of lag coil.
- The adjustment can be done by moving the shading band along the axis of the limb.
- As the shading bands are move up the limb it can provide more amount of flux.
- By the adjusting the position of shading band the phase angle can be made approximately equals to  $90^\circ$ .

### Note:-

This lag adjustment is also known as power factor adjustment, inductive lag adjustment & quadrature adjustment.

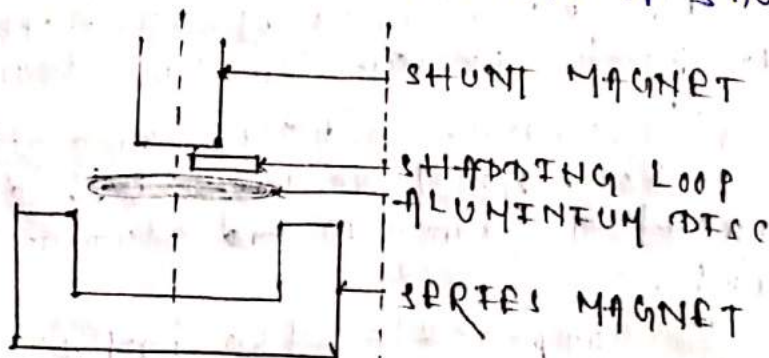


## Energy compensations:

- 1- Light load compensation / friction
- 2- creep compensation
- 3- overload compensation
- 4- temperature compensation
- 5- voltage compensation.

### (a) Light load compensation:-

- The jewelled pivot-bearings for the spindle provides sum-friction to the movement of aluminium disc.
- During low load & light load supply we need to generate an extra small torque to overcome this friction.
- This is because during light load very small amount of driving torque is generated which is not sufficient to overcome the friction & move the aluminium disc.
- To compensate this light load condition small shading pole or loop is added in between the centre of the pole of the shunt magnet & the disc.
- This shading loop is slightly placed towards one side of the centre line of shunt magnet.



- The interaction between the portion of shaded & unshaded flux & the current induced in the disc generates a torque which can be used for friction compensation.

### (b) Creep compensation!

- In sum meters a slow rotation of the disc is obtained even when there is no current flowing through the current coil. This is known as creeping.
- In order to prevent this creeping to diametrical holes are drilled in the disc.
- The disc comes to rest when one of the holes is under the edge of the pole.

### (c) Overload compensation!

- During overload condition to compensate the driving torque which is proportional to breaking torque an extra magnetic shunt are used in the device.
- The magnetic shunt approaches & saturate & divert the series magnetic flux to the disc air gap. Due to this action the driving torque during the overload condition can be controlled.

### (d) Temperature compensation!

- An increase in atmospheric temperature can increase the resistance of all copper & aluminium parts present in the device.
- Due to this change in resistance the lag between the supply voltage 'V' & 'I' changes, also varies.
- In order to compensate the impact of increase in temperature the meter should be installed with proper shielding.
- A special material metemp can be used to make different parts of the device which is very less sensitive to temperature.

## (e) Voltage Compensation!

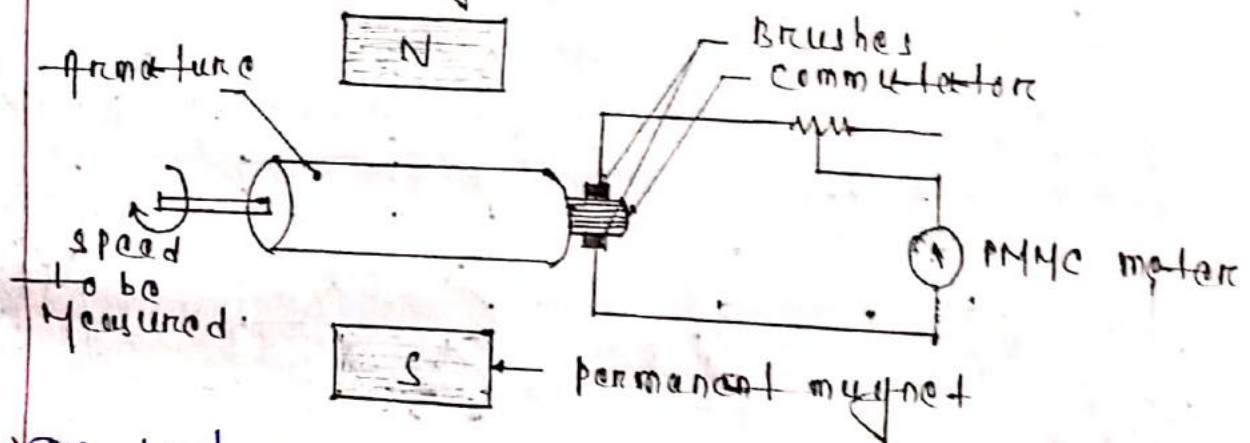
Compensation for voltage variation can be provided by using magnetic shunt to divert the flux into the disc airgap & to the active part if it is required.

## chapter-5

### TACHOMETER

- Tachometer is an equipment which is used for the speed measurement of any device.
- It is classified into 2 types.
  - 1- DC tachometer generator
  - 2- AC tachometer generator.

#### (A) DC Tachometer generator!

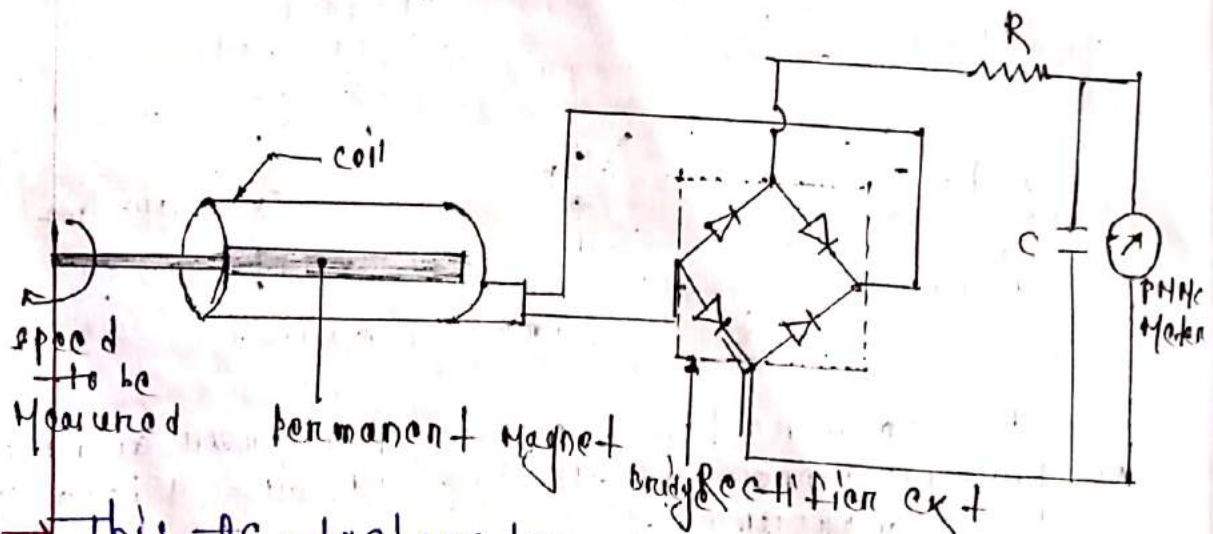


- DC tachogenerator consists of a small armature which is connected to the machine whose speed is to be measured.
- This armature coil revolves within the permanent magnetic field.
- Due to the rotation of armature within the magnetic field an emf is induced in the armature coil.
- This induced emf is proportional to flux & speed of rotation.
- Since for this permanent magnetic field flux is constant, so the generator voltage is proportional to speed of rotation.

- Induced emf can be collected by the help of commutator & brush arrangements.
- This voltage can be measured by using a PMMC type meter which can be calibrated with speed.

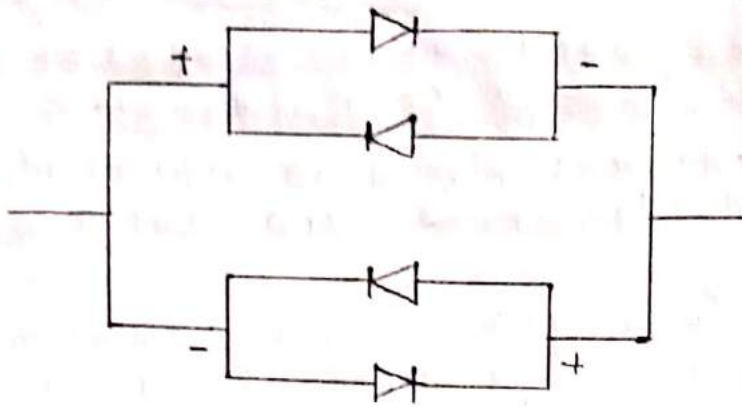
### AC tachometer generator:-

- The AC tachometer generator contain rotating magnet which can be either an permanent magnet & electro-magnet.
- The AC tachometer generator generally generate AC voltage with the measured speed.
- The diagram of AC tachometer generator is shown below.



- This AC tachometer generator generates output AC voltage which can be converted to equivalent DC voltage with the help of rectifier circuit.
- A PMMC type meter can be used to indicate this generated voltage which can be calibrated with measured speed.

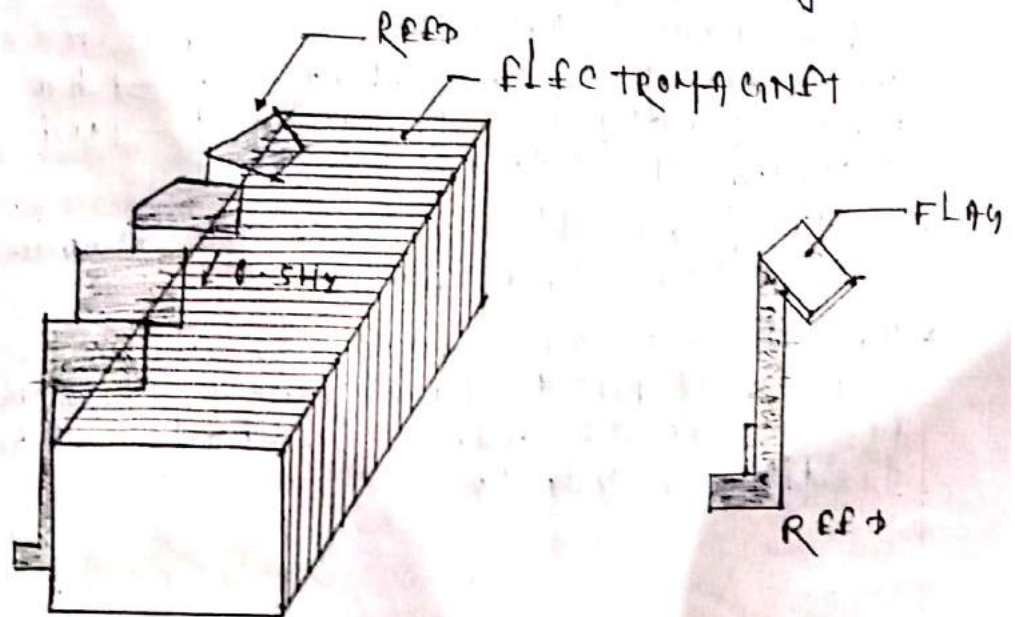
(Fig: AC tachometer generator)



- so we can easily identified the direction of speed using dc tachometer.
- dc tachometer requires regular maintenance as the carbon brushes used in it, easily get damage.
- ac tachometer generator requires very less maintenance as compare to dc tachometer generator.

### Frequency Meter:-

- frequency meter is a device which is used to measure the frequency of supplied input signal.
- Here we are going to discuss 2 type of frequency meter devices.
  1. Mechanical resonance type of frequency meter
  2. Electrical resonance type of frequency meter.



(Fig: vibrating reed type frequency meter)

- This frequency meter consists of a no. of thin steel strips known as reeds.
- These reeds are placed in a row along side of an electromagnet as shown in the above fig.
- The electromagnet has a laminated iron pole & its coil is connected across the supply whose frequency is to be measured.
- The reeds are sliding in direction of vibration & carry direction weight on their tongue known as flag.
- The natural frequency of vibration of the reeds depends on their weight & dimension.
- These reeds are arranged in ascending order of their natural frequency with a difference between two adjacent reeds natural frequency of 0.5 Hz.
- The reeds are fixed at the bottom & free to move towards the tongue.

#### Working:-

- When the frequency meter is connected across the supply the coil of the electromagnet carries a current with respect to the supply frequency.
- The force exerted in each reed varies in every half cycle. So the reed which has natural frequency twice of the supply frequency is satisfied the resonance condition & vibrates the most.
- All the reeds tend to vibrate due to the supplied frequency. The reed which is resonance with the supply frequency vibrates the most & is distinctly visible.

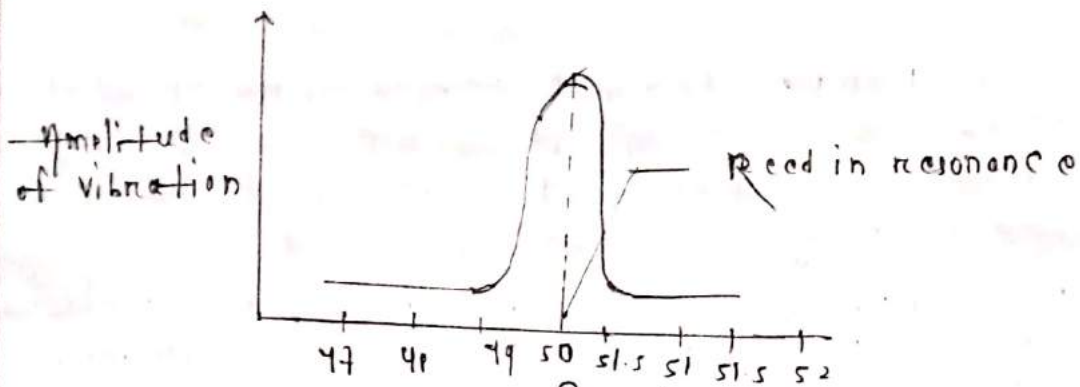


Fig: - variation of amplitude of vibration with respect to frequency.

Disadvantages:-

- ~~Because~~ the instrument can't be used to read small fractional frequency value between two adjacent reeds.
- The reliability of the reading depends upon the accuracy with which the reeds are turned.

Electrical Resonance type of frequency meter:-

- There are 2 type of electrical resonance type of frequency meter.
- 1. Ferrodynamic type frequency meter
- 2. Electrodynamic type frequency meter.

Ferrodynamic type frequency meter:-

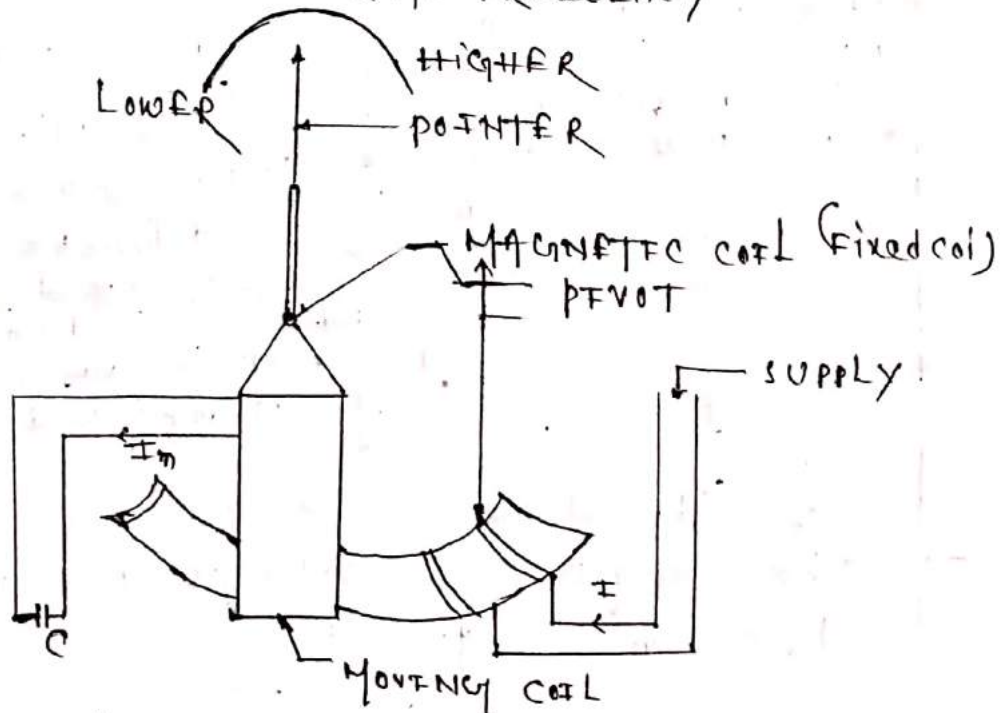
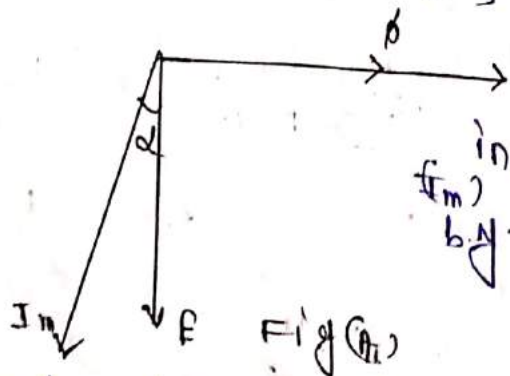


Fig: Ferrodynamic electrical resonance frequency meter.

- The Ferrodynamic type frequency meter consists of a fixed coil which is connected across the supply whose frequency is to be measured.
- This fixed coil is known as magnetising coil & this coil is mounted on a laminated iron core.
- The iron core cross-section varies gradually over the length, it is maximum near the end where the magnetising coil is mounted.
- A moving coil is pivoted over this iron core which is attached to a pointer.
- The moving coil terminal are connected across a capacitor (C).

### Phasor: -

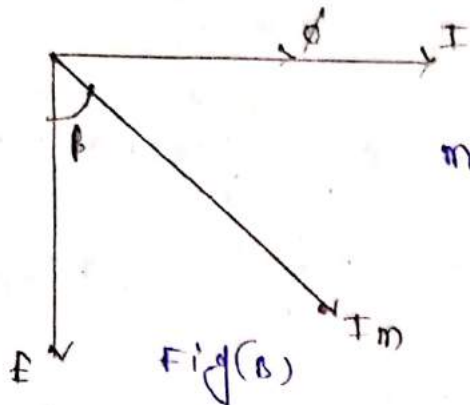
- The operation of the instrument can be understood from the phasor diagram.
- The magnetising coil carries supply current (I) which produces a flux (Φ).
- Flux is in same phase with current (I).
- An emf (E) is induced in the moving coil flux by  $90^\circ$ .
- Due to the induced emf a current (I<sub>m</sub>) flows in the moving coil.
- The phase of the current (I<sub>m</sub>) depends upon inductance (L) & capacitance (C).



→ If the moving coil is assumed to be more inductive then current (I<sub>m</sub>) lags the voltage (E) by an angle α.

- The torque developed in the moving coil is  $T_d = I_m \cos(\phi + \alpha)$ .

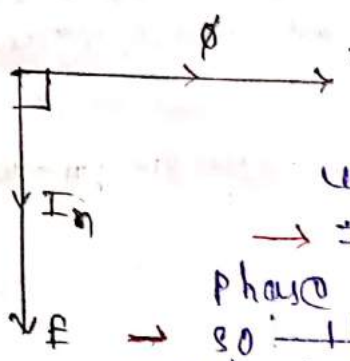




→ If the moving coil circuit is assumed to be more capacitive than the current  $I_m$  leads the current  $I$  by an angle  $\beta$ .

Fig (b)

→ the torque developed in the moving coil  $T_d \propto I_m \cos(\phi - \beta)$



→ when the inductive reactance the capacitive reactance then the circuit is under the resonance condition.

→ In this case current  $I_m$  is in phase with voltage 'E'

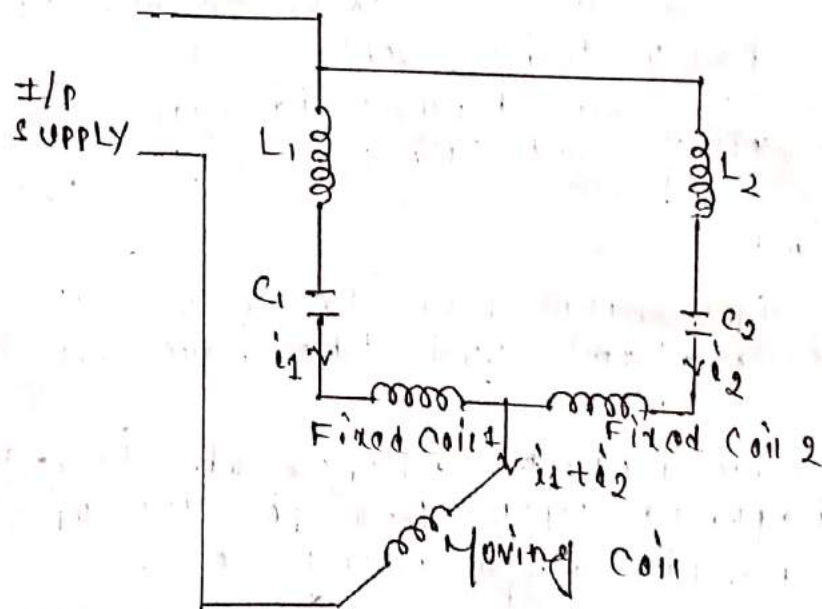
→ so the torque developed in the moving coil  $T_d \propto \cos(\phi - 0)$

Working:-

- The capacitive reactance of the moving coil is constant but the inductive coil is not constant.
- The inductive reactance of the moving coil depends upon the position of the moving coil over the magnetising coil.
- Suppose the frequency of the supply increases then the inductive reactance ( $X_L$ ) of the coil also increases ( $\because X_L = 2\pi fL$  or  $\propto f$ ).
- To achieve the resonance condition a torque is developed in the moving coil are described in fig (a) phasor diagram.
- If the supply frequency 'f' decreases from its normal value then the inductive reactance ( $X_L$ ) decrease &  $X_L < X_C$ .

- Here the torque is developed to move the moving coil to achieve the resonance condition as describe in fig (b).
- The moving coil will come to rest when  $X_L$  will be equal to  $X_C$  or  $f = \frac{1}{2\pi\sqrt{LC}}$ .
- The torque moves the coil to a position where inductive reactance = capacitive reactance & due to moment of the moving coil the pointer also moves over the scale to show a particular frequency value.

### Electrodynamometer type frequency meter:



#### Construction:-

- In this type frequency meter one fixed coil is present which is divided into 2 parts i.e. fixed coil 1 & fixed coil 2.
- Fixed coil 1 is in series with inductance  $L_1$  & a capacitance  $C_1$ . The value of  $L_1$  &  $C_1$  are chosen that its resonant frequency  $F_1$  is slightly lower than the instrument frequency.

→ the fixed coil 2 is in series with an inductance  $L_2$  & a capacitance  $C_2$ . The value of  $L_2$  &  $C_2$  are so chosen that its resonant frequency of the circuit  $f_2$  is slightly higher than the instrument frequency.

→ Ex: - If 50 Hz is the middle scale of the instrument frequency then  $f_1 \approx 40$  Hz,  $f_2 \approx 60$  Hz

$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}, \quad f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

→ one moving coil is connected in between the fixed coil 1 & fixed coil 2 through which  $i_1 + i_2$  current flows.

### Working:-

→ For an applied frequency of the ext of fixed coil 1 operates above the resonant frequency, as  $X_L > X_C$ , so here current  $i_1$  lags the voltage.

→ At that time the circuit of fixed coil 2 operates below the resonant frequency, as  $X_C > X_L$ , so here current  $i_2$  leads the voltage.

→ since one circuit is inductive & the other is capacitive, therefore the 2 currents  $i_1$  &  $i_2$  generates to opposite torque on the moving coil.

→ The resultant torque which acts on the moving coil is a function of frequency of the applied voltage.

→ the movement of the moving coil can be calibrated over a scale in terms of frequency.

## power factor meter:-

- The instrument which indicates the power factor of a load circuit to which it is connected, is called as power factor meter.

$$P = VI \cos \phi$$

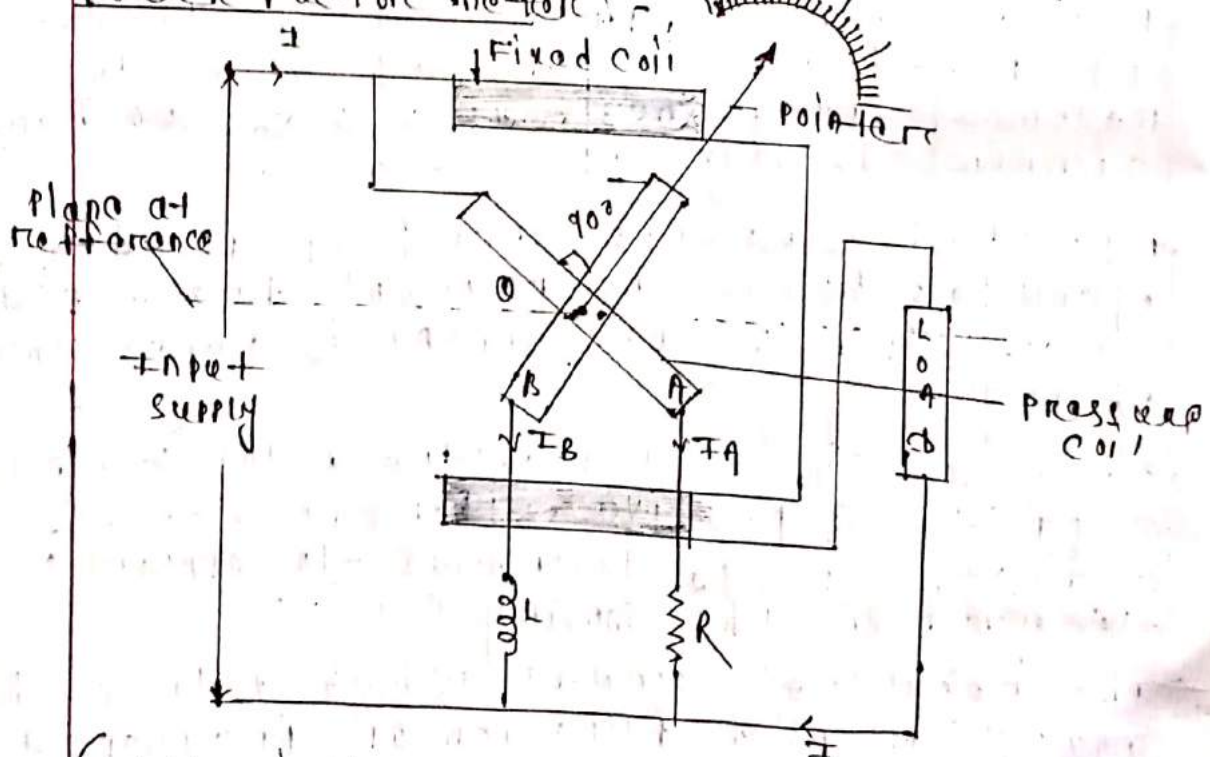
$$\Rightarrow \cos \phi = \frac{P}{VI}$$

Where,

$\cos \phi$  is known as power factor of the load.

- So, to measure the power factor, current, voltage & power consume in an AC circuit may be calculated, so the design of power factor meter is similar to wattmeter.

## Single phase electrodynamicometer type power factor meter



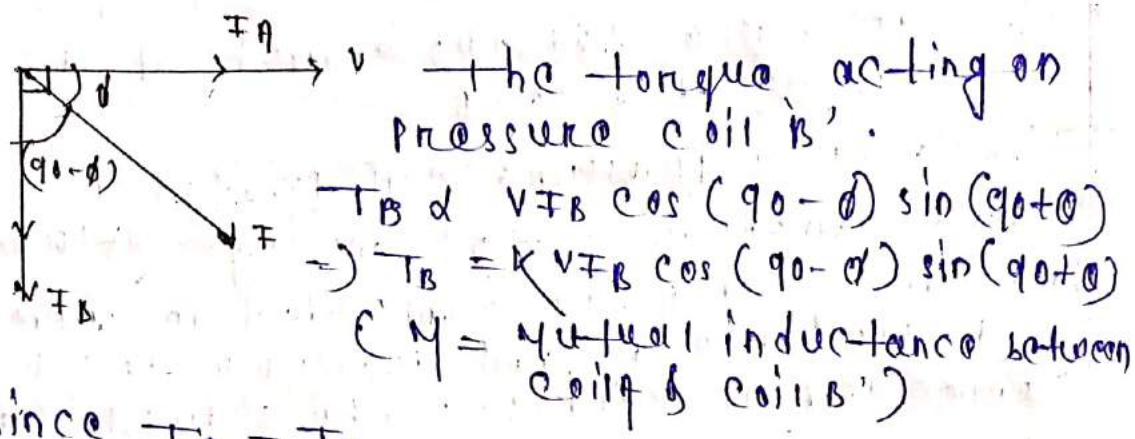
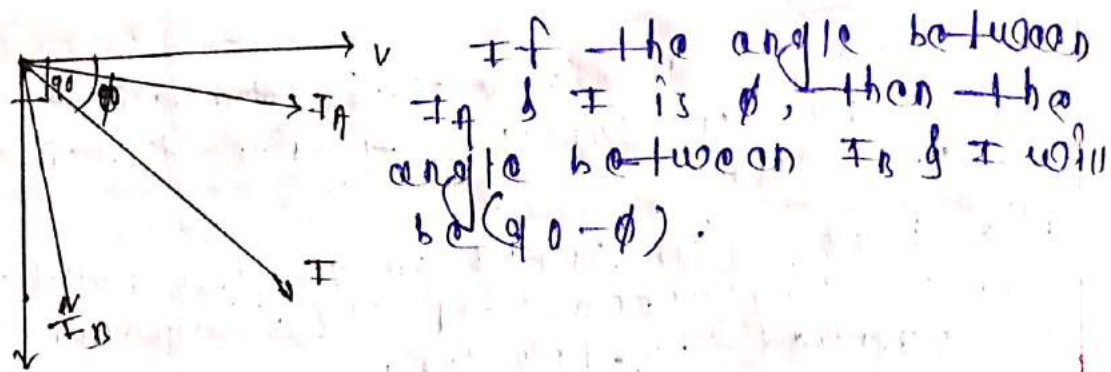
(Fig: single phase electrodynamicometer type power factor meter)

## Construction:-

- The single phase electrodynamicometer type power factor consists of fixed coil & pressure coil.
- The fixed coil splits into two parts and carries the current of the circuit which is supplied to the load.
- The pressure coil also consists of two coils that is coil A & coil B.
- These two pressure coil pivoted on a spindle which constitute the moving system.
- Pressure coil 'A' is connected in series with a resistance 'R' and pressure coil 'B' is connected in series with inductance 'L'.
- The current proportional to the voltage drop across the load flows through the pressure coil.
- The value of R & L are so adjusted that  $R = 2\pi f L$
- The angle between the plane of the coil is made equal to  $90^\circ$ . The current  $I_B$  lags the voltage by  $90^\circ$ , current  $I_A$  is in phase with voltage. (is in same phase).

## Working / operation:-

- In this case two deflecting torque are produce which acts on coil A & coil B.
- The coils are so design that the torque acting on it, are equal & opposite in direction. so the pointer takes a position where the two torques are equal.
- The deflecting torque acting on coil A, proportional to  $V I_A M \cos \phi \sin \theta$   
 $T_A \propto V I_A M \cos \phi \sin \theta$   
 $\Rightarrow T_A = K V I_A M \cos \phi \sin \theta$



since  $T_A = T_B$

$$\Rightarrow k V I_A M \cos \phi \sin \theta = k V I_B \cos(90 - \theta) \sin(90 + \theta)$$

$$\Rightarrow \cos \phi \sin \theta = \cos(90 - \theta) \sin(90 + \theta)$$

$$\Rightarrow \cos \phi \sin \theta - \sin \phi \cos \theta = 0$$

$$\Rightarrow \cos \phi \sin \theta - \sin \phi \cos \theta = 0$$

$$\Rightarrow \sin(\theta - \phi) = 0$$

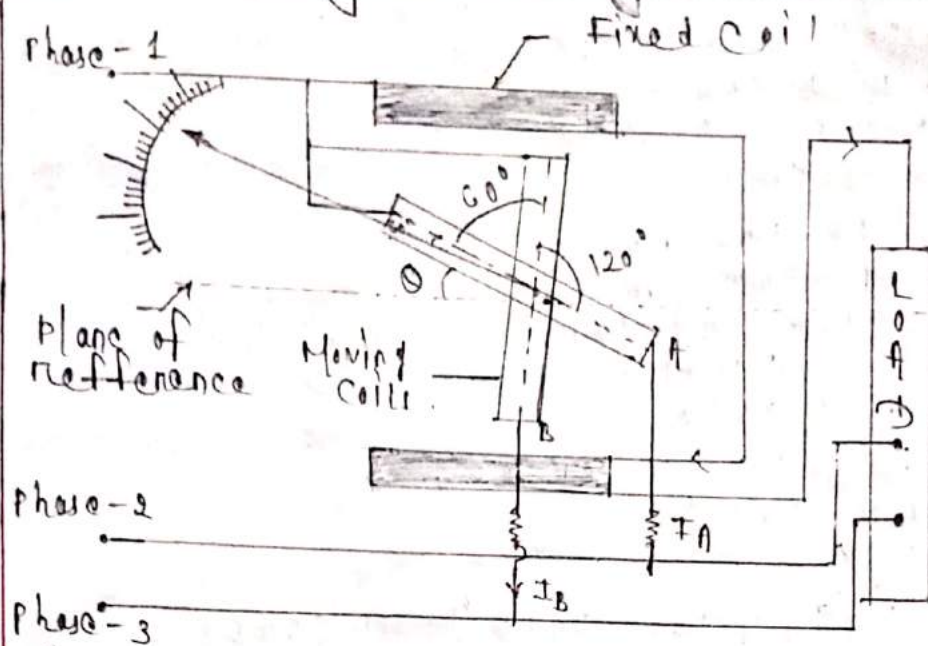
$$\Rightarrow \sin(\theta - \phi) = \sin 0$$

$$\Rightarrow \theta - \phi = 0$$

$$\Rightarrow \boxed{\theta = \phi}$$

- so the deflection of the instrument with respect to pressure coil 'A' is the measure of phase angle of the circuit.
- the scale of the instrument can be calibrated cosine of the phase angle that is power factor.

### 3- $\phi$ Electrodynamicometer type power factor meter:-



(Fig: 3- $\phi$  Electrodynamicometer type power factor meter)

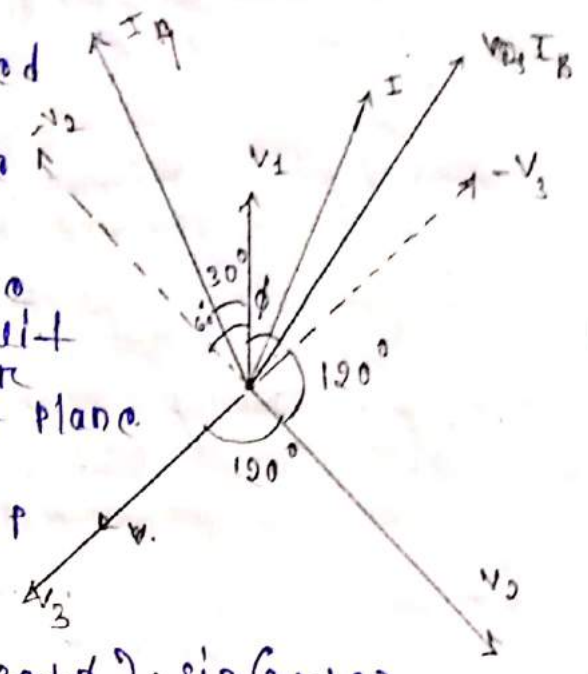
#### Construction:-

- The above figure shows the construction & connection of a 3- $\phi$  electrodynamicometer type power factor meter.
- The two moving coils of the meter are so placed that the angle between their plane is  $120^\circ$ . These two coils are connected across two different phases of the supply circuit.
- Each of these two coils has series resistance through which it is connected to the phase angle.
- The voltage applied across coil 'A' is  $V_{12}$  & the current flowing through it is  $I_A$ .
- The voltage applied across coil 'B' is  $V_{13}$  & the current flowing through it is  $I_B$ .
- These two moving coils are placed in between the segments of fixed coils.

Working:-

→ The torque developed in coil 'A' & coil 'B' are equal & opposite in direction.

→ Let ' $\phi$ ' is the phase angle of the circuit & ' $\theta$ ' is the angular deflection with reference plane.



→ The torque developed in coil 'A' can be given by,

$$T_A = k V_{12} I_A M \cos(30 + \phi) \cdot \sin(60 + \theta)$$

→ The torque developed in coil 'B' can be given by,

$$T_B = k V_{13} I_B M \cos(30 - \phi) \cdot \sin(120 + \theta)$$

→ This two torque are equal in magnitude, so we can write,  $T_A = T_B$

$$k V_{12} I_A M \cos(30 + \phi) \cdot \sin(60 + \theta) = k V_{13} I_B M \cos(30 - \phi) \cdot \sin(120 + \theta)$$

since the voltage drop & the current flowing through the two coils are same, so  $V_{12} = V_{13}$  &  $I_A = I_B$

$$\Rightarrow \cos(30 + \phi) \cdot \sin(60 + \theta) = \cos(30 - \phi) \cdot \sin(120 + \theta)$$

$$\Rightarrow (\cos 30 \cdot \cos \phi - \sin 30 \cdot \sin \phi) (\sin 60 \cdot \cos \theta + \cos 60 \cdot \sin \theta) = (\cos 30 \cdot \cos \phi + \sin 30 \cdot \sin \phi) (\sin 120 \cdot \cos \theta + \cos 120 \cdot \sin \theta)$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2} \cdot \cos \phi - \frac{1}{2} \cdot \sin \phi\right) \left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta\right) = \left(\frac{\sqrt{3}}{2} \cos \phi + \frac{1}{2} \sin \phi\right) \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta\right)$$

$$\Rightarrow \frac{3}{4} \cos \phi \cdot \cos \theta - \frac{\sqrt{3}}{4} \sin \phi \cdot \cos \theta + \frac{\sqrt{3}}{4} \cos \phi \cdot \sin \theta - \frac{1}{4} \sin \phi \cdot \sin \theta = \frac{3}{4} \cos \theta \cdot \cos \phi + \frac{\sqrt{3}}{4} \cos \phi \cdot \sin \theta + \frac{\sqrt{3}}{4} \sin \phi \cdot \cos \theta - \frac{1}{4} \sin \phi \cdot \sin \theta$$

$$\Rightarrow -\frac{\sqrt{3}}{4} \sin \phi \cdot \cos \theta + \frac{\sqrt{3}}{4} \cos \phi \cdot \sin \theta = \frac{\sqrt{3}}{4} \cos \phi \cdot \sin \theta - \frac{\sqrt{3}}{4} \sin \phi \cdot \cos \theta$$

$$\Rightarrow \cos \phi \cdot \sin \theta - \sin \phi \cdot \cos \theta = \sin \phi \cdot \cos \theta - \cos \phi \cdot \sin \theta$$

$$\Rightarrow \sin(\theta - \phi) = \sin(\phi - \theta)$$



since the angular deflection of the pointer from the plane of reference is equal to the phase angle of the circuit, so the pointer movement can be calibrated in terms of power factor ( $\cos\phi$ ).

## CHAPTER-6

### MEASUREMENT OF RESISTANCE, INDUCTANCE & CAPACITANCE, RESISTANCE MEASUREMENT

Resistance are categorised into 3 category:-

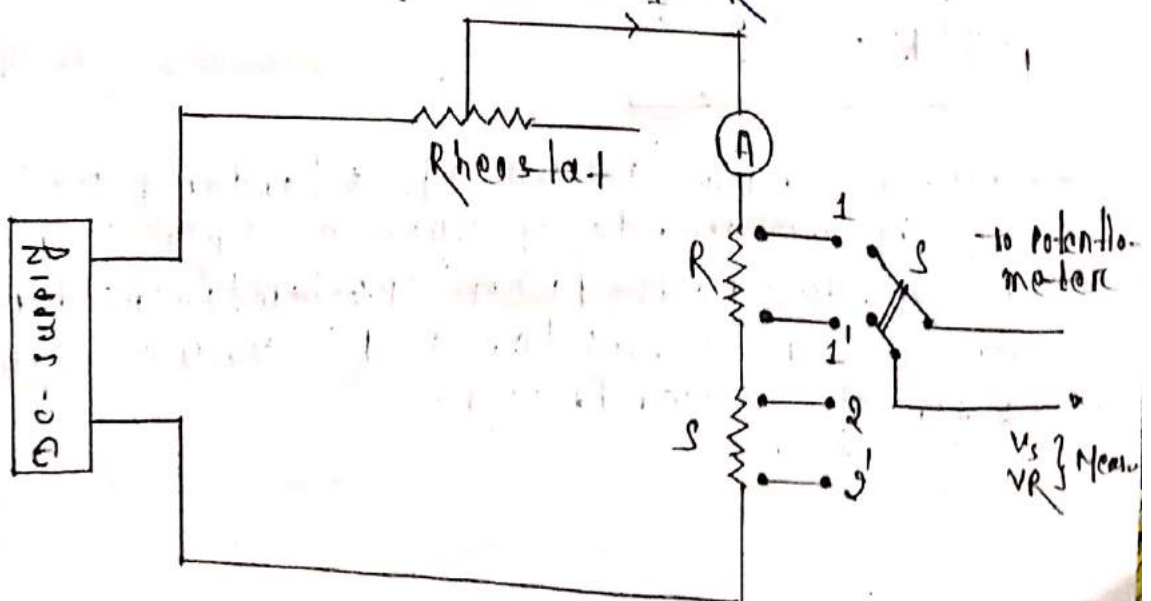
- 1 Low resistance (below  $1 \Omega$  or  $\leq 1 \Omega$ )
- 2 Medium resistance (1 -  $100k \Omega$ )
- 3 High resistance (More than  $100 \Omega$  to  $100k \Omega$ )

Low resistance Measurement by potentiometer

Method:-

$$V_s = I \times S \Rightarrow I = \frac{V_s}{S} \Rightarrow R = \frac{V_R \times S}{V_s}$$

$$V_R = I \times R \Rightarrow I = \frac{V_R}{R} \quad \frac{V_s}{S} = \frac{V_R}{R}$$

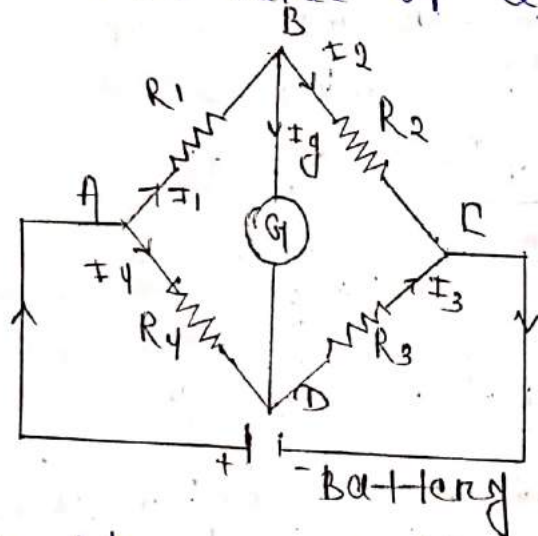


- The above ckt is used to measure the unknown resistance with the help of a potentiometer.
- Potentiometer is a device which can detect variable unknown voltages.
- In the above ckt 'R' is the unknown resistance whose value is to be measured & 'S' is the known standard resistor.
- The ckt current is control with the help of a rheostat.
- A double-throw switch is used to connected 1 1' or 2 2' points.
- The double-throw switch is connected to a potentiometer to measure the voltage drop in unknown resistor 'R' ( $V_R$ ).
- When the switch is connected 1 1' than voltage drop  $V_R = I \times R \Rightarrow I = \frac{V_R}{R}$  — eqn ①
- When the switch is connected 2 2' than voltage drop  $V_S = I \times S \Rightarrow I = \frac{V_S}{S}$  — eqn ②
- From eqn ① & ② we get that,
 
$$\frac{V_S}{S} = \frac{V_R}{R}$$

$$\Rightarrow R = \frac{V_R \times S}{V_S}$$
- Since the value of standard resistance 'S' is accurately known,  $V_R$  &  $V_S$  value can be detected from potentiometer, the unknown resistance 'R' value can be easily calculated.

## Medium resistance Measurement by wheatstone bridge method:-

- Wheatstone is an electrical ckt in which 4 no. of resistors are connected in a bridge structure.
- out of this 4 resistances, 3 resistances are known & 1 resistance is unknown.
- The wheatstone bridge ckt can be used to calculate the value of unknown resistance.



- Consider the 4 resistances  $R_1, R_2, R_3$  &  $R_4$  which are connected in the wheatstone bridge.
- Let,  $R_1, R_2$  &  $R_3$  are known resistances &  $R_4$  is unknown resistance.
- Let  $I_1, I_2, I_3$  &  $I_4$  are the current flowing through the resistances.
- A null type galvanometer is connected between B & D junction & ' $I_g$ ' is the current flowing through the galvanometer.
- Consider ' $R_3$ ' is the variable resistor, whose value is show adjusted that the galvanometer show null deflection. At this condition  $I_g = 0$ .

→ At this balancing condition:-

At junction B,

$$I_1 = I_g + I_2$$

$$\Rightarrow I_1 = I_2 \quad \text{--- eqn (1)}$$

$R_g$  is the internal resistance of the galvanometer.

At junction D,

$$I_4 + I_g = I_3$$

$$\Rightarrow I_4 = I_3 \quad \text{--- eqn (2)}$$

Applying KVL in ABCD loop,

$$I_1 R_1 + I_g R_g - I_4 R_4 = 0$$

$$\Rightarrow I_1 R_1 = I_4 R_4 \quad \text{--- eqn (3)}$$

Applying KVL in BCED loop,

$$I_2 R_2 - I_3 R_3 - I_g R_g = 0$$

$$\Rightarrow I_2 R_2 = I_3 R_3 \quad \text{--- eqn (4)}$$

Dividing eqn (3) with eqn (4) we get:-

$$\frac{I_1 R_1}{I_2 R_2} = \frac{I_4 R_4}{I_3 R_3}$$

Using eqn (1) & (2)

$$\frac{I_1 R_1}{I_1 R_2} = \frac{I_4 R_4}{I_4 R_3}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{R_4}{R_3}$$

This condition is known as balancing condition of wheatstone bridge.

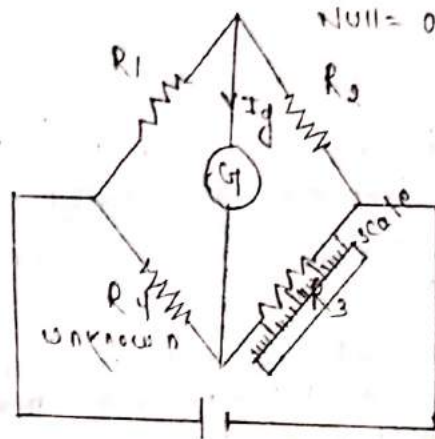
$$R_4 = \frac{R_1 \times R_3}{R_2}$$

From the above expression the value of unknown resistances  $R_4$  can be calculated.

There are two types of wheatstone bridge  
ext: -

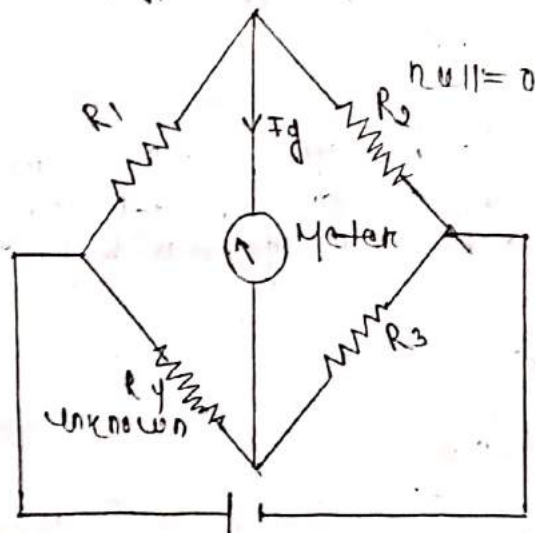
1. Null balance type
2. Deflection type.

Null balance type: -



→ The adjustment of variable resistor  $R_3$  is calibrated in terms of unknown resistance  $R_4$ .

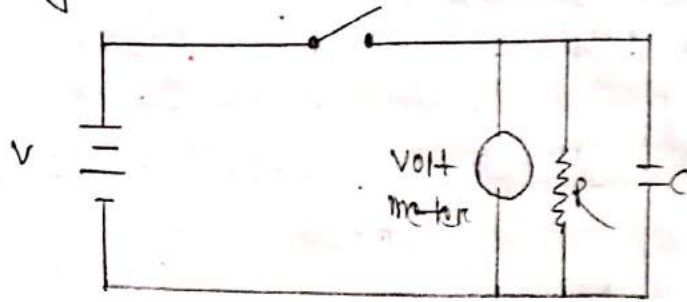
Deflection type: -



→ The current  $I_g$  is proportional to the unknown resistance  $R_4$ , when the bridge is unbalanced.

→ A PMMC type instrument can be connected to measure the ' $I_g$ ' current & the meter is calibrated in terms of unknown resistance  $R_4$ .

# High resistance measurement by loss of charge method! -



- The high resistance can be measured by using loss of charge method.
- The high resistance is also known as insulation resistance.
- In this method the insulation resistance whose value is to be measured is connected in parallel with a capacitor 'C' & a voltmeter.

## Working: -

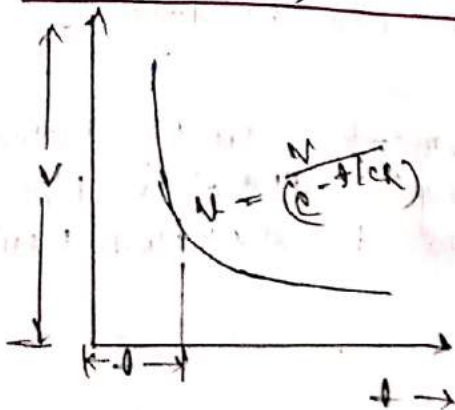
- The capacitor is charged to a suitable voltage by using a battery.
- After that the capacitor is allowed to discharge.
- During the discharge the terminal voltage across the capacitor or at any instant of time 't', can be given by,

$$v = V e^{-t/CR}$$

$$\Rightarrow \frac{v}{V} = e^{-t/CR}$$

$$\Rightarrow \ln\left(\frac{v}{V}\right) = -t/CR$$

$$\Rightarrow R = \frac{-t}{C \ln\left(\frac{v}{V}\right)} = \frac{-t}{C \times 2.3 \log\left(\frac{v}{V}\right)} = \frac{0.434 t}{C \log\left(\frac{v}{V}\right)}$$



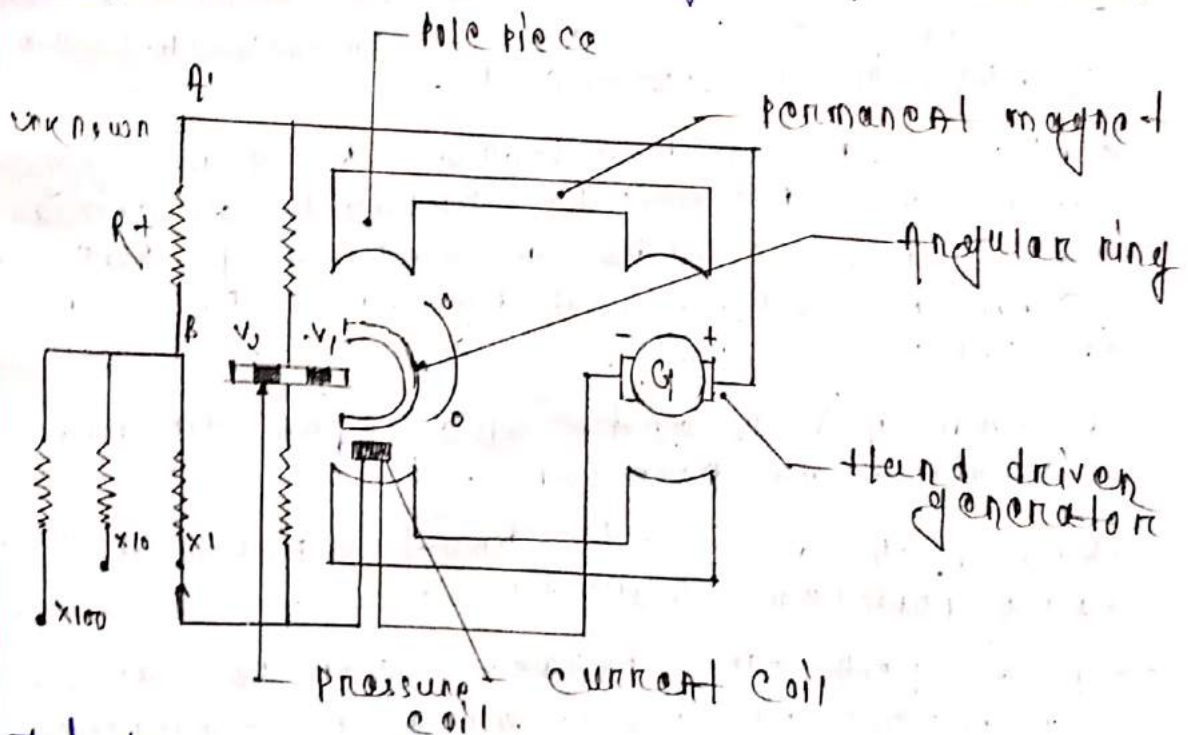
→  $V$  is the instantaneous value of voltage across a capacitor at any instant of  $t$ .

$$R = \frac{-0.4343t}{C \log\left(\frac{V}{V_0}\right)} \quad \text{--- eqn (1)}$$

→ In eqn (1) if  $V$ ,  $t$ ,  $V_0$  &  $C$  values are known to us then the unknown resistance  $R$  can be calculated.

Megger :-

→ Megger is an instrument which is used to measure insulation resistance of very high resistance. It is also known as insulation tester.



→ It is a modified PMMC type instrument.

→ This instrument contains 1 current coil & 2 pressure coils.

→ The pressure coils are  $V_1$  &  $V_2$ , these 2 coils are so located that when the magnetic field gradually becomes stronger the pointer moves from  $\infty$  to 0.

- The current coil also controls the pointer movement by its magnetic field.
- When the current in the current coil is the large, than the pointer indicates '0', which means  $R_x$  is very small.
- Similarly when the current in current coil is low, it indicates ' $\infty$ ' over than scale, which means  $R_x$  value is very large.
- The voltage range of the instrument can be controlled by using variable resistor switch, which is connected in series with current coil.
- The generator is use to generate the testing voltage while measuring the unknown resistance.
- The unknown resistance  $R_x$  can vary the current flowing through 'c' coil. so the movement of pointer can be affected by the unknown resistance ' $R_x$ '.
- When A & B ends are upon circuited than the pointer indicates ' $\infty$ '.
- When A & B end short circuited than the pointer indicates '0'.
- The pointer movement can be calibrated in terms of resistance to measure the ' $R_x$ ' value.
- A centrifugal clutch is incorporated in the generator to drive it, at a constant speed while generate in the voltage.



## Earth tester:-

- Earth tester is an instrument which is used to measure earth resistance.
- While earthing the earth electrode should be present in a low resistance soil, so that it can carry the excess current to the earth without any dislocation.
- The earth soil resistance is affected by the moisture content of the soil. So, periodic testing of earth resistance is required to make the earthing system more effective.

## Construction:-

The earth tester is a special type of megger with two additional features.

- ① current reversal
- ② Rectifier.

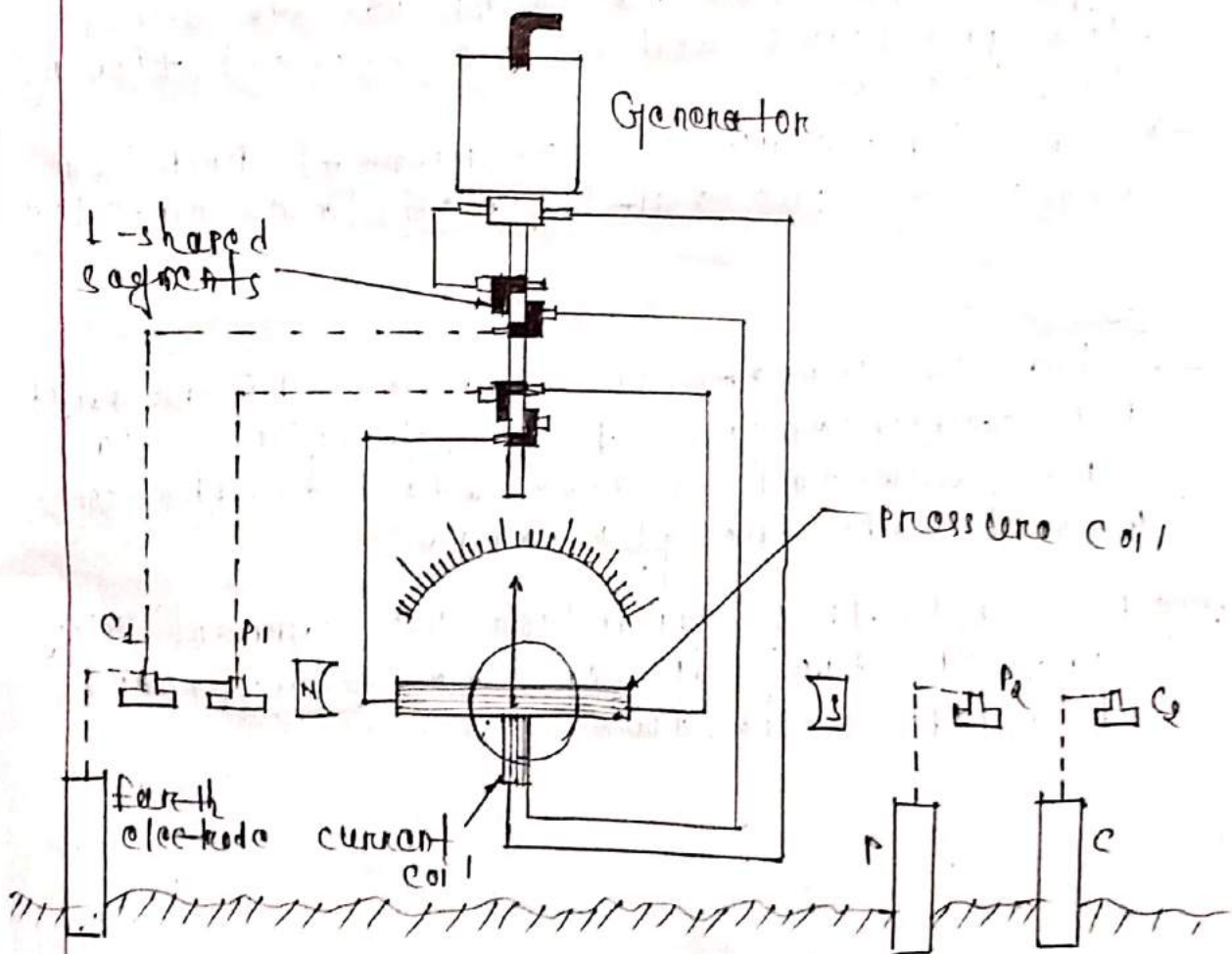


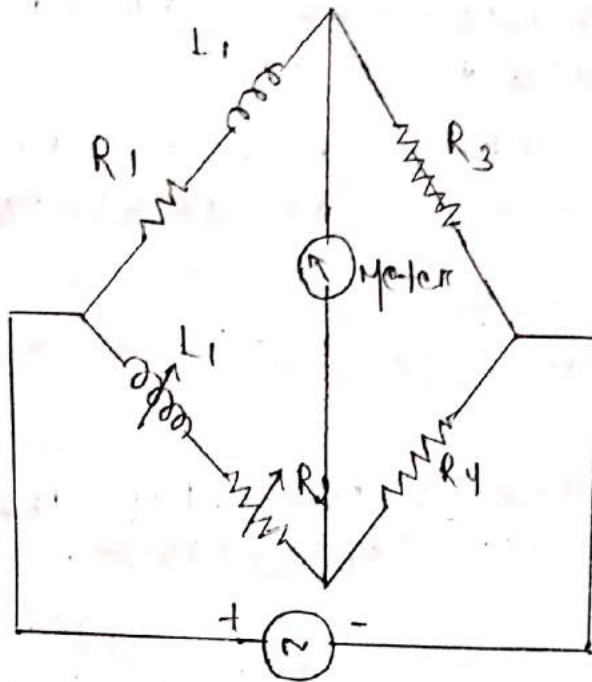
Fig: Earth tester.

- This instrument consists of commutators made up of L-shaped segments.
- These segments are mounted on the shaft of the generator.
- The commutator has four brushes, their brushes are positioned that one pair contact alternately with one segment, while the second pair fixedly contact to the same point, when the commutator rotates.
- The earth tester has four terminals  $P_1, P_2, C_1$  &  $C_2$ .
- Two terminals  $P_1$  &  $C_1$  are shorted and connected to earth electrode.
- The other two terminals  $P_2$  &  $C_2$  are connected to auxiliary electrodes 'p' & 'c'.
- The indication of earth tester instrument depend upon the ratio of voltage across the pressure coil & the current flowing through it.
- The deflection of instrument pointer indicates the earth resistance directly.

#### Note:-

- When DC current is supplied to the earth for measurement of resistance, then the back emf is generated in the soil, due to electrolytic effect.
- To avoid this condition AC current supply through the soil for the measurement of earth resistance.

## Maxwell bridge for (Inductor Measurement):-

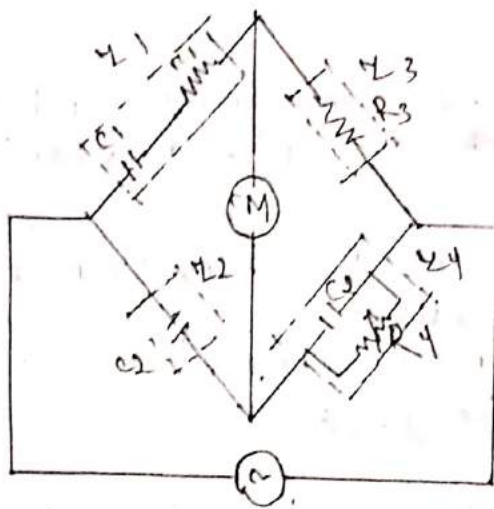


- The Maxwell bridge measures inductance by comparing with a variable standard inductance.
- In the above circuit  $L_1$  = unknown inductance  
 $L_2$  = variable inductance  
 $R_1, R_3, R_4$  = known resistance  
 $R_2$  = variable resistance.
- The balancing condition of the bridge can be given by,  
$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$
$$\Rightarrow Z_1 Z_4 = Z_2 Z_3$$
$$\Rightarrow (R_1 + j\omega L_1) R_4 = (R_2 + j\omega L_2) R_3$$
$$\Rightarrow R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_3 L_2 R_3$$
- Equating the imaginary part of the above eqn we get,  
$$\Rightarrow j\omega L_1 R_4 = j\omega L_2 R_3$$
$$\Rightarrow L_1 R_4 = L_2 R_3$$
$$\Rightarrow L_1 = \frac{L_2 R_3}{R_4}$$

- In the above eqn the value of  $L_2, R_2, R_4$  are known, so the unknown inductance  $L_1$  can be calculated.
- By equating the real parts  $R_1 R_4 = R_2 R_3$ , this condition also has to be satisfied.

### Measurement of capacitance by Schering bridge:

Schering bridge is used for the measurement of unknown capacitance.



$C_1$  = capacitor whose capacitance is to be measured.

$r_1$  =  $\eta$  series resistance representing loss in capacitor

$C_2$  = standard capacitor

$C_4$  = variable capacitor

$R_4$  = variable resistor

in parallel with  $C_4$

$R_3$  = standard resistance

The balancing condition of the bridge can be given by,

$$\frac{V_1}{V_3} = \frac{V_2}{V_4}$$

$$\Rightarrow V_1 V_4 = V_2 V_3$$

$$\Rightarrow \left( r_1 + \frac{1}{j\omega C_1} \right) \left( \frac{R_4}{1 + j\omega C_4 R_4} \right) = \frac{1}{j\omega C_2} \times R_3$$

$$\Rightarrow \left( r_1 + \frac{1}{j\omega C_1} \right) = \frac{R_3}{j\omega C_2} \times \frac{(1 + j\omega C_4 R_4)}{R_4}$$

$$\Rightarrow r_1 - j \frac{1}{\omega C_1} = \frac{R_3 + j\omega C_4 R_3 R_4}{j\omega C_2 R_4}$$

$$= \frac{R_3}{j\omega C_2 R_4} + \frac{j\omega C_4 R_3 R_4}{j\omega C_2 R_4}$$

$$= -j \times \frac{R_3}{\omega C_2 R_4} + \frac{C_4 R_3}{C_2}$$

$$\Rightarrow r_1 - j \frac{1}{\omega C_1} = \frac{C_4 R_3}{C_2} - j \frac{R_3}{\omega C_2 R_4}$$

comparing the real part from the above eqn we get,  $\pi_1 = \frac{C_2 R_3}{C_2}$  — eqn (i)

comparing the imaginary part from the above eqn we get  $\frac{-1}{\omega C_1} = \frac{R_3}{\omega C_2 R_4}$

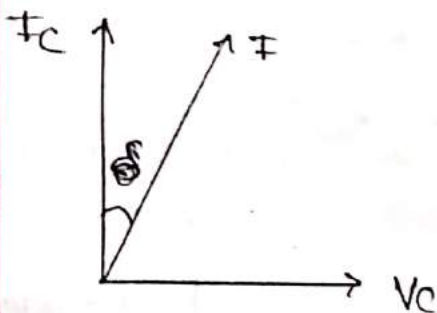
$$\Rightarrow C_1 = \frac{\omega C_2 R_4}{\omega R_3}$$

$$\Rightarrow C_1 = \frac{C_2 R_4}{R_3} \quad \text{--- (ii)}$$

so, the unknown resistance 'C<sub>1</sub>' can be determined from eqn (ii). C<sub>2</sub>, R<sub>4</sub> & R<sub>3</sub> values are known, so 'C<sub>1</sub>' can be calculated.

this dissipation factor can be given by

$$\begin{aligned} \tan \delta &= \omega C_1 \pi_1 \\ &= \omega C_1 \times \frac{C_2 R_3}{C_2} \end{aligned}$$



### Multimeter:-

→ This is a measuring instrument which is used for measurement of multiple quantities like voltage, current, resistance etc.

→ There are 2 types of multimeter available:-

1. Analog multimeter
2. Digital multimeter.

## Analog Multimeter:-

- An analog multimeter is an PMMC-type meter which works on 'd' Arsonval movement principle.
- It consists of a needle or pointer to indicate the measured value over a graduated scale.
- The PMMC-type meter acts as ammeter when shunt resistors are connected.
- The meter acts as voltmeter when multiplier resistors are connected.
- It acts as ohmmeter when a battery & a resistance network is connected.

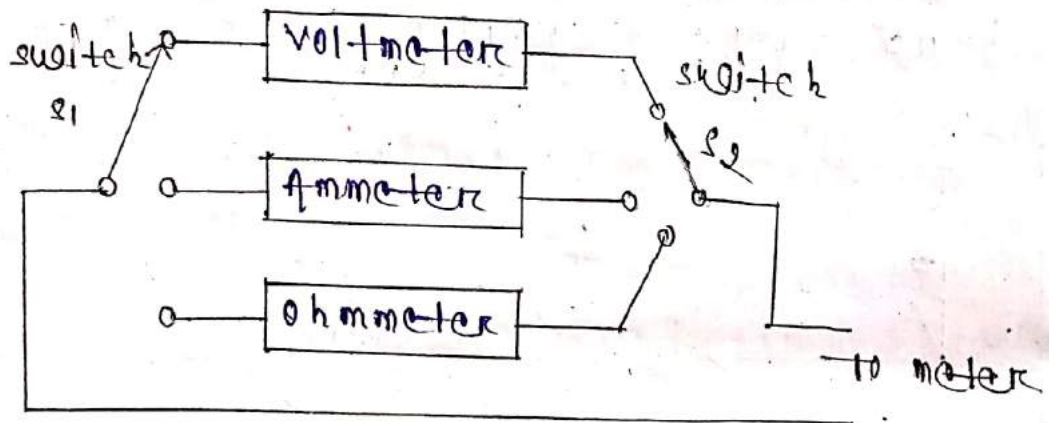
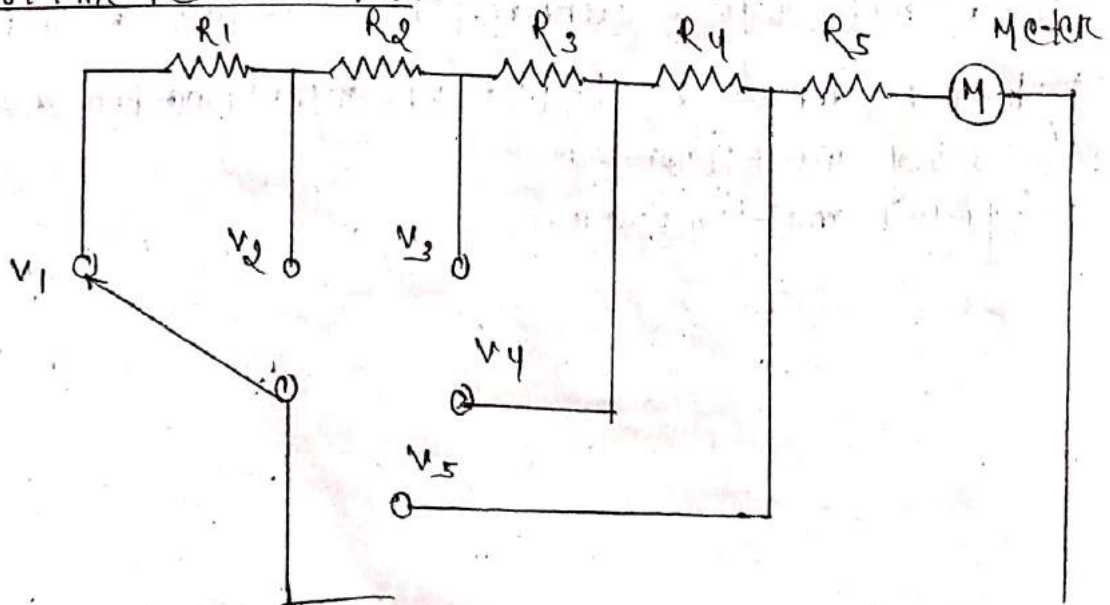


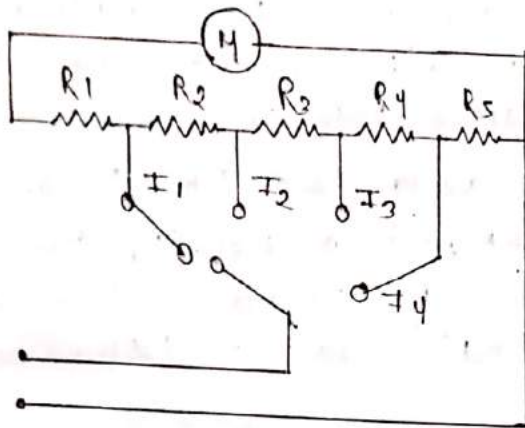
Fig: Block diagram of multimeter

## Voltmeter section:-



- Multipliers are connected in series with the PMMC type wattmeter.
- In the above figure  $V_1, V_2, V_3, V_4$  &  $V_5$  are the different voltage ranges for measurement.

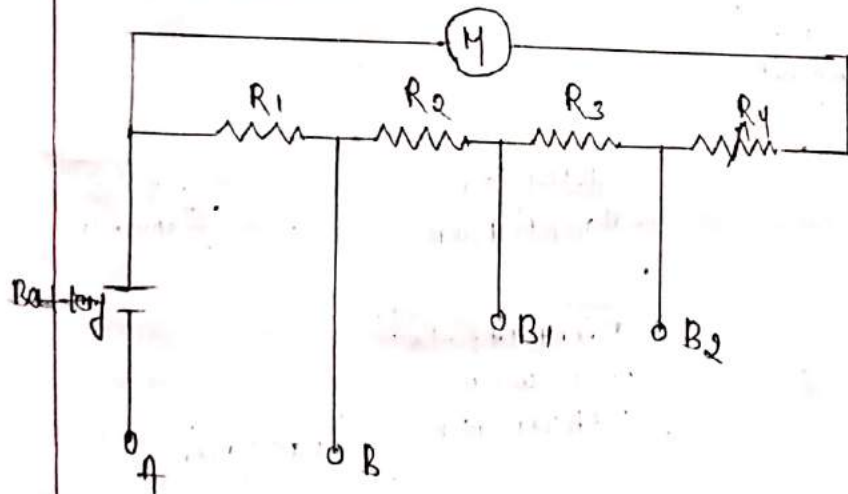
### ammeter section



### ohmmeter section

- shunt resistors are connected parallelly with the meter. In the above fig  $I_1, I_2, I_3$  &  $I_4$  are different current ranges for measurement.

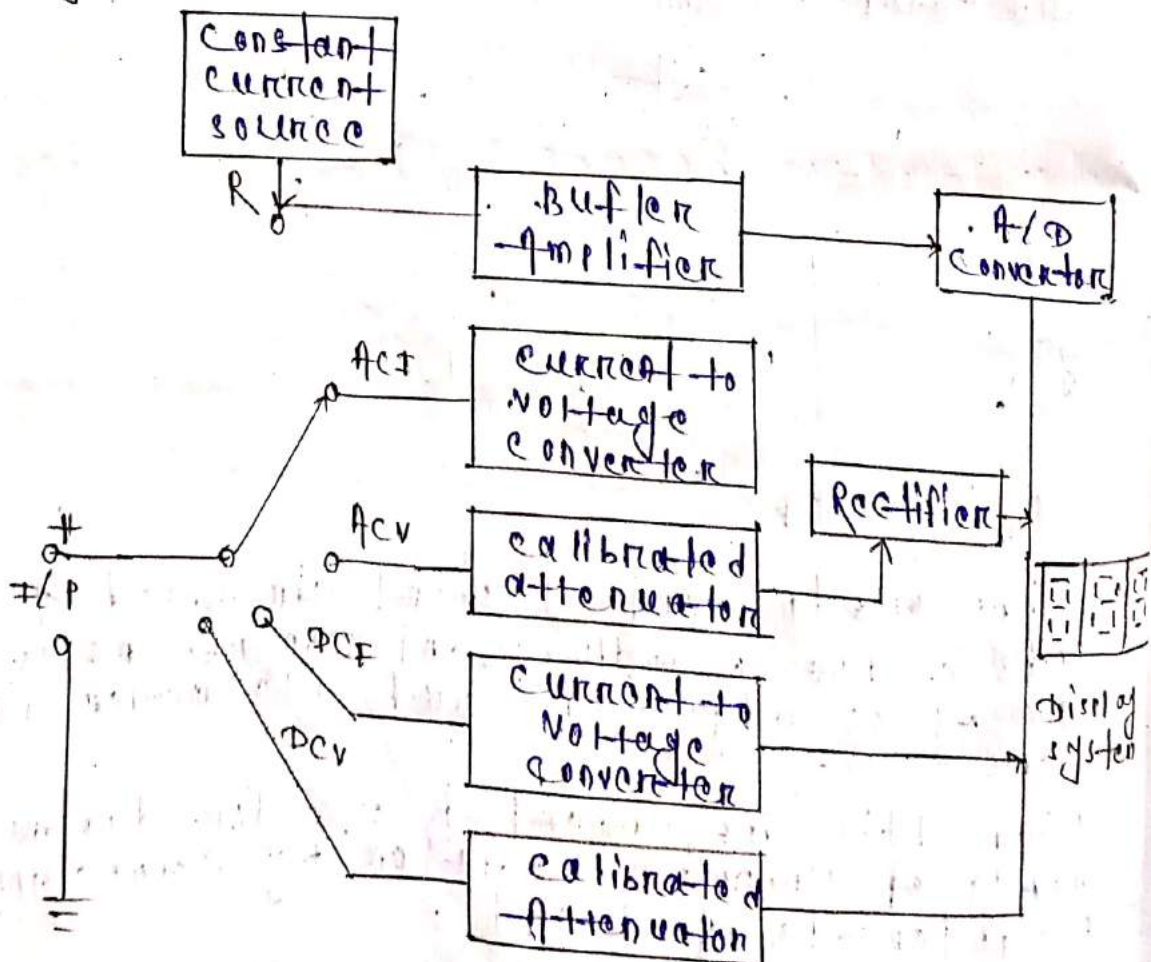
### ohmmeter section:-



- This instrument is short circuited at B ends & the '0' adjustment control resistor are it so adjusted that the meter reads zero.
- Then this instrument is use for the measurements of unknown resistor by connecting the resistor to A-B end.

- The range of the measurement can be varied by varying the position of 'B' end. In the above figure B, B<sub>1</sub> & B<sub>2</sub> can be used to achieve different resistance ranges from measurement.
- A switch can be used to select AC/DC quantity for measurement. If DC quantity is selected by the switch then the input is direct fed to the PMMC meter for indication.
- If AC quantity is selected then the input is passed through a rectifier ext, which converts AC quantity to DC and then fed to PMMC meter for indication.

### Digital Multimeter:-





→ Digital multimeter is the instrument which is used to measure multiple quantity like :- voltage, current, resistance etc & display the measure quantity in terms of digits.

→ In digital multimeter the +ve input probe is connected to a rotary switch through which different measurements can be selected like :- resistance, ac current, dc current, ac voltage & dc voltage.

→ The ac quantities after converting to particular voltage range is passed through a rectifier ckt for ac to dc conversion.

→ ac current & dc current are passed through current to voltage converter ckt which converts the current into proportional voltage.

→ ac voltage & dc voltage are attenuated (decrease strength) within a particular voltage range before giving it to addy converter.

→ A constant current source is used to generate equivalent voltage w.r.t unknown resistance while resistance measurement.

→ All the quantities are converted to dc voltage from by using proper ckt & then it is given to addy converter.

→ Addy converter converts the analog signal into digital forms (0 to 1).

→ The digital data is then provided to display system.

→ A micro-controller chip is present with in the display system, which control the digit display on segment LEDs.

# CHAPTER - 7

## SENSORS & TRANSDUCERS

What is transducers:-

- Transducers is a device which can convert or transduce one form of energy into another form.
- sensors are special type of transducer which are used to sense or detect physical parameters & provides output generally used electrical form.
- Ex! - speaker, potentiometer, turbine etc.

Classification of transducers:-

Basic upon the output produced by transducers elements, transducers are categorise into two type.

Mechanical Transducers:-

- This transducers produced output in term of mechanical energy i.e, displacement, speed.
- Ex! - turbine, Bourdon tube.

Electrical Transducers:-

- This transducers produced output in terms of electrical energy.
- Ex! - LVDT transducer, strain gauge, piezoelectric sensor etc.

## Active Transducers:-

→ The transducers which can generate electrical output in terms of voltage or current without any external power supply are known as active transducers.

→ Ex:- Thermocouple

## Passive Transducers:-

→ The transducers which requires external power supply to generate output in terms of voltage & current are known as passive transducers.

→ Ex:- LVDT, strain gauge, potentiometer etc.

→ The passive transducers produces output in terms of resistance, inductance & capacitance w.r.t input parameter. According to this the passive transducers are categorised into 3 types.

### 1. Resistive Transducer:-

→ The output resistance of this transducers changes w.r.t input parameters.

Ex:- (1) Potentiometer

(2) Thermistor

Resistance thermometer  
strain gauge.

## 2- Inductive transducers:-

The o/p inductance of this transducer changes w.r.t input parameter.  
Ex:- LVDT.

## 3- Capacitive transducers:-

The o/p capacitance of this transducers changes w.r.t input parameter.

- (i) Variable area type capacitive transducer
- (ii) Variable air gap type capacitive transducer

Note:-

→ The transducer which is directly connected to the physical parameter being measured is known as primary transducer & the transducer which are connected to the primary transducer are known as secondary transducer.

## Resistive transducer:-

- Already here*
- The resistance of this transducer changes w.r.t change in i/p parameter.
  - These are passive transducer i.e. external supply is required to generate voltage or current as o/p.

## 1. Potentiometer:-

- Potentiometer is a type of displacement sensor.
- Simply it is turn as POT stands for potentiometer.
- POT meter consists of a uniform resistive element & a sliding contact. This sliding contact is known as slider or wiper.

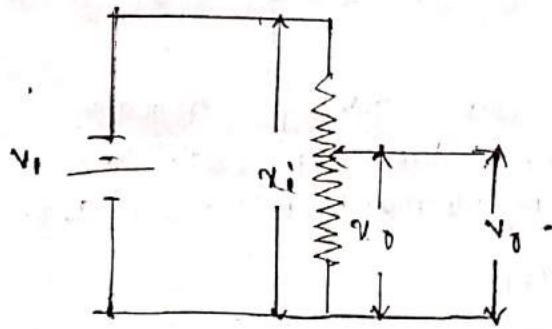
- the motion of the sliding contact may be translational or rotational.
- Depending upon the movement of the slider the potentiometer are classified into 2 categories:-

- Linear potentiometer
- Angular potentiometer

Note! -

- 3rd type pot is also available where the slider can move in both translational & rotational direction. This pot is known as helipot.

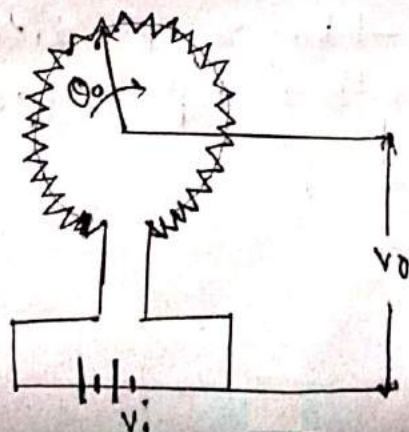
### A. Linear potentiometer:-



(Fig. 1 - Linear potentiometer)

- In this transducer the resistive element is linear in shape so it is called as linear potentiometer.
- Linear potentiometer is used to measure linear displacement.

### B. Angular potentiometer:-



(Fig. 2 - Angular potentiometer)

→ In this transducer the resistive element is present in a circular shape.

→ This angular potentiometer is used to measure angular displacement.

$$V_o = \frac{x_o}{x_i} \times V_i$$

Theory:-

→ The resistive of the potentiometer is a very clean wire made up of platinum & nickel alloy.

→ If, from fig 1, if  $V_i$  is the I/P supply voltage,  
 $V_o$  is the O/P voltage  
 $x_i$  is the total length of resistive element.

→  $x_o$  is the displacement made by the wiper then, the O/P voltage generated by the potentiometer can be given by

$$V_o = \frac{x_o}{x_i} \times V_i$$

→ For angular potentiometer the full turn of the wiper may be approximately  $300^\circ$ . If  $\theta_i$  is the total angular turn by the wiper, then in fig 2 the O/P voltage developed by the potentiometer can be given by,

$$V_o = \frac{\theta_o}{\theta_i} \times V_i$$

→ The O/P of the potentiometer varies linearly w.r.t the I/P displacement. The characteristics of potentiometer can be given by.

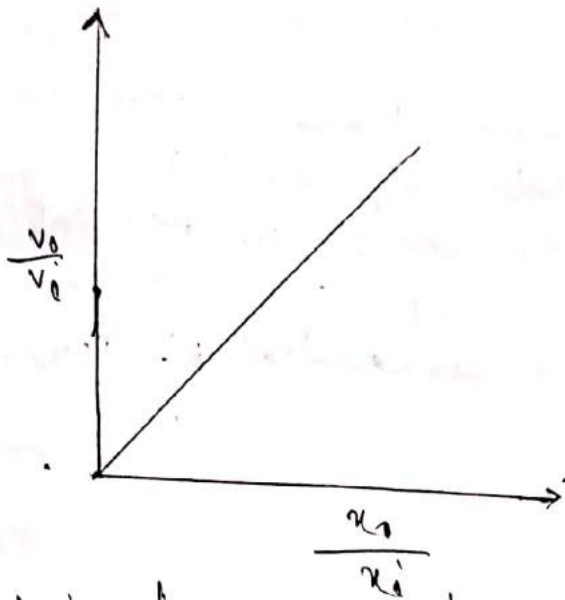


Fig:- characteristic of the potentiometer.

## 2. Thermistor:-

- Thermistor is a temperature sensor.
- It measures temp. & provides o/p in terms of resistance.
- It is a passive transducer.
- It is also called as thermal resistor.
- Two types of thermistor are generally available.

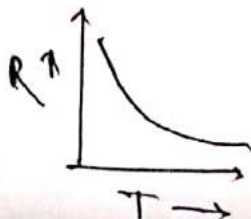
### 1. PTC (Positive coefficient temp.)

- When i/p temp. increases o/p resistance also increases.



### 2. NTC (Negative temp. coefficient)

- When i/p temp. increases o/p resistance decreases gradually.



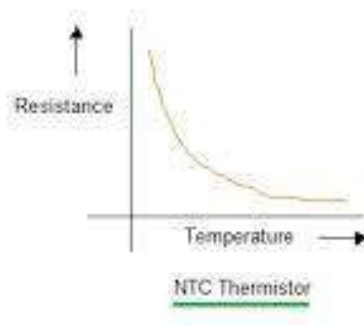
## Note:-

→ Generally all the thermistor which are used ~~are~~ of NTC type. This NTC type thermistor can detect very small change in temp. As it gives high resistance ~~at~~ at low temp.

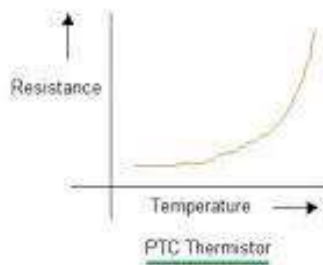


## 2. Thermistor:

- Thermistor is a resistive transducer whose resistivity depends upon surrounding temperature. For this reason it can be used as Temperature sensor.
- The term Thermistor is a combination of “thermal” and “resistor”
- It is made up of semiconductor material. Thermistor devices are generally made from oxides of certain metals like Manganese, Cobalt & Nickel etc.
- There are two types of thermistors: Negative Temperature Coefficient (NTC) and Positive Temperature Coefficient (PTC).  
With an NTC thermistor.
  - NTC Type:  
In this type when temperature increases, resistance decreases. Similarly, when temperature decreases, resistance increases. This type of thermistor is used the most.



- PTC Type:  
In this type when temperature increases, the resistance increases, and when temperature decreases, resistance decreases.



- **Working Principle:**
  - As the temperature of a thermistor increases its resistance decreases exponentially.
  - The mathematical expression for the relationship between resistance of thermistor and temperature is

$$R_{T1} = R_{T2} \exp\left[\beta \left(\frac{1}{T1} - \frac{1}{T2}\right)\right]$$

Where,

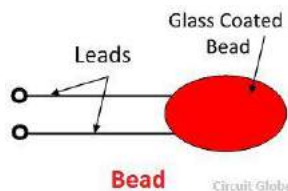
$R_{T1}$  = resistance of the thermistor at temperature T1

$R_{T2}$  = resistance of the thermistor at given temperature T2

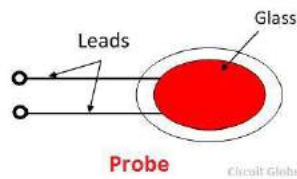
$\beta$  = constant, its value depends upon the material used in the construction of thermistor, typically its value ranges from 3500 to 4500.

**This above equation is known as characteristic equation of Thermistor**

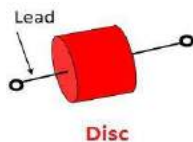
- Thermistor can be made in different shape and sizes. It is available in the form of the bead, probe, rod and disc etc. The different types of the thermistor are shown in the figure below.



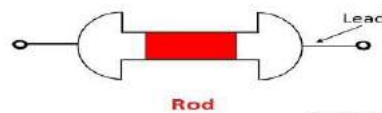
The bead form of the thermistor is smallest in shape.



Bead type is enclosed inside the solid glass rod to form probes.



The disc shape is made by pressing material under high pressure with diameter range from 2.5 mm to 25mm



It is shaped as a long vertical rod 0.250-2.0 inches (0.63-5.1 centimetres) long and 0.050-0.110 inch (0.13-0.28 centimetre) in diameter, of oxide-binder mix and sintered; ends are coated with conducting paste and leads are wrapped on the coated area.

### ➤ Advantages

- They are compact and inexpensive.
- They have good stability and high sensitivity.
- Their response is very fast.

- They are not affected by stray magnetic and electric fields. Due to all these advantages, thermistors are preferred over other temperature detecting devices like RTDs and thermocouples.

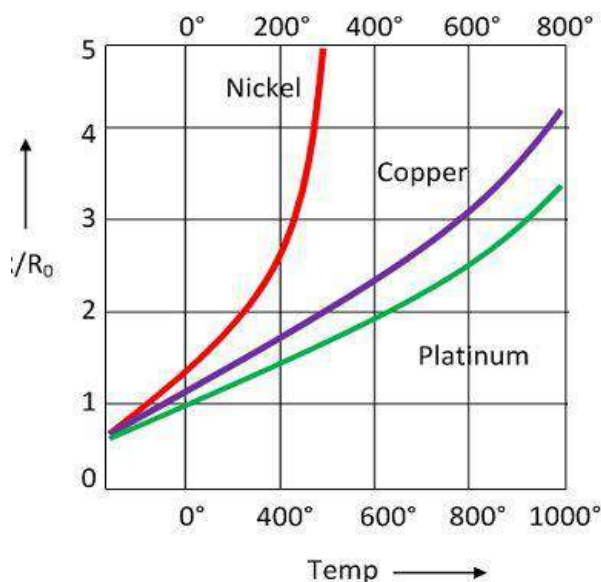
### 3. Resistance Thermometer:

- Resistance thermometers are based on the principle that the electrical resistance of a metal wire varies with temperature.
- The resistance thermometer is also known as Resistance Temperature Detector (RTD)
- It uses the resistance of electrical conductor for measuring the temperature.
- If  $R_0$  is the resistance at  $0^\circ\text{C}$ , then the resistance  $R_T$  at  $T^\circ\text{C}$  is:

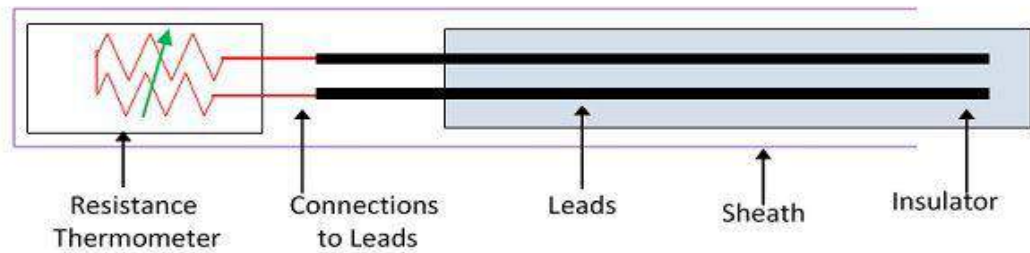
$$R_T = R_0 (1 + \alpha T)$$

Where,  $\alpha$  = temperature coefficient of resistance of a particular material.

- The resistance thermometer uses a sensitive element made of extremely pure metals like platinum, copper or nickel.
- RTD is a PTC type transducer.



- **Construction of Resistive Thermometer**



### Resistance Thermometer

- The resistance thermometer is placed inside the protective tube for providing the protection against damage.
- The resistive element is formed by placing the platinum wire on the ceramic bobbin.
- This resistance element is placed inside the tube which is made up of stainless steel or copper steel.
- The lead wire is used for connecting the resistance element with the external lead. The lead wire is covered by the insulated tube which protects it from short circuit.
- The ceramic material is used as an insulator for high-temperature material and for low-temperature fibre or glass is used.

#### ➤ Advantages:

- It provides highly accurate results.
- RTD provides a vast operating range.
- Due to its high accuracy
- RTD is used in all such applications where precise results are needed.

#### ➤ Disadvantages:

- The sensitivity of platinum RTD is very less for the minor variation in temperature.
- RTD possess slower response time.

## 4. Strain Gauge:

- A strain gauge is a device used to measure strain on an object.
- Resistance of the device varies with respect to applied force. It converts force, pressure, tension, weight, etc., into a change in electrical resistance which can then be measured.
- When an external force is applied on an object, due to which there is a deformation occurs in the shape of the object. This deformation in the shape is both compressive or tensile is called strain, and it is measured by the strain gauge.
- **Working Principle:**

Resistance of any conductor wire is directly dependent on the length and the cross-sectional area of the conductor, given by:

$$R = \rho L / A$$

Where,

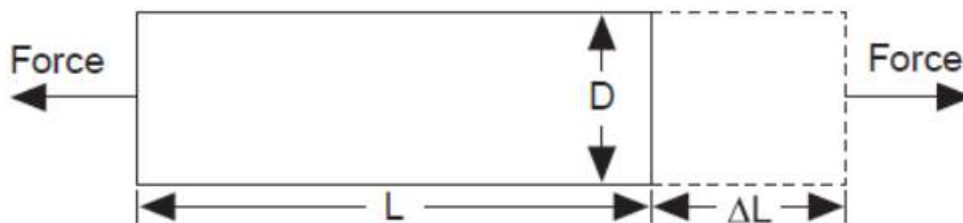
R = Resistance

L = Length

A = Cross-Sectional Area

$\rho$  = Resistivity of the material

The change in the shape and size of the conductor also alters its length and the cross-sectional area which eventually changes its resistance.



If  $\Delta L$  is the change in length of the wire by the application of force or stress then strain ( $\epsilon$ ) is given by:

$$\text{Strain } (\epsilon) = \frac{\Delta L}{L}$$

- Sensitivity of the strain gauge material is given by a parameter known as **Gauge factor** (G). The Gauge Factor is the sensitivity coefficient of strain gauges
- **Gauge factor** is defined as the ratio of fractional change in electrical resistance to the fractional change in length (strain):

$$G = \frac{\Delta R / R}{\Delta L / L} = \frac{\Delta R}{R} \cdot \frac{L}{\Delta L} = \frac{\Delta R}{R} \cdot \frac{1}{\epsilon}$$

Where,

R = original Resistance of wire

$\Delta R$  = change in Resistance

L = original Length of wire

$\Delta L$  = change in Length

$$\epsilon = \frac{\Delta L}{L}$$

➤ **Construction:**

The metallic strain gauge consists of a very fine wire or, more commonly, metallic foil arranged in a grid pattern.

The grid is bonded to a thin backing, called the carrier, which is attached directly to the test specimen (object). Therefore, the strain experienced by the test object is transferred directly to the strain gauge and changes the resistance of the strain gauge.

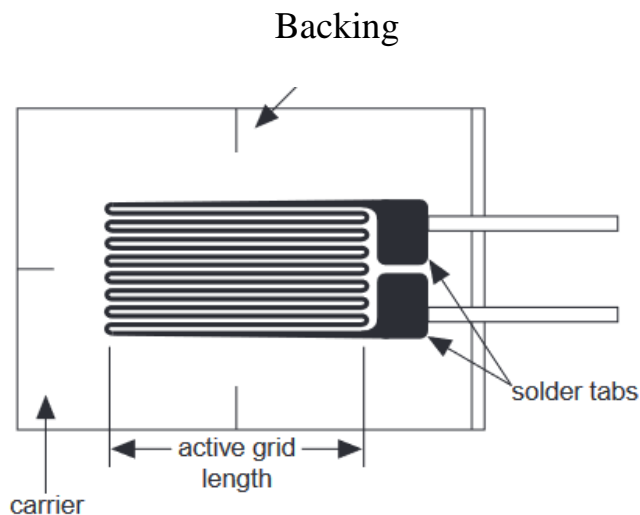
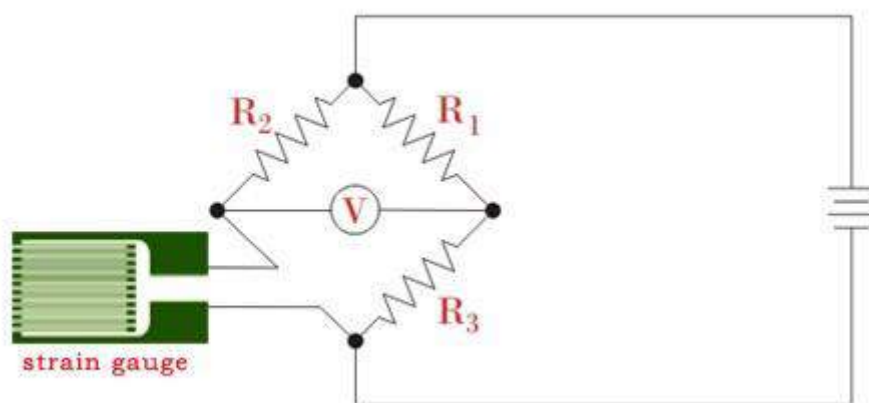


Figure: Wire type bonded strain gauge

➤ **Measurement by using Bridge circuit:**

The change in resistance in strain gauge can be measured in terms of change in voltage by connecting the strain gauge in a Wheatstone bridge circuit.



- In this circuit,  $R_1$  and  $R_3$  arms are equal to each other, and  $R_2$  is the rheostat arm and its value equal to the strain gauge initial resistance.

- When no force is applied, the gauge is unstrained and the bridge is balanced. Voltmeter shows zero value at this condition
- When force is applied on the strain gauge resistance of the gauge changes. As there is a change in resistance of strain gauge, the bridge gets unbalanced and produces a voltage indication at the voltmeter.

➤ **Application:**

- It can be used as Weight, Force, Pressure or Stress sensor.

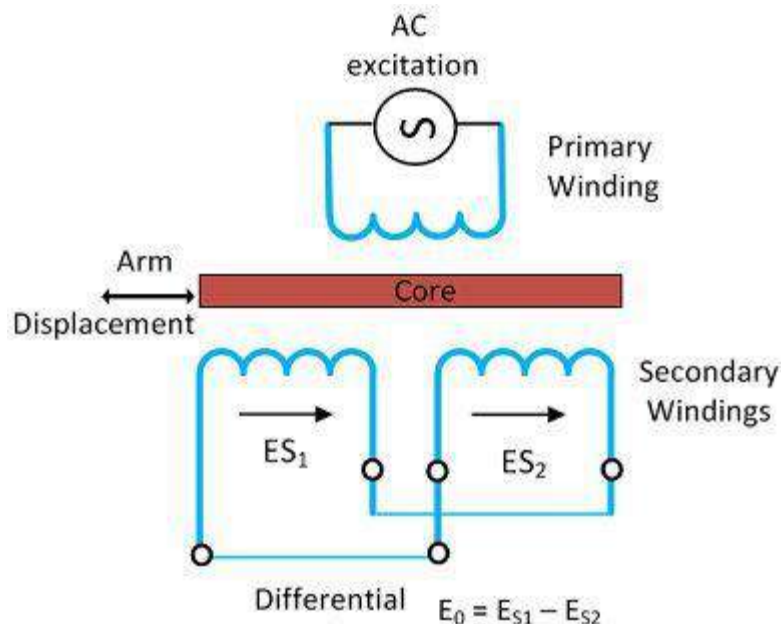
## I. Inductive Transducers

The transducer whose inductance changes with respect to change in input parameter is known as inductive transducer.

### 1. LVDT(Linear Variable Differential Transformer):

- The Linear Variable Inductive Transformer converts the linear displacement into an electrical signal.
- It works on the principle of mutual induction, i.e., the flux of the primary winding is induced to the secondary winding. The output of the transformer is obtained because of the difference of the secondary voltages, and hence it is called a differential transformer.

➤ **Construction of LVDT:**



- The basic construction of the LVDT is shown above in the figure. LVDT consist of a primary winding and two secondary windings  $S_1$  and  $S_2$ . The secondary winding is wound on the cylindrical former.
- The secondary windings have an equal number of turns, and it is placed identically on both the side of the primary winding.

- The output voltage of the secondary winding  $S_1$  is  $ES_1$  and that of the  $S_2$  is  $ES_2$ .
- The secondary voltage signal is converted into an electrical signal by connecting the secondary winding in series opposition as shown in the figure above.
- The output voltage of the transducer is determined by subtracting the voltage of the secondary windings.

$$\text{Output voltage (E}_0\text{)} = \text{ES}_1 - \text{ES}_2$$

➤ **Working:**

The change in output voltage is directly proportional to the displacement of the core. Any displacement will increase the flux of one of the secondary winding and on the other hand, reduces the other which develops a differential voltage at the output. There could be three possible conditions which are described below:

**Condition-I:**

- When the soft core moved towards left, the flux linked in  $S_1$  is more as compared to  $S_2$ .
- The output voltage of the winding  $S_1$  is more than the  $S_2$ .
- Since  $\text{ES}_1 > \text{ES}_2$ ,  $\text{E}_0$  is **positive**. So  $\text{E}_0$  is in phase with the primary voltage.

**Condition-II:**

- When the soft iron core move towards right the magnitude of the flux linked  $S_2$  is more than  $S_1$ .
- The output voltage of the winding  $S_1$  is less than the  $S_2$ .
- Since  $\text{ES}_1 < \text{ES}_2$ ,  $\text{E}_0$  is **negative**. The output voltage  $\text{E}_0$  is  $180^\circ$  out of phase with the primary winding.

**Condition-III:**

- When the soft iron core is at the centre of  $S_1$  and  $S_2$ , the flux linked in  $S_1$  and  $S_2$  are same.
- The output voltage of the winding  $S_1$  is equal to  $S_2$ .
- So  $\text{E}_0 = \text{ES}_1 - \text{ES}_2$   
 $= 0 \text{ V}$



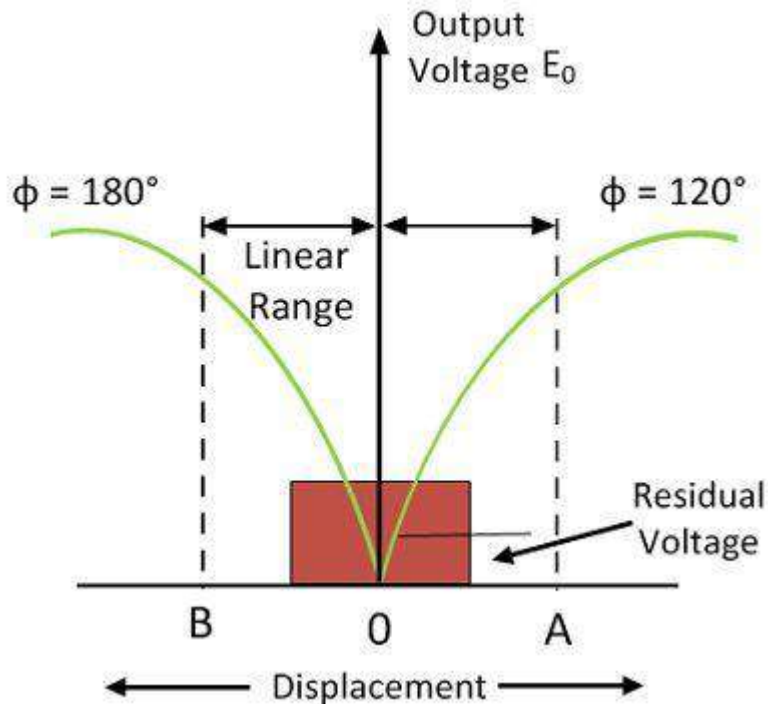


Figure: LVDT Characteristics

The curve between the output voltage and the input displacement is shown in the figure above.

The curve is linear for small displacement between A & B.

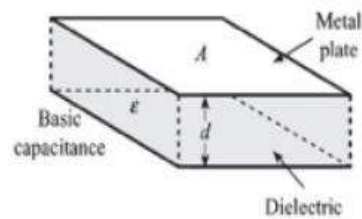
➤ **Uses of LVDT:**

- It is used for measuring the displacement having a range from few mm to cm. The LVDT directly converts the displacement into an electrical signal.
- The LVDT is used as a device for measuring the force, weight and pressure. Some of the LVDT used for measuring the load and pressure.

## II. Capacitive Transducers

The transducer whose capacitance changes with respect to change in input parameter is known as capacitance transducer.

- The working principle of a capacitive transducer is variable capacitance. It consists of two parallel metal plates which are separated by dielectric medium (such as air).



- The capacitance of the variable capacitor can be measured by this formula.

$$C = A \frac{\epsilon_0 \epsilon_r}{d}$$

$$C = A \frac{\epsilon}{d}$$

**C** = capacitance of the variable capacitor

$\epsilon_0$  = permittivity of free space

$\epsilon_r$  = relative permittivity

$\epsilon = \epsilon_0 \epsilon_r$

**A** = overlapping area between the two plates

**d** = distance between the two plates

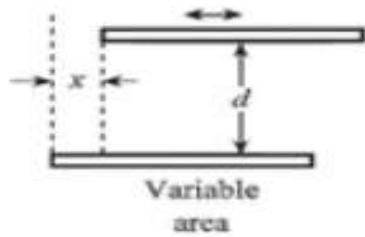
By varying the parameters like **A**, **d** &  $\epsilon_r$  of the variable capacitor the capacitance can be changed.

So the capacitive transducer is of three types:

1. Variable Area(**A**) Capacitive Transducer
2. Variable distance between two plates (**d**) type capacitive Transducers
3. Variable dielectric constant ( $\epsilon$ ) type capacitive Transducers

## 1. Variable Area Capacitive Transducer:

- In this type capacitive transducer the overlapping area (A) between the two plates changes due to the application of Displacement, Force or Pressure.
- Since parameter 'A' changes, the capacitance 'C' also changes, as 'C' is directly proportional to 'A'.



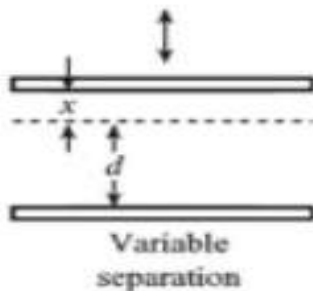
$$C = \frac{\epsilon (A - wx)}{d}$$

Where, 'x' is the displacement of the plate and 'w' is the width of the plate

- It can be used as Displacement, Force or Pressure sensors.

## 2. Variable distance between two plates type capacitive Transducers

In this type capacitive transducer the distance (d) / separation between the two plates changes due to the application of Displacement, Force or Pressure.



- Since parameter 'd' changes, the capacitance 'C' also changes, as 'C' is inversely proportional to 'd'.

$$C = A \frac{\epsilon}{d+x}$$

Where, 'x' is the displacement of the plate

- It can be used as Displacement, Force or Pressure sensors.

## Active Transducers

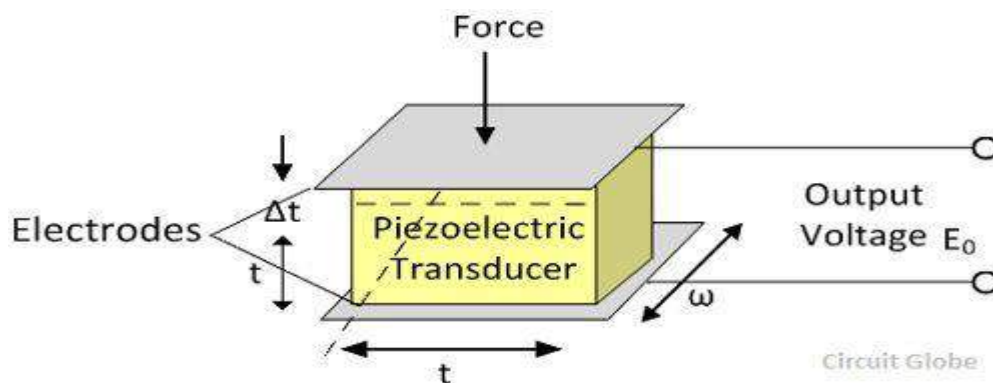
The transducers which do not require any external power supply for the generation of electrical output (V or I) is known as Active Transducer.

### I. Piezoelectric Transducers

- A piezoelectric transducer is a device which can convert mechanical energy like Force or Pressure into an electrical energy.
- It uses piezoelectric effect for the generation of electric charge.
- It is an active transducer.

#### Construction and Working:

- Piezoelectric materials like Quartz, Rochelle salt etc can be used to make the transducer.



- The faces of piezoelectric material, usual quartz, is coated with a thin layer of conducting material such as silver known as Electrode.
- When stress is applied, the ions in the material move towards one of the conducting surface while moving away from the other. This results in the generation of charge.
- This charge is used for calibration of stress. The polarity of the produced charge depends upon the direction of the applied stress/force.
- If  $F$  is the applied force and  $Q$  is the charge developed due to it then  $Q \propto F$ .

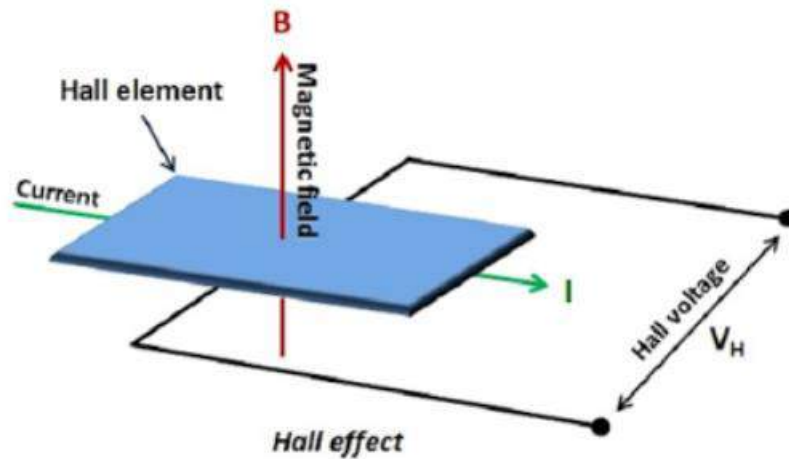
$$\Rightarrow Q = d \times F$$

Where,  $d$  is known as piezoelectric coefficient of the material.

- Due to the charge  $Q$ , potential difference  $V_0$  developed between the electrodes which can be taken as output.
- This transducer is used as Force, Pressure or Stress sensor.

### II. Hall effect Transducers

- A Hall Effect sensor is a transducer that varies its output voltage in response to a magnetic field.
- This transducer works on the principle of Hall Effect.
- **Hall Effect:** If a current carrying strip of the conductor is placed in a transverse magnetic field, then an EMF is developed on the edge of the conductor. The magnitude of the developed voltage depends on the density of flux. This property is known as Hall Effect.



The output voltage of Hall Effect sensor  $V_H = K \frac{B \times I}{t}$

Where, K= Hall Effect coefficient

B=Magnetic flux density

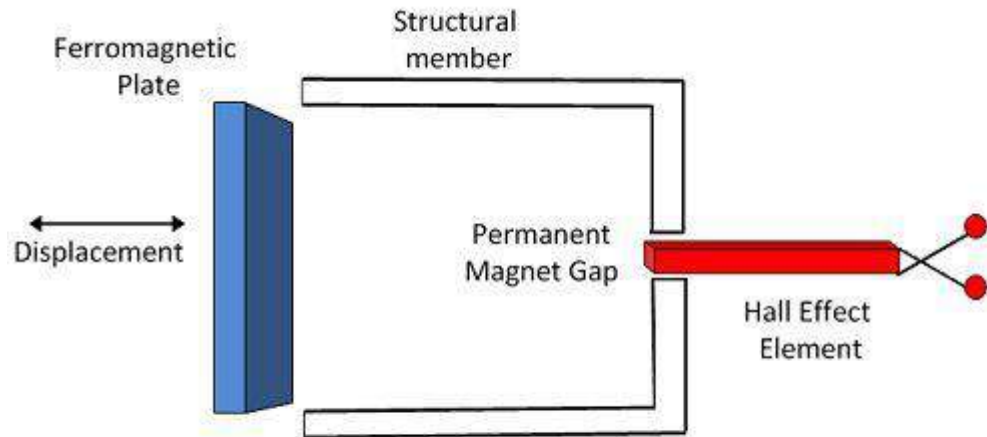
I= Circuit current

t = Thickness of the conductor strip (Hall Element)

The strip of the conductor is called as Hall element.

#### Applications of Hall Effect Transducer:

- Magnetic to Electric Transducer** – The Hall effect element is used for converting the magnetic flux into an electric signal.
- Measurement of Displacement** – The Hall Effect element measures the displacement of the structural element.



### Measurement of Displacement Using Hall Effect Transducer

Circuit Globe

Consider the ferromagnetic structure which has a permanent magnet. The hall effect transducer placed between the poles of the permanent magnet. The magnetic field strength across the Hall Effect element changes by changing the position of the ferromagnetic field. So output voltage of the transducer changes with respect to input displacement.

- c. **Measurement of Current** – The Hall Effect transducer is also used for measuring the current.

The AC or DC is applied across the conductor for developing the magnetic field. The strength of the magnetic field is directly proportional to the applied current. The magnetic field develops the EMF across the strips.

\*\*\*\*\*

## Chapter-8: OSCILLOSCOPE

### CATHODE RAY OSCILLOSCOPE (CRO)

- The cathode ray oscilloscope (CRO) is an electrical instrument which is used for display, measurement and analysis of waveforms and others and electrical phenomenon.
- A cathode ray oscilloscope is a very fast X-Y plotter that can display an input voltage signal versus time.

#### Working:

- The CRO has the cathode ray tube which acts as a heart of the oscilloscope.

- In an oscilloscope, the CRT produces the electron beam which is accelerated, decelerated and focus with the help of accelerating and focusing anode at a high velocity and brings to the focal point on a fluorescent screen.
- After the collision of the electron on the screen, it produces a visible spot where the electron beam strikes with it and this spot is seen on another side of the screen.
- This collision or bombarding of electrons continually done on the screen which shows the electrical signal, this electron beam like an electrical pencil of light which produces a light where it collides with the screen.

## Major Components of Cathode Ray Oscilloscope

The main blocks of CRO are

- Cathode Ray Tube (CRT)
- Vertical amplifier
- Delay Line
- Trigger circuit
- Time base generator
- Horizontal amplifier
- Blanking circuit
- Power supply

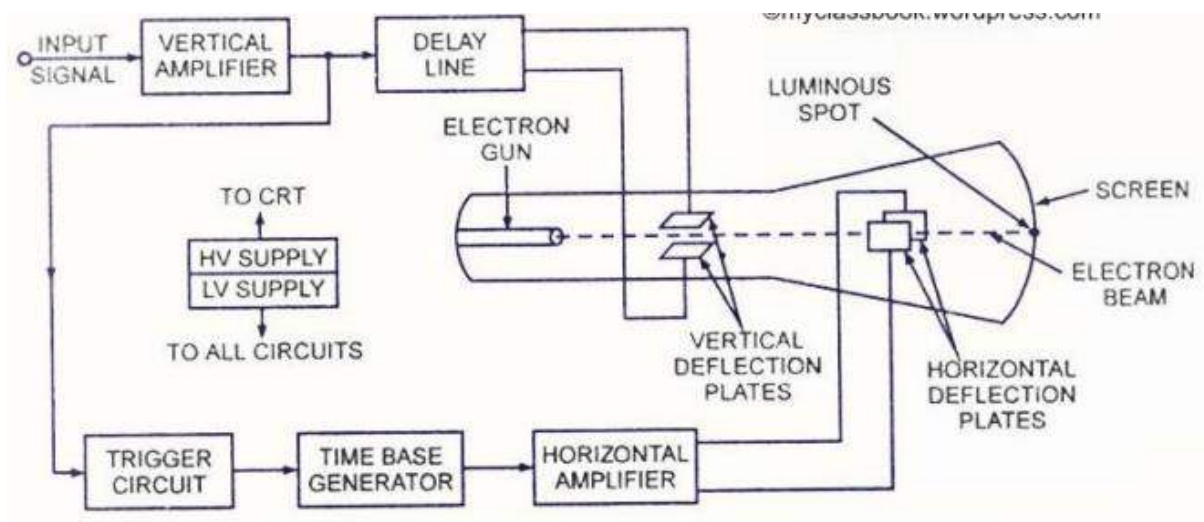
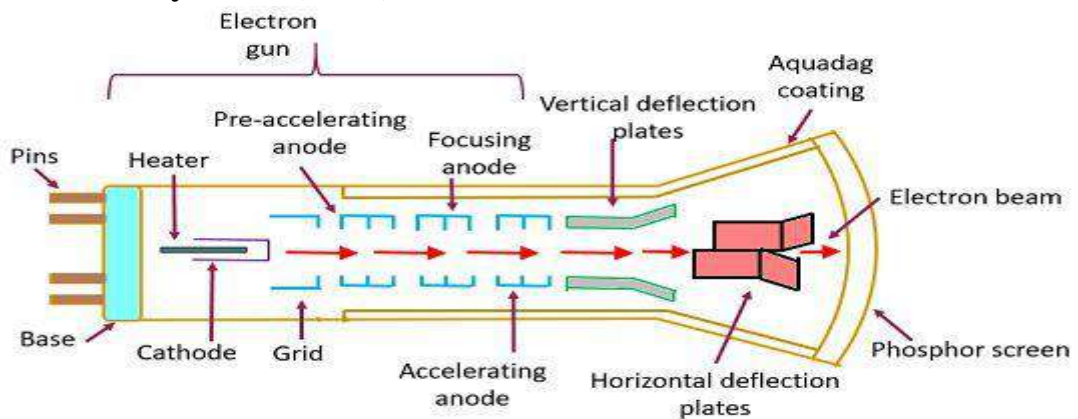


Figure: Block diagram of CRO

## 1. Cathode Ray Tube (CRT):



**Internal structure of CRT**

Electronics Coach

- CRT Produces a sharply focused beam of electrons and accelerate it to a very high velocity.
- CRT consist of the following parts:
  - Electron gun
  - Deflection plate assembly
  - Glass envelope
  - Fluorescent screen
  - Base, for connections
- This electron beam travels from the electron gun to the screen. The electron gun consists of a filament, cathode, control grid, accelerating anodes and focusing anode.
- While travelling to the screen, electron beams passes between a set of vertical deflecting plates and a set of horizontal deflection plates. Voltages applied to these plates can move the beam in vertical and horizontal plane respectively.
- The electron beam then strikes the fluorescent material (phosphor) deposited on the screen with sufficient energy to cause the screen to light up in a small spot.

## 2. Vertical Amplifier:

The input signal is applied to the vertical amplifier. The gain of this amplifier can be controlled by VOLT/DIV knob. Output of this amplifier is applied to the delay line.

## 3. Delay Line:

The delay Line delays the arrival of the input waveform at the vertical deflection plates until the trigger and time base circuits start the sweep of the beam.



#### 4. Trigger Circuit:

A sample of the input waveform is fed to a trigger circuit which produces a trigger pulse at some selected point on the input waveform. This trigger pulse is used to start the time base generator.

#### 5. Time base (Sweep) Generator:

- This produces a saw-tooth waveform that is used as horizontal deflection voltage of CRT.
- The rate of rise of a positive going part of the sawtooth waveform is controlled by TIME/DIV knob.
- The sawtooth voltage is fed to the horizontal amplifier if the switch is in the INTERNAL position. If the switch is in EXT. position, an external horizontal input can be applied to the horizontal amplifier.

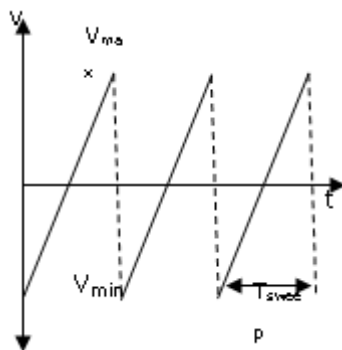


Figure: Sawtooth Waveform

- It is responsible for horizontal sweep of CRT spot from left hand side of the screen to right hand side.
- When a sawtooth voltage is applied to horizontal plates and an input signal is applied to vertical plates, display of vertical input signal is obtained on the screen as a function of time.

#### 6. Horizontal Amplifier:

This amplifies the saw-tooth voltage. As it includes a phase inverter two outputs are produced. Positive going sawtooth and negative going sawtooth are applied to right – hand and left – hand horizontal deflection plates of CRT.

#### 7. Blanking Circuit:

The blanking circuit is necessary to eliminate the retrace that would occur when the spot on CRT screen moves from right side to left side.

This retrace can cause confusion if it is not eliminate. The blanking voltage is produced by sweep generator. Hence a high negative voltage is applied to the control grid during retrace period.

## 8. Power Supply:

A high voltage (HV supply) section is used to operate CRT and a low voltage section (LV supply) is used to supply electronic circuit of the oscilloscope.

## Measurement of Voltage, Current, frequency, phase by CRO

### Measurement of Voltage:

- The oscilloscope is mainly a voltage measuring device.
- The number of divisions on the voltage axis (Y-axis) is measured and it is multiplied by the value indicated by the Volts/Div knob on the CRO.

**Voltage measured= Total no of Y-axis division × Volts/Div**

### AC Voltage:

- It is measured from peak-to-peak amplitude which measures the absolute difference between the maximum point of signal and its minimum point of the signal
- The sine wave is supplied to the Y input of CRO. By adjusting the Volt/div knob, obtain a sufficiently large display of signal on the CRO screen.
- The vertical length of the waves from the negative maximum to the positive maximum is read on the graphic scale of the screen.
- This reading (in div.) is multiplied by the volt/div knob reading to give peak to peak voltage  $V_{p-p}$ .
- The voltage  $V_{p-p}$  is divide by 2 to give peak ac voltage of the signal.

### DC Voltage:

- The DC power supply is connected to Y input of CRO taking care that positive lead of the cable is connected to +ve terminal and negative to the -ve of the dc power supply.
- The Volt/div knob is set and the dc power supply is switched ON. A sufficiently large display of signal (vertical line) on the CRO screen is obtained by setting Volt/div knob.
- The vertical length of the waves is read on the graphic scale of the screen.
- This reading (in div.) is multiplied by the volt/div knob reading to get the DC voltage.

### Measurement of Current :

- Electrical current cannot be measured directly by an oscilloscope. However, it could be measured indirectly within scope by attaching probes or resistors.

- Resistor measures the voltage across the points and then substituting the value of voltage measured and resistance in Ohm's law formula and calculates the value of electrical current.

$$\text{Current} = \frac{\text{Measured Voltage}}{\text{Resistance}}$$

### Measurement of frequency (Direct Method):

- The sine wave is given to the Y input of CRO whose frequency is to be measured.
- By adjusting the **time/div** knob, obtain a sufficiently large display of signal on the CRO screen.
- Measure the width of one full wave in no of divisions.
- Multiply this measured division with reading of time /div knob. This gives the time period of applied signal.

**Time Period = Total no of X-axis division × Time/Div**

- Reciprocal of time period will be the frequency of the applied signal.

$$\text{Frequency} = \frac{1}{\text{Time period}}$$

### Measurement of Phase & frequency by Lissajous figure method

- A Lissajous figure is displayed pattern on the screen when sinusoidal signals are applied to both horizontal & vertical deflection plates of CRO.
- This Lissajous figure pattern can be used for the measurement of Phase difference and frequency of applied signals.

Measurement of Phase:

- When two equal voltages of equal frequency but with a different phase shift ( $\phi$ ) are applied to a CRO we obtain different patterns of Lissajous figure in the below figure.

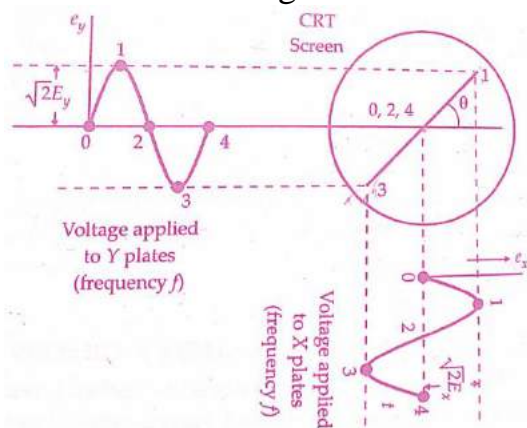


Figure: Lissajous figure for  $0^\circ$  phase shift

- When two sinusoidal voltages of equal frequency which are in phase with each other are applied to the horizontal and vertical deflection plates, the pattern appearing on the screen is a straight line.

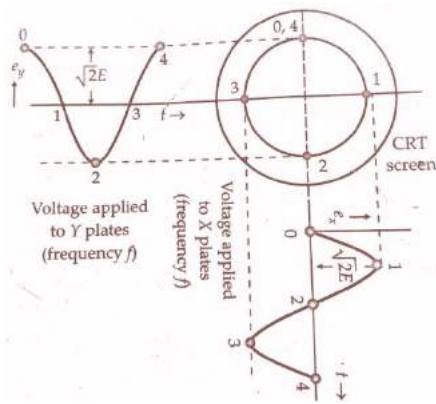
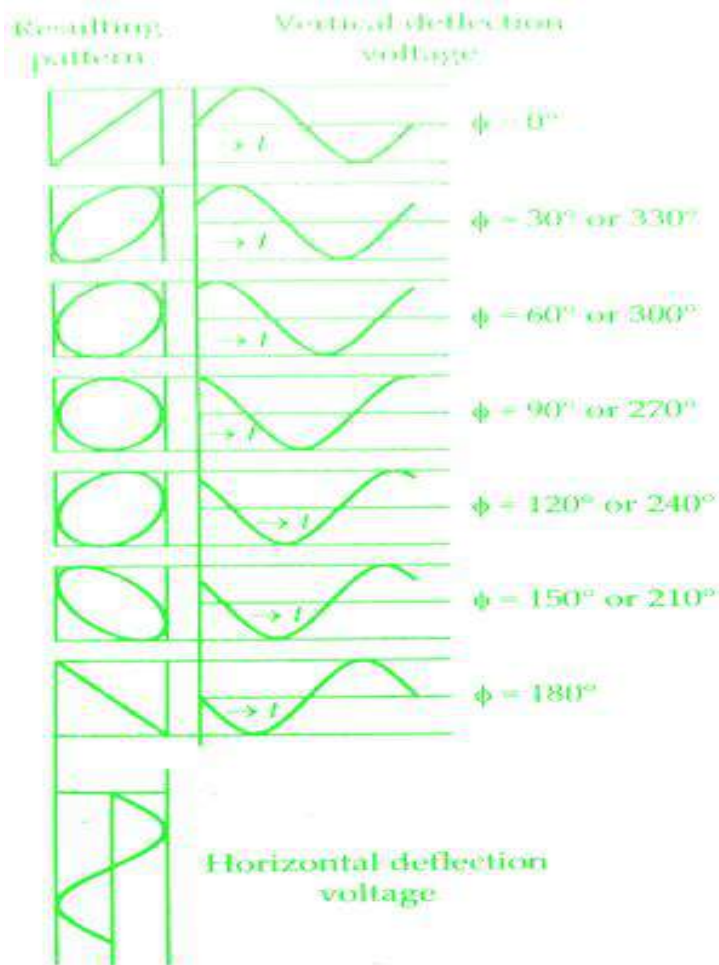
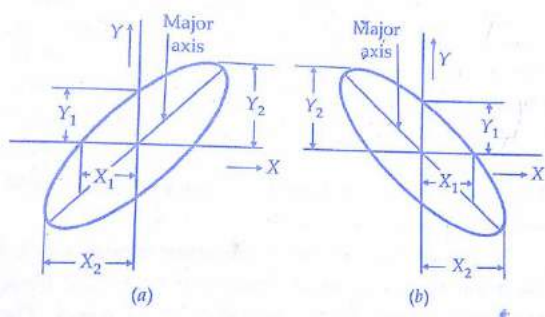


Figure: Lissajous figure for 90° phase shift

- Thus when two equal voltages of equal frequency but with 90° phase difference are applied to a CRO, the trace on the screen is a circle
- Similarly for different phase differences different type of pattern appears. Some of them are given below.



- The ellipse pattern of Lissajous figure provides a simple means of measuring phase difference between two voltages.



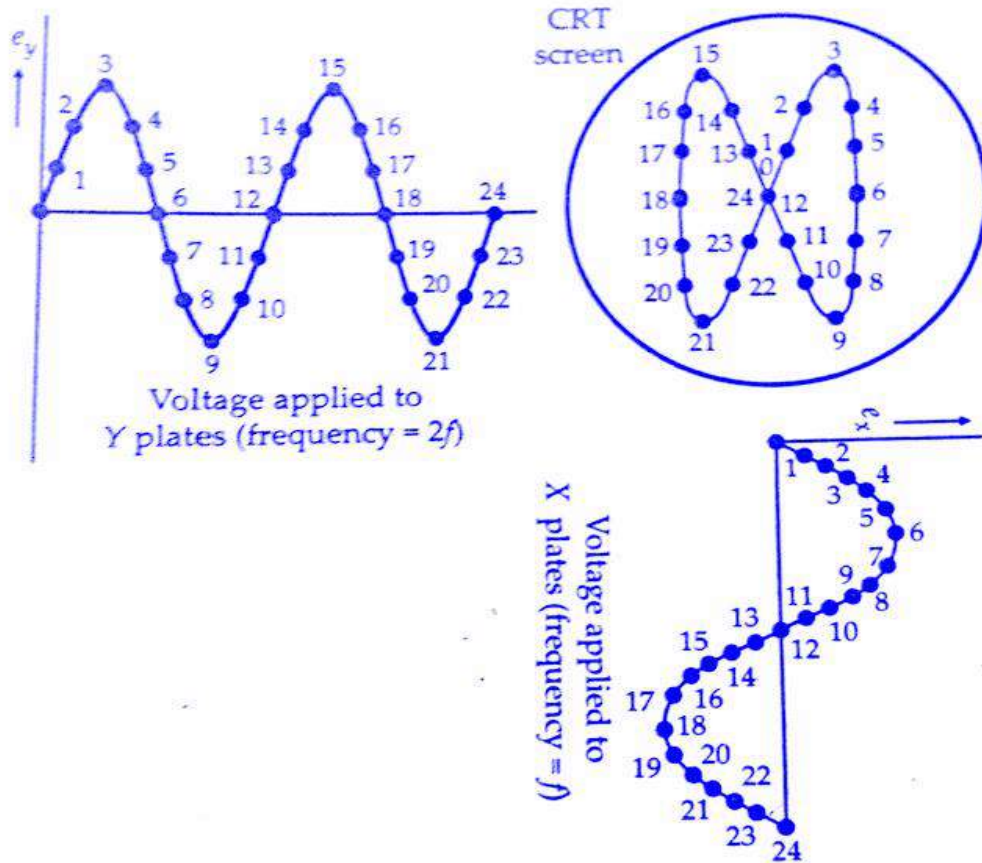
Referring to figure, the sine of the phase angle between the voltages is given by:

$$\sin\phi = \frac{Y_1}{Y_2} = \frac{X_1}{X_2}$$

$$\Rightarrow \phi = \sin^{-1}\left(\frac{Y_1}{Y_2}\right) = \sin^{-1}\frac{X_1}{X_2}$$

### Measurement of Frequency Lissajous Patterns

- Lissajous patterns may be used for accurate measurement of frequency.
- The signal, whose frequency is to be measured, is applied to the Y plates. An accurately calibrated standard variable frequency source is used to supply voltage to the X plates.
- Suppose sine waves are applied to X and Y plates as shown in the figure below. Let the frequency of wave applied to Y plates is twice that of the voltage applied to X plates. This means that the CRT spot travels two complete cycles in the vertical direction against one in the horizontal direction.



- In the above case Frequency of Y signal is 2 times (twice) of the X signal so two loop of pattern appear on the CRO screen.
- Similarly number of loop increases if Y signal frequency increases, which is indicated below.

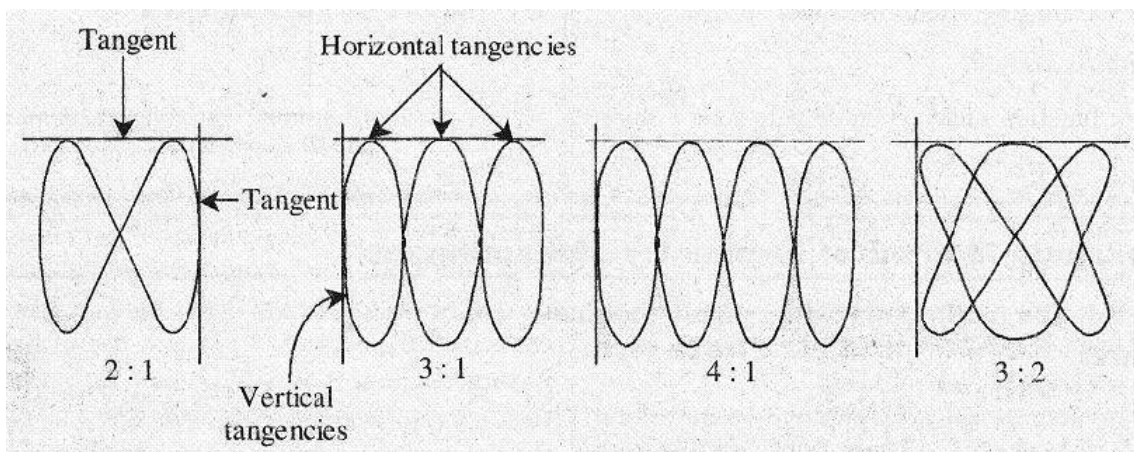


fig 6.2 Lissajous patterns allowing different frequencies ratios

The ratio of frequency can be calculated by drawing tangent at top/bottom and left/right sides.

The ratio of the two frequencies can be given by:

$$\frac{f_x}{f_y} = \frac{\text{Number of times tangent touches top or bottom}}{\text{Number of times tangent touches either left or right side}}$$

$$\Rightarrow \frac{f_x}{f_y} = \frac{\text{Number of horizontal tangencies}}{\text{Number of vertical tangencies}}$$

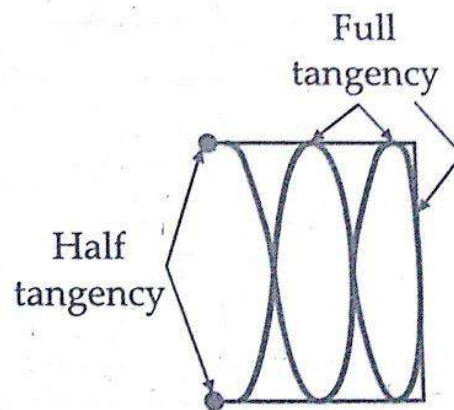
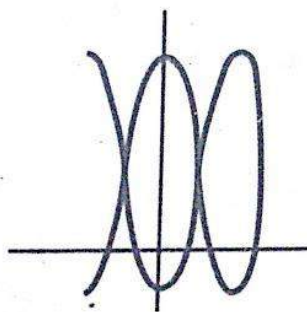
Where,  $f_x$  = frequency of signal applied to X

$f_y$  = frequency of signal applied to Y

- The ratio of frequencies when open-ended Lissajous patterns are obtained can also be found by treating the open ends as half tangencies as shown in the below

$$\therefore \frac{f_y}{f_x} = \frac{\text{number of horizontal tangencies}}{\text{number of vertical tangencies}}$$

$$= \frac{2 + 1/2}{1} = \frac{5}{2}$$



\*\*\*\*\*